"It falls somewhat short of logical precision" Bolzano on Kant's Definition of Analyticity

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Summary: Kant's famous definition of analyticity states that a judgement is analytic if its subject contains its predicate. Bolzano objects that (i) Kant's definiens permits an interpretation too wide, (ii) the definiens is too narrow, (iii) the definiendum is too limited, and (iv) the definition does not capture the proper essence of analyticity. Objections (i), (iii) and (iv), however, can be countered. Objection (ii) remains because, among other things, the Kantian definition has an eye only for an analysis of the *subject* within a judgement.

In a short manuscript titled "Zur Lebensbeschreibung", Bolzano relates that he began to study Kant's *Kritik der reinen Vernunft* when he was eighteen. Although he was immediately attracted by the distinction between analytic and synthetic judgements, as well as the one between judgements a priori and a posteriori, he could not accept Kant's explanations of them.[1] Bolzano's quadripartite objection to Kant's definition of analyticity can be found in the *Wissenschaftslehre* and in the *Neuer Anti-Kant*, a collection of Bolzano's critical remarks on Kant which was put together by his pupil Franz Příhonský.[2] Kant's definition in the Introduction to the *Kritik* states, roughly, that a judgement is analytic if its predicate is contained in its subject. Bolzano demurs that (i) Kant's definiens permits an interpretation too wide, (ii) the definiens is too narrow, (iii) the definiendum is too limited, and (iv) the definition does not capture the proper essence of analyticity.

In section 1, I shall introduce Kant's account of analytic judgements, while sections 2-5 deal with Bolzano's four objections in the given order. Even though my general philosophical sympathies lie more with Bolzano than with Kant, I must acknowledge that Bolzano's criticisms are often a bit hasty. Their significance primarily consists in the fact that, by expanding on them, one encounters serious problems every now and then. In other words, Bolzano sometimes focused on the right target even if his own arrows miss it.

The volume at hand is devoted to Wolfgang Künne, teacher, friend and mentor. Without him, my philosophical life would have taken place in a possible

[1] See *Bernard Bozano-Gesamtausgabe* 2 A 12/1, ed. by J. Berg, Frommann und Holzboog: Stuttgart-Bad Canstatt 1977, 67f. Wolfgang Künne (2006, 184) preludes his "Analyticity and Logical Truth" with a citation of this passage.

[2] I refer to the four-volume *Wissenschaftslehre* (Bolzano 1837) by 'WL' plus number of volume and paragraph, to the *Neuer Anti-Kant* (Příhonský 1850) by 'NAK', and to the *Kritik der reinen Vernunft* (Kant 1789) by 'KrV'. 'AA' abbreviates the *Akademie-Ausgabe* of Kant's writings (Kant 1902ff.).

world not philosophically accessible from the actual one. And even if it is accessible, I do not want to know what this world looks like. My contribution benefitted especially from two of Wolfgang Künne's papers: "Constituents of Concepts: Bolzano vs. Frege" (2001) and "Analyticity and Logical Truth: From Bolzano to Quine" (2006). Many of the following considerations rest on the insights found in these papers. Moreover, I extracted a crucial methodological maxim from the latter:

> Unlike 'true' and 'necessary', the word 'analytic' is a philosopher's term of art. Memories of doctrines associated with this term (be they Kantian, Fregean, Carnapian, or whatever) should not be mistaken for pre-theoretical 'intuitions' concerning analyticity. There simply are no such intuitions one could appeal to. (Künne 2006, 219)

In order not to get lost in fruitless discussions on what analyticity "really" or "truly" or "actually" is, I shall follow the maxim: Read Bolzano's objections as *inner-Kantian* objections! For example, Bolzano's second demur should not be understood as saying that Kant's definiens is too narrow because there are judgements which do not satisfy the definiens but are analytic in some *external*, pretheoretical or whatever, sense. The demur rather is that the definiens is too narrow from *Kant*'s perspective because some judgements do not satisfy it although *Kant* would take them to be analytic.

1 Kant's definition of analyticity

Kant's famous definition of analyticity in terms of conceptual containment reads as follows:[3]

> In all judgements in which the relation of a subject to the predicate is thought (if I consider only affirmative judgements, since the application to negative ones is easy) this relation is possible in two different ways. Either the predicate B belongs to the subject A as something that is (covertly) contained in it; or B lies entirely outside the concept A […]. In the first case I call the judgement analytic, in the second synthetic. […] the former do not add anything to the concept of the subject, but only break it up by means of analysis into its component concepts, which were already thought in it (though confusedly); while the latter, on the contrary, add to the concept of the subject a predicate that was not thought in it at all, and could not have been extracted from it through any analysis. (KrV, B 10f.)

Kant's best-known example is 'All bodies are extended', but he could as well have offered 'All drakes are male' or 'Foals are horses'. In all these cases, the predicate(-concept) seems to be contained in the subject(-concept).

But what does Kant mean by 'judgement', 'subject(-concept)' and 'predicate(-concept)'? Firstly, even though Kant quite often talks about analytic and synthetic "sentences", he cannot allude to *linguistic expressions* when using the terms 'subject' and 'predicate'. Elsewise, 'All bodies are extended' would not be analytic because the letter combination 'extended' is patently not contained in the letter combination 'bodies'. Secondly, Kant does not mean *subjective mental*

[3] Hintikka (1973, 125) and Morscher (2006, 251) point out that Kant's account is closely similar to Thomas Aquinas' characterisation of "self-evidence" in *Summa Theologica* I, 2, 1. Compare also Locke's "trifling propositions" in his *Essay* (1690, IV.VIII) and Leibniz's "frivolous sentences", including "identical" and "semi-identical sentences", in the *Nouveaux essais* (1705, IV.VIII).

representations, i.e., what immediately comes to the mind of a thinker when she imagines the given objects, or what she regularly associates with the given expressions. Otherwise, 'Every bird can fly' would be analytic for many people because their prototype of a bird includes the ability to fly.

I assume (cum Bolzano) that Kantian "judgements" and "sentences" strongly resemble Bolzanian "sentences in themselves", Fregean "thoughts" or, in today's terminology, "propositions". They are neither mental nor linguistic entities, but the contents of such things; and the same holds for what Kant refers to by "subject(-concept)" and "predicate(-concept)".[4] Thus, the sentence 'All drakes are male' expresses an analytic proposition because the subject-notion within this proposition, the concept of a drake, contains the predicate-notion, the concept of maleness. In other words, drakes are *defined* by being male (as well as by being ducks); maleness is part of the *definition* of drakes.[5] I shall freely vacillate between 'judgement', 'proposition' and 'statement' in the following; and I shall use single quotation marks to refer to words and sentences as well as concepts and propositions.

The initial sentence of the above-quoted passage points out that Kant's definition is restricted to (a) *affirmative* judgements of (b) *subject-predicate form*. In short, it is restricted to judgements in which a property is assigned to some object(s). Because of (a), Kant's characterisation applies to statements of the form 'All A are B' and, apparently, 'Some A are B' and 'The A is B', but not to 'No A is B', 'Some A are not B' and 'The A is not B'. On account of (b), the characterisation is hardly applicable to 'If all humans are mortal and Socrates is a human, then Socrates is mortal' or 'It is raining'. Including these constraints on the intended range of application, Kant's definition reads as follows:

(KA1) An affirmative subject-predicate proposition x is analytic $=_{\text{df.}}$ the predicate-concept of x is contained in its subject-concept.

As to restriction (b), van Cleve (1999, 19) adds that Kant's definition "does not apply to existential judgments (such as 'there are lions'), which (if we accept the dictum that existence is not a predicate) are not of subject-predicate form". However, there is reason to think that Kant complies with Bolzano's understanding of existential statements, which anticipates the Fregean view. According to this understanding, existential statements *are* of subject-predicate form, albeit the subject is a higher-order notion representing a concept and the predicate is not the notion of *existence* but the one of *instantiation*.[6] Thus, 'There are lions' translates into 'The concept of a lion is instantiated'. Since the notion of

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[4] Cf. Künne 2006, 212. Davis (2003, chs. 12-15) takes propositions and concepts in the given sense to be types of mental events, which is problematic for many reasons (cf. Siebel 2008, 420-422). More on Fregean "thoughts" and Bolzanian "sentences in themselves" in Künne 1997.

[5] Cf. KrV, B 746: "what I actually think in my concept of a triangle […] is nothing further than its mere definition".

[6] Cf. *Der einzig mögliche Beweisgrund zu einer Demonstration des Daseins Gottes*, AA 2, 72f.; KrV, B 626f.; WL II, § 137; and Frege 1884, § 53. Bennett (1974, § 72) refers to the above-mentioned account of existential statements as "the Kant-Frege view" and Wiggins (1994) as "the Kant-Frege-Russell view". Rosefeldt argues in "Kants Begriff der Existenz" (2008) and his contribution to the volume at hand that Kant is rather a Meinongian because he takes existence to be a property some objects fail to have.

instantiation is not contained in the notion 'concept of a lion', such a judgement is synthetic. This conforms to Kant's position that "every existential sentence is synthetic" (KrV, B 626; cf. Proops 2005, 592f.).

But are there not existential statements whose subjects include the predicates, namely, the ones expressed by sentences of the following ilk?

> The instantiated concept of an A is instantiated.
> There are existing As.

I assume that Kant would regard propositions of the first type as analytic. But it is far from obvious that they are existential statements in the "pure" sense Kant probably had in mind, i.e. statements of the form 'The concept of an A is instantiated' *without* any adjectival qualification in front of 'concept of an A' (cf. Morscher 2006, 252f.). Propositions of the second type, on the other hand, are existential propositions. Since they translate into 'The concept of an existing A is instantiated', however, they are synthetic, given that the concept of existence does not include the concept of instantiation (but rather the other way round).

Speaking of existence, it should be emphasised that Kant's account of analyticity would be jeopardised if his reading of universal affirmative statements agreed with the one of Bolzano and Aristotle. The latter assume that 'All A are B' is true only if there exists at least one A. But then 'All drakes are male' would fail to be analytic. For while analytic judgements are a priori in Kant's view (cf. KrV, B 9-12), 'All drakes are male' would entail the existence of drakes and thus could not be shown to be true without recourse to experience. However, unlike Bolzano and Aristotle, Kant does not take 'All A are B' to have existential import. In the section of the *Kritik* in which he presents his animadversion on the ontological proof of God's existence, we are told:

> Every sentence of geometry, e.g., 'a triangle has three angles', is absolutely necessary […]. The unconditioned necessity of judgements, however, is not an absolute necessity of things. For the absolute necessity of the judgement is only a conditioned necessity of the thing, or of the predicate in the judgement. The above sentence does not say that three angles are absolutely necessary, but rather that under the condition that a triangle exists (is given), three angles also exist (in it) necessarily. (KrV, B 621f.)

According to this passage, 'All drakes are male' would be true even if there were no drake, so that mere conceptual analysis suffices to recognise its truth. More generally, if statements of the form 'All A are B' do not imply the existence of an A, there is no longer any obstacle to describing them as analytic in case their subject contains the predicate.[7]

In *Kant and the Foundations of Analytic Philosophy*, Hanna (2001, sect. 3.1.1) ignores Kant's confinement to affirmative judgements and, in return, augments his definition with the stipulation that analytic judgements are *necessary*. Kant in

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[7] Cf. also B 314 where Kant says that, in recognising an analytic truth, the understanding "leaves it undecided whether the concept even has any relation to objects". – Bolzano objects to Kant's view on the origin of analytic cognition that even propositions of the form 'A which is B is B' are known to be true not until one can be sure that the concept of an A which is B is instantiated. "The question is therefore […] how we come to the […] synthetic judgement 'Some A are B'" (WL III, § 305, 178). Evidently, Bolzano cannot imagine that Kant might plead for an account of affirmative universal judgements according to which they have no existential supposition.

fact assumes that analytic judgements are a priori and hence necessarily true (cf. KrV, B 3f., 9-12). But I see no reason to assert that this is part of his *definition* of analytic judgements. It is simply a corollary. Similarly, I follow de Jong (1995, 619) and Proops (2005, 603f.) in their interpretation of Kant's reference to the principle of contradiction in a later passage of the *Kritik*: "if the judgment is analytic, whether it be negative or affirmative, its truth must always be able to be cognised sufficiently in accordance with the principle of contradiction" (B 190). This remark is not meant to offer a *defining* characteristic of analytic statements. Its point is rather an *epistemological* one: in the case of analytic statements the principle of contradiction provides an effectual basis for *proving* that they are true.

I have not incorporated into (KA1) the bracketed addition in Kant's formulation "the predicate B belongs to the subject A as something that is (*covertly*) contained in it" (KrV, B 10; my emph.). Furthermore, I have omitted Kant's suggestion to call analytic judgements *judgements of clarification* and synthetic ones *judgements of amplification* (B 11). Whereas on (KA1) 'All male ducks are male' expresses an analytic proposition just as much as 'All drakes are male', this is not so obvious against the background of Kant's reference to covertness and clarification. After all, the wording 'All male ducks are male' does not conceal that the predicate-concept is part of the subject-concept; and the triviality of 'All male ducks are male' casts its clarificatory power into doubt.

For several reasons, however, one should take the reference to covertness and clarification with a pinch of salt. Regarding covertness, firstly, one should bear in mind that Kant bracketed the word 'covertly' (as well as 'though confusedly' in the same paragraph), implying thereby that these attributes are possessed only by a subclass of analytic judgements. Secondly, if these features were considered necessary for analyticity, one would run the risk of ending up with a subjectively relativised version of the analytic-synthetic distinction. Consider a person who immediately and clearly recognises the concept of extension within the concept of a body. For her, the predicate of 'All bodies are extended' is not covertly but openly contained in the subject. Does this mean that this proposition is not analytic for such a person, although it is analytic for the man on the street?

Concerningclarification, the first reason for taking it with a grain of salt is that Kant uses the fairly tentative formulation that one *could* call judgements of the analytic ilk judgements of clarification. Secondly, he continues in the same sentence by explaining this proposal in terms of nothing other than the containment idea. Thirdly, later on in the *Kritik* Kant offers as an example of a negative analytic judgement 'No unlearned person is learned' (B 192). This suggests that, despite the triviality of the equivalent 'Every learned person is learned', as well as the triviality of 'All male ducks are male', he would consider the voiced judgements analytic. Finally, if one still wants to insist on the general clarificatory power of analytic statements, note that even 'All male ducks are male' provides an elucidation insofar as it makes clear that the function of the concept 'male' in 'male duck' differs from the one of 'putative' in 'putative duck'. In the former but not in the latter compound, the constituent administered by the adjective serves as a conjunctive part: a male duck is a duck which is male, but it makes no sense to say that a putative duck is a duck which is putative.

But what is to be done with cases where subject- and predicate-expression are *identical*? At first glance, Kant's examples of analyticities encompass 'Man is man' ('Der Mensch ist ein Mensch'), which is mentioned in the *Jäsche-Logik* (AA 9, 111), and 'a = a' ('The whole is equal to itself'), on which he touches in the *Kritik* (B 16f.). In order to integrate these judgements, Kant's containment definition must be read in such a way that the predicate-concept can also be an *improper* part of the subject-concept, that is, be identical with it.[8] However, as will be discussed further in section 3, it is not clear that Kant is, and should be, serious about the analyticity of 'a = a'. For if the concept of equality is part of the predicate, the predicate exceeds the subject. As to 'Man is man', some adepts have reservations about the authenticity of the *Jäsche-Logik*.[9] The analyticity of this statement is thus open to question. But note also that denying its analyticity has a serious consequence. After all, if Kant does not want to call such propositions synthetic either – and why should he? –, he has to narrow down the scope of his analytic-synthetic classification further. Then it does not apply to all affirmative subject-predicate propositions because it does not apply to 'Man is man' and the like.

Whatever Kant's position on such statements in the *Kritik der reinen Vernunft* might be, he is quite explicit in the *Preisschrift über die Fortschritte der Metaphysik*, which he began around 1793:

> Judgements are analytic, we may say, if their predicate merely presents clearly (*explicite*) what was thought, albeit obscurely (*implicite*), in the concept of the subject; e.g., any body is extended. If we wanted to call such judgements identical, we should merely cause confusion; for judgements of that sort contribute nothing to the clarity of the concept which all judging must yet aim at, and are therefore called empty; e.g., any body is a bodily (or in other words material) entity. Analytic judgements are indeed *founded* upon identity, and can be resolved into it, but they *are* not identical, for they need to be dissected and thereby serve to elucidate the concept; whereas by identical judgements, on the other hand, *idem per idem*, nothing whatever would be elucidated. (AA 20, 322)

Proops (2005, 602) takes passages like this one to show that Kant revised his conception of analyticity by coming to draw the analytic-synthetic division only within the class of *knowledge-advancing* judgements. In this spirit, we read in Gotthilf Busolt's notes of Kant's lectures on logic, which are dated to 1788-90 (AA 24, 667): "Propositions which explain *idem per idem* expand knowledge neither analytically nor synthetically. They are tautological propositions. By them I have neither an increase in distinctness nor a growth in cognition." Perhaps, the Kant who seems to speak here would even dispute that 'All male ducks are male' is analytic, thereby replacing definition (KA1) with something to the effect that a *knowledge-advancing* affirmative subject-predicate judgement is analytic if and only if its predicate is *covertly* contained in its subject.

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[8] In Bolzano's view (WL I, § 60), subject- and predicate-concept in 'Man is man' are *not* identical. The latter is rather a proper part of the former because the former reads 'something which is man' ('etwas, welches ein Mensch ist').

[9] See Hinske 2000, 90f.; Boswell 1988 and 1991. Stuhlrnann-Laeisz (1976) does nowhere allow for the *Jäsche-Logik* in his standard work *Kants Logik*.

I shall concentrate, however, on (KA1) as the characterisation which is most strongly suggested by the Introduction to the *Kritik*. It does not bear the risk of a subjectively relativised variant of the analytic-synthetic dichotomy because it does not mention covertness, clarification or something similar; and it is the characterisation towards which Bolzano's objections are directed. Incidentally, these objections are also applicable to a Kantian definition of analyticity which is restricted to knowledge-advancing judgements.

(KA1) tells us only under which conditions an *affirmative* subject-predicate judgement is analytic since, as Kant avers, "the application to negative ones is easy" (KrV, B 10). In that case, how about Kant's own example 'No unlearned person is learned' (B 192)? Why is it analytic? Kant's answer may be found in the section "Analytik der Grundsätze":

> In the analytic judgement I remain with the given concept in order to discern something about it. If it is an affirmative judgement, I only ascribe to this concept that which is already thought in it; if it is a negative judgement, I only exclude the opposite of the concept from it. (KrV, B 193; cf. *Metaphysik Mrongovius*, AA 29, 789)

In slightly improved words, 'No unlearned person is learned' excludes the property of being learned from unlearned persons. Since the predicate-concept 'learned' is the opposite of a part of the subject-concept 'unlearned person', the judgement is analytic.

But what exactly does Kant mean by "the opposite"? Proops (2005, 598) uses the formulation that the predicate *contradicts* a part of the subject. Contradiction, however, admits of too broad an interpretation because one can also acknowledge it in case of the following synthetic proposition:

(1a) No triangle has an angular sum different from two right angles.

In a later section of the *Kritik* (B 744f.), Kant outlines the corresponding Euclidean proof (*Elements*, prop. 32) in order to show that

(1b) Every triangle has the same angular sum as two right angles.

is a synthetic truth a priori (cf. also WL III, § 305, 185f.). According to the official definition, a triangle is a three-sided polygon. Since this definition does not mention the sum of the angles, the concept of having the same angular sum as two right angles is not contained in the concept of a triangle. Nevertheless, the sum of the angles in a triangle *necessarily* coincides with two right angles. Hence, the predicate-notion of (1a) – 'has an angular sum different from two right angles' – contradicts the subject-notion – 'triangles' – insofar as nothing can fall under both notions. But then 'No triangle has an angular sum different from two right angles' would be analytic according to the proposal in question, which does not coincide with Kant's perception that its equivalent 'Every triangle has the same angular sum as two right angles' is synthetic.[10]

To rule out that (1a) is analytic, 'contradiction' must be understood in a more direct or, in other words, a more explicit or purely logical way. Proops

[10] Arguments of the same ilk can be carried out with 'No event is without a cause', 'No straight line between two points is the longest' (cf. KrV, B 13, 16) and 'No object which is red all-over is green all-over'.

(2005, 591) heads in the right direction when explaining that a negative judgement is analytic if the predicate is the *negation* of one of the subject's constituents. To be sure, 'negation' must be understood in a broad sense here. It is natural to treat 'unlearned' as the negation of 'learned', such that 'No *learned* person is *unlearned*' is analytic because the predicate is the negation of a part of the subject. However, Kant's example was 'No *unlearned* person is *learned*', entailing that we must also regard 'learned' as being the negation of 'unlearned'. Let us define that a notion *negates* another notion if and only if one of them is composed of the other and the notion expressed by 'not', the prefix 'un-' or the like. Then 'No unlearned person is learned' is analytic because 'learned' negates 'unlearned' (even though the former might not be the negation of the latter in a stricter sense). And 'No event is without a cause' is synthetic because 'without a cause' does not negate 'event' (even though the former contradicts the latter). Thus Kant's definition of the analyticity of subject-predicate propositions in general, i.e. affirmative and negative ones, reads (cf. Proops 2005, 598):

(KA2) A subject-predicate proposition x is analytic $=_{df.}$
either x is affirmative and its predicate-concept is contained in its subject-concept, or x is negative and its predicate-concept negates a constituent of its subject-concept.

This could be called the *containment-or-negation* conception of analyticity. If Kant allows for analyticities such as 'No man is not a man', 'constituent' must be read as meaning a proper *or improper* part.

There is a traditional objection to Kant's definition which Bolzano rebuts. It was raised by Johann Gebhard Maaß (1789) in the journal *Philosophisches Magazin*, and it says that Kant's conception makes the analytic-synthetic distinction subjective because one and the same judgement could be both analytic and synthetic, depending on which ideas a person happens to associate with the subject-expression. If Susan means 'figure with three angles and three sides' by 'triangle' whereas Tom means 'figure in which the angle sum is two right angles', then the judgement that the sum of the angles in a triangle is two right angles appears to be synthetic for Susan but analytic for Tom. Bolzano correctly observes that this objection rests on a confusion between words and their sense or content: "I have a different opinion on the matter. Since what I mean by a proposition is not a mere combination of *words* that asserts something, but the *meaning* of this assertion, I do not admit that the proposition, the sum of the angles in a triangle, etc., remains the same when we attach to the word, triangle, now this concept and now another" (WL II, § 148, 89). Even if Susan and Tom utter the same sentence, they articulate different judgements, one of them analytic, the other synthetic.

Before I move on to Bolzano's four objections, it should be mentioned that he also praises Kant's division between the analytic and the synthetic. Firstly, he acclaims that it "is one of the most felicitous and influential discoveries made in the field of philosophical research" (NAK, 34). "[E]ven if it is true that this distinction was mentioned before at times, nevertheless it was never properly pinned down and fruitfully applied. The merit of having been the first to have done that indisputably belongs to Kant" (WL II, § 148, 87). Secondly, although

Bolzano wants to replace Kant's conception of analyticity with his own, in some places he explicitly makes use of the Kantian conception.[11] Among other things, he argues that, by allowing for synthetic judgements a priori, Kant embraces the fact "that there are properties possessed by an object, and necessarily possessed by it according to the concept we form of it, without being thought in this concept as constituents" (WL I, § 65, 288;).[12] This holds for Kant's example:

(1b) Every triangle has the same angular sum as two right angles.

Although triangles necessarily possess the property assigned to them by this judgement, it is not part of the definition of a triangle, entailing that the concept of having the same angular sum as two right angles is not contained in the concept of a triangle. Bolzano himself instances propositions quite close to (1b) (cf. WL I, § 64, 271, 274):

In every square the side is related to its diagonal as $1 : \sqrt{2}$.
Equilateral triangles are equiangular.

In addition, as Morscher (2006) strongly emphasised, we must not overlook that Bolzano is far from being hostile to the synthetic a priori even when it comes to his *own* understanding of these terms. In concert with Kant, he offers (1b) as an exemplar (cf. WL IV, § 447, 116), and he could as well have mentioned the last two examples.[13]

The praise for Kant notwithstanding, Bolzano is dissatisfied with Kant's explication of analyticity because it "fall[s] somewhat short of logical precision" (WL II, § 148, 87). The following four sections will expand on Bolzano's discontent.

2 The definiens permits an interpretation too wide

In "Two Dogmas of Empiricism" Quine (1951, 21) criticised Kant's definition by stating that "it appeals to a notion of containment which is left at a metaphorical level". He does not hold forth about this issue, but there could be something at the back of his mind which is close to Bolzano's much more precise plea:

> If it is said […] that in analytic judgements the predicate is contained in the subject (in a concealed manner), or does not lie outside of it or already occurs as a component of it; […] these are in part merely figurative forms of expression that do not analyse the concept to be explained, in part expressions that admit of too wide an interpretation. For everything that has been said here can also be said of

[11] Cf. Künne 2006, 235; and de Jong 2001, 334.

[12] Cf. WL I, § 120; *Über eine Entdeckung, nach der alle neue Kritik der reinen Vernunft durch eine ältere entbehrlich gemacht werden soll*, AA 8, 229f., 241f.; and de Jong 1995, 632-638.

[13] Cf. Frege (1884, § 89): "I consider Kant did great service in drawing the distinction between synthetic and analytic judgements. In calling the truths of geometry synthetic and a priori, he revealed their true nature. If Kant was wrong about arithmetic, that does not seriously detract, in my opinion, from the value of his work. […] His point was that there are such things as synthetic judgements a priori; whether they are to be found in geometry only, or in arithmetic as well, is of less importance." I assume that Kant, Bolzano and Frege agree in taking some trivial arithmetical and geometrical truths to be analytic, e.g., 'Prime numbers are numbers' and 'Triangles are polygons'.

> propositions no one would take for analytic, e.g., The father of Alexander, King of Macedon, was King of Macedon; A triangle similar to an isosceles triangle is itself isosceles […].” (WL II, § 148, 87f.; cf. NAK, 34f.)

Bolzano does not bluntly criticise Kant's characterisation for being wrong, but for being too unspecific because Kant's talk of containment could be understood in such a way that synthetic propositions had to be counted among the analytic ones. Here is Bolzano's first paradigm, supplemented by two further examples:

(2) The father of Alexander, King of Macedon, was King of Macedon.
(3) Every son of a bachelor is a bachelor.
(4) No putative bachelor is married.

The linguistic meaning of 'King of Macedon' is enclosed in the meaning of 'father of Alexander, King of Macedon'. Similarly, it is not possible to know what a son of a bachelor is without knowing what a bachelor is. There is thus a sense of 'contained' according to which (2) and (3) satisfy the Kantian definitions because their predicate-concepts are contained in their subject-concepts. Furthermore, since the subject-notion of (4), i.e. 'putative bachelor', includes the notion 'bachelor' and therefore 'unmarried' on this interpretation of containment, the predicate-notion 'married' negates one of the subject-notion's constituents. It thus appears that (4) is analytic according to the Kantian containment-or-negation conception. But no one would classify these judgements as analytic.[14] Kant in particular would deny that they are analytic because he considers analyticities to be a priori and hence necessary, while (2) is a contingent truth and (3) and (4) are contingent falsities (cf. KrV, B 3f., 9-12).

In § 65 of the *Wissenschaftslehre*, Bolzano emphasises that logicians might not associate the same meaning with the phrase that there are complex notions. He explains his own conception by saying that "everything which must necessarily be *thought* in order to really think a given notion is a constituent of it" (WL I, 282f.; my emph.). When talking about analyticity, Kant frequently makes use of similar formulations, e.g., "in the case of an analytic proposition the question is only whether I actually *think* the predicate in the presentation of the subject" (KrV, B 205; my emph.). In the light of the traditional distinctions between clear and opaque and between distinct and obscure ideas, such characterisations in terms of what people have in mind when they grasp a notion are delicate. Bolzano explicitly admits in § 56 of his *Wissenschaftslehre* that acts of thinking and their conceptual contents could differ with respect to their constituents. A subjective act of thinking might lack parts of the corresponding objective concept, and it might contain parts which are not contained in the latter (WL I, 246; cf. § 64, 273).

No matter how Bolzano's sense of 'containment' or 'constituent' is to be spelled out, he is surely justified in blaming Kant for using unclear formulations. But he overlooks that the theory behind these formulations resists his examples. When noting in § 65 of the *Wissenschaftslehre* that logicians conceive of complex notions in disparate ways, Bolzano refers to the notion 'man who has no integrity'. Many logicians, so Bolzano observes, "say that the concept of integrity is not connected with the concept man in this notion, but is rather sepa-

[14] Cf. de Jong 2001, 334f.; Künne 2001, 272f.; 2006, 213; and Lapointe 2007, 229f.

rated from it, and therefore must not be regarded as a constituent of the whole notion" (WL I, 282). Had he wondered what kind of account this denial is based on, he might have hit upon the so-called "traditional theory of concepts". De Jong (1995) and Lanier Anderson (2004, 2005) called attention to this theory which is in the background of Kant's definition of analyticity. Within this theory we do not only find the relational expression 'is contained *in*' but also 'is contained *under*' (see Friedman 1992, 67). A concept which is contained *in* a concept x is taken to be part of x's *intension* or *content*; and if something is contained *under* a concept x, then it is part of x's *extension* or *sphere*.

From the perspective of modern logic, one can hardly resist the temptation to read 'is contained under' as meaning *falls under*. It would thus denote what Frege called *subsumption*, i.e., a relation usually holding between objects and concepts.[15] In this sense, Donald Duck and his uncle would be contained under the concept of a drake because they fall under, or are subsumed under, this concept. Kant himself makes use of this sense at some places (see, e.g., *Wiener Logik*, AA 24, 910f., 925). When Bolzano criticises "the canon that content and extension stand in an inverse relationship" in § 120 of his *Wissenschaftslehre*, he assumes this reading; and Wolfgang Künne (2001, 268f.) follows him.

However, a closer look reveals that containment-under in Kant's primary use is not a relation between *objects* and concepts but solely between *concepts*. Here are two representative passages:[16]

> Now one must […] think of every concept as a representation that is contained *in* an infinite set of different possible representations (as their common mark), which thus contains these *under* itself […]. (KrV, B 39f.; my emph.)

> Let us consider a series of several concepts subordinated to each other, e.g. iron, metal, body, substance, entity […]. The lower concept is not contained *in* the higher, for it contains more in itself than does the higher; but it is contained *under* the latter […]. (*Jäsche-Logik*, AA 9, 97f.; my emph.)

The (higher) concept of metal is contained in the (lower) concept of iron. Since 'iron' includes further constituents besides 'metal', 'iron' is not contained *in* 'metal'. But the former is contained *under* the latter. In this sense, it is not Donald who is contained under the concept of a drake but, say, the concept of a clumsy drake. Note that Kant talks about *subordination* when a concept is contained under another concept, thereby using the term Frege contrasts with 'subsumption'. And note also that he calls a concept a *mark* (Merkmal) of another concept when the former is contained in the latter. Interestingly enough, there seem to be close similarities between Kant's 'is contained in' and Frege's 'is a mark of'; and the same might hold for Kant's 'is contained under' and Frege's 'is subordinated to'.

The key for the answer to Bolzano's challenge is that, in the traditional theory of concepts, containment-in is the inverse of containment-under:[17]

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[15] Wolfgang Künne (2001, 274) stresses that there are some places where Frege, contrary to his official definition in "Funktion und Begriff" (1891), does not mean by 'concept' the *reference* of a predicate (a function whose arguments are truth-values) but its *sense* (a part of a thought).
[16] See also KrV, B 94; *Logik Pölitz*, AA 24, 568; and *Wiener Logik*, AA 29, 910.

The concept x is contained in the concept y ↔ y is contained under x.

For example, 'metal' is contained in 'iron', entailing that 'iron' is contained under 'metal'. Now consider proposition (8), 'Every son of a bachelor is a bachelor'. Bolzano is right in pointing out that there is a weak sense of 'contained in' on which the notion 'bachelor' is contained in the notion 'son of a bachelor'. But the Kantian sense, according to which containment-in is the inverse of containment-under, appears to be stronger. It thus might introduce further restrictions to the effect that 'son of a bachelor' is not contained under 'bachelor', and hence 'bachelor' not contained in 'son of a bachelor', so that (8) would *not* be analytic on the proper understanding of Kant's definition. In this spirit, I assume it that Kantian containment-in coincides with Bolzanian containment-in plus an extra:

x is contained in y in the strong sense, that is, y is contained under x $=_{\text{df.}}$
x is contained in y in the weak sense & …

The crucial question then is: what does this extra consist in which turns weak containment-in into strong containment-in and thus containment-under?

Kant's reference to the series 'iron', 'metal', 'body', 'substance', 'entity' suggests as a minimal constraint for a concept y being contained under a concept x that everything represented by y is also represented by x. For example, 'iron' would not be contained under 'metal' if there were pieces of iron which are not pieces of metal. According to what may be called the *extensional* interpretation of the sought-after extra, it thus demands that the extension of y is a (proper or improper) subset of the extension of x:

There is nothing falling under y without falling under x.

Bell (1982, 458) and Wiggins (1998, 142) suggest to interpret Fregean 'marks' in this way. Prima facie, Hanna (2001, 127-141) reads Kant in the extensional manner when claiming that y is contained under x if the comprehension of y is part of the comprehension of x. However, Hanna takes a Kantian comprehension to include not only the actual objects falling under the concept but also *possible* ones. Thereby he subscribes to the modal conception to be discussed in a moment.

Quite obviously, the extensional reading does not take us very far. It has the benefit of excluding (3) and (4) from the analytic realm. Since there are sons of a bachelor who are not themselves bachelors, 'son of a bachelor' is not contained under 'bachelor', and hence 'bachelor' is not contained in 'son of a bachelor'. Furthermore, on this conception of containment-under 'putative bachelor' is not contained under 'unmarried', and thus does not include the latter, because some putative bachelors are married. However, the extensional reading does not filter out Bolzano's example (2), 'The father of Alexander, King of Macedon, was King of Macedon'. For the object represented by the subject-notion, i.e. Philip II, falls under the predicate-notion, i.e. was King of Macedon. More gen-

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[17] Cf. de Jong 1995, 627; and Lanier Anderson 2004, 507; 2005, 27. The left-to-right half of this equivalence was brought forward by Kant in the above-quoted passage from the *Kritik* (KrV, B 39f.).

erally, subject and predicate already satisfy the extra condition at hand when the judgement is *true*. This extra is thus suited only for excluding *false* synthetic judgements. Kant's definition of analyticity would still be too wide if we added that the predicate represents the objects falling under the subject because there would remain synthetic truths still satisfying the definiens.

Next there is the *modal* reading of our key notion, stating that a concept is contained under another concept only if the extension of the former *must* be a part of the extension of the latter:

> *Necessarily*, there is nothing falling under y without falling under x.

There are various places in Frege's writings suggesting that he had something on these lines in mind when using the term 'marks'. Wolfgang Künne (2001, 272f.; 2006, 213) coined the name 'Port Royal Constraint' for the given condition, and he supposes that Kant's characterisation of analytic judgements includes this proviso.

Unfortunately, the modal reading does not take us far enough either, because the given constraint is met by every *necessary* truth.[18] Since it is not necessary that Alexander's father was King of Macedon, the concept 'father of Alexander, King of Macedon' would not be contained under, and thus would not include, the concept 'King of Macedon'. Consequently, we need no longer accept the corresponding judgement (2) as analytic. The following proposition, however, cannot be ruled out:

> (5) Every square of a number greater than 1 is greater than 1.

The predicate-concept is contained in the subject-concept in Bolzano's broad sense. Furthermore, (5) is a mathematical and hence a necessary truth: nothing can fall under 'square of a number greater than 1' without falling under 'greater than 1'. But this means that (5) would be analytic because the predicate-notion 'greater than 1' was contained in the subject-notion 'square of a number greater than 1' even according to the strong sense given by the modal conception of the sought-after extra. Frege would definitely embrace the outcome that (5) is an analyticity, but remember that Kant cannot approve of it because he regards such arithmetical truths as synthetic.

Or consider the following statements (the subjects are in italics for ease of understanding):

> (1b) Every *triangle* has the same angular sum as two right angles.
> (1c) Every *triangle which is similar to polygons having the same angular sum as two right angles* has the same angular sum as two right angles.

Both (1b) and (1c) satisfy the condition that that nothing can fall under the subject-concept without falling under the predicate-concept. But only (1c) satisfies the further proviso that the subject-concept contains the predicate-concept in Bolzano's wide sense: 'has the same angular some as two right angles' is not part of 'triangle', but it is part of 'triangle which is similar to polygons *having the same*

[18] Necessary truths like 'Every triangle has the same angular sum as two right angles' are not analytic because their subjects do not contain their predicates in the *weak* sense.

angular sum as two right angles'. Hence, a highly insignificant addendum to the subject-notion would dictate that a synthetic truth mutates into an analytic one.

How to conceive of the crucial extra if not even the modal variant comes to grips with all examples? Interestingly, Bolzano himself provides the essential clue for the proper reading of Kantian containment-in. Subsequent to his objection that Kant's definiens admits of too wide an interpretation, we find the following idea for improvement:

> This unfortunate state of affairs could be avoided if […] one made use of the expression that in analytic judgements the predicate is one of the essential parts of the subject or (which comes to the same thing) constitutes one of its essential marks, understanding these to be constitutive marks, i.e. such as are present in the concept of the subject. (WL II, § 148, 88; cf. NAK, 35)

In an earlier passage of the *Wissenschaftslehre* the term 'essential mark' is explained in a bit more detail: "many logicians distinguish between *essential* or *constitutive* and *inessential* or *derivative* marks; [that is] between properties of an object which are thought as constituents in its concept and others where this is not the case" (WL I, § 65, 290f.).

Like Kant and Frege, Bolzano uses the term '(essential) marks' both for particular *properties* and *concepts*.[19] If we apply it to concepts, the idea is that 'bachelor' is contained in 'blonde bachelor' as an essential mark because the former is not only a constituent of the latter in the weak sense but, in addition, represents one of the defining properties of the objects given by 'blonde bachelor'. In contrast, 'bachelor' is neither an essential mark of 'son of a bachelor' nor of 'putative bachelor'. For even though both notions contain 'bachelor' in the Bolzanian sense, its function within these notions is not to determine their extension by specifying properties of the objects falling under 'son of a bachelor' or 'putative bachelor' (cf. Frege 1893, xiv, 3, 150). Frege (1903, 271) also called such extension-determining constituents *logical* parts. And Kant, in the *Handwritten Remains on Logic*, makes comments to the effect that marks are partial notions constituting a *ground of the recognition* (Erkenntnisgrund) of the corresponding objects (cf. AA 16, 297f.). In accordance with the aforementioned interpretation, this could mean that, by detecting the specified properties in an object, we have a sufficient reason for subsuming it under the notion composed of the marks.

So, let us assume that Kant refers to marks in the given sense when talking about concepts being contained in other concepts:

> x is contained in y in the strong sense, that is, y is contained under x $=_{df.}$
> x is contained in y in the weak sense, where x specifies properties of the objects falling under y

On this account of containment the judgement voiced by (5), 'Every square of a number greater than 1 is greater than 1', is not analytic. It is true that being a number greater than 1 is a necessary property of the objects falling under the subject-concept of (5). But this does not entail that 'greater than 1' is contained in 'square of a number greater than 1' in the Kantian sense. After all, it does not belong to those properties which 'square of a number greater than 1' lists as

[19] As for Frege, cf. the sentences cited by Wolfgang Künne (2001, 274f.); as for Kant, cf. the *Handwritten Remains on Logic*, AA 16, 297f.

properties of the objects in its extension.[20] That is to say, in contrast to a number greater than 1, the *square* of a number greater than 1 is not defined by being greater than 1; it is rather defined but by being the result of multiplying a number greater than 1 by itself.

Moreover, both (1b), 'Every *triangle* has the same angular sum as two right angles', and (1c), 'Every *triangle which is similar to polygons having the same angular sum as two right angles* has the same angular sum as two right angles', turn out to be synthetic propositions. The subject of the second proposition includes the concept 'has the same angular sum as two right angles' in the weak sense Bolzano called our attention to. But its function within the subject is not the extension-determining function which is relevant to Kant's conception of containment.

In this way, Bolzano himself inspires a solution to the difficulty that Kant's specification of analyticity permits too wide a reading: since Kant reverts to the notion of essential marks, all of the seemingly problematic judgements emerge as synthetic. In Ayer's view, this pro must be seen as a con because Ayer (1936, 77f.) considers geometrical and arithmetical truths, such as (1b) and (5), to be analytic. But remember that we are interested in *inner*-Kantian objections. From this perspective, Bolzano's first objection to Kant's definition is somewhat superficial. There is no denying that the central word 'contains' admits of too wide an interpretation. But the reading arising from the traditional theory of concepts comes to terms with Bolzano's examples.

Nonetheless, when suspecting Kant's characterisation of being too broad, Bolzano's line of attack could have been proper even if the attack itself did not hit the mark. Consider *particular* affirmative statements, viz. statements of the form 'Some A are B'. At first glance, many of them conform to Kant's definition of analyticity because the given concept of an A contains the concept of a B as an essential mark. Just consider

(6) Some drakes are male.

On the other hand, 'Some A are B' appears to be loaded with an existential supposition so that (6) entails that there is at least one drake. Since the existence of a drake cannot be proved without falling back on experience, 'Some drakes are male' seems to be a posteriori and therefore not analytic. Taken together, there is reason for assuming that Kant's characterisation is too wide because it does not exclude a posteriori truths like (6).

Singular affirmative judgements of the type 'The A is B' raise the same problem because they seem to have the same existential import as 'Some A are B': if there exists no A, they are not true. But then

(7) The drake on the neighbouring farm is male.

is a posteriori and hence not analytic because one needs experience in order to assure that there is a drake on the neighbouring farm.[21] The problem here is,

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[20] Cf. De Jong (1995, 632-638) on *propria* and *essentialia*.

[21] Even though they have the form 'The A is B', too, the Bolzanian translations of existential and affirmative particular statements might differ in not being a posteriori because their subject is a concept. Or is it necessary to invoke experience in order to show that a *concept* exists?

again, that (7) appears to meet Kant's regulations for analyticity because the notion 'male' is contained in the notion 'drake on the neighbouring farm' even in the strong sense of 'contained'. This substantiates the suspicion that Kant's definiens is too wide.

Bolzano provides a resort for Kant. In Bolzano's view, particular affirmative judgements are to be treated in the same way as simple existential judgements: 'Some A are B' states that there is at least one A which is B and is thus synonymous with 'The concept of an A which is B is instantiated' (see WL II, § 137). If Kant agrees with this, as Morscher (2006, 252) relates, then his containment account of analyticity is put in a position to keep analyticity back from (6). For if the proposition expressed by 'Some drakes are male' is

> The concept of a (male) drake is instantiated.

then subject and predicate are not 'drake' and 'male' anymore but 'concept of a (male) drake' and 'instantiated'. Since the latter notion is not contained in the former, 'Some drakes are male' would be synthetic according to Kant's definition. Propositions of this type could therefore not be used to show that Kant's specification of analyticity in terms of containment is too broad.

The same holds for 'The A is B'. Such singular affirmative statements could be seen as differing from particular affirmative statements only insofar as the definite article makes for a uniqueness condition. 'The A is B' thus translates into 'The concept of a unique A which is B is instantiated'. But then Kant's containment account does not mark out (7) as analytic anymore. For 'The drake on the neighbouring farm is male' would mean

> The concept of unique (male) drake on the neighbouring farm is instantiated.

Since the concept of instantiation is not even in the broad Bolzanian sense included in 'unique (male) drake on the neighbouring farm', there is no analyticity according to Kant's definition.

Note, however, that this is a Pyrrhic victory. If Kant complies with Bolzano's understanding of particular affirmative statements (as well as with my proposal for 'The A is B'), one would expect him also to approve of Bolzano's analogous interpretation of universal negative statements. In Bolzano's view, we prevalently read 'No A is B' as claiming that the concept of an A which is B is uninstantiated (cf. WL II, § 138). But then Kant's paradigm of a negative analyticity would fail to be analytic on his own standards because 'No unlearned person is learned' translated into

> The concept of an unlearned person who is learned is uninstantiated.

If we conceive of this judgement as a negative judgement with the predicate 'instantiated', then it is not analytic because the subject 'concept of an unlearned person who is learned' does not contain a notion which negates 'instantiated'. And the same holds if it is an affirmative statement with the predicate 'uninstantiated'. Then it is not analytic either because the notion of non-instantiation is not included in 'concept of an unlearned person who is learned'. Viewed from any angle, 'No unlearned person is learned' does not emerge as analytic in Kant's sense if we assume the Bolzanian reading of 'No A is B'. Moreover, *no* universal negative statement would be analytic. Curiously enough, Kant should thus be cautious of unanimously accepting this reading and the analogous one of particular affirmative statements. For although the latter promises help in allaying the suspicion that Kant's conception of analyticity is too *wide*, the former has the equally unwelcome consequence that it is too *narrow*.

But there is a further loophole. One of the premises in our argument was the seemingly obvious claim that 'Some A are B' (as well as 'The A is B') is true only if there is an A. That premise lead to the conclusion that 'Some drakes are male' (as well as 'The drake on the neighbouring farm is male') is not analytic because the given existential supposition renders it a posteriori. As surprising as that may be, there are some grounds to believe that Kant would withhold consent to the premise in question.

It was argued in section 1 that Kant prefers an interpretation of 'All A are B' under which it does not imply the existence of an A. Prima facie, this does not go well with § 53 of the *Jäsche-Logik* where we are told that universal affirmative propositions imply the corresponding particular affirmative propositions (cf. AA 9, 118f.). For if 'All A are B' entails 'Some A are B' and the latter has the aforementioned existential import, then the former must also have it. This contradiction could be explained away by denying § 53 of the *Jäsche-Logik* the status of authenticity; but the conflict could also be resolved by attributing to Kant the view that 'Some A are B' comes in without an existential supposition.[22] This opens up the possibility of conferring the title 'analytic' on particular affirmative statements like 'Some drakes are male'. They can then be regarded as true even if there are no drakes. Like in the case of 'All drakes are male', conceptual analysis alone would be sufficient to determine that they are true, so that we would be allowed to file them under 'analytic'.

Moreover, if we take it at face value, 'Some drakes are male' expresses a proposition whose predicate-concept, namely 'male', is contained in its subject-concept, namely 'drake', entailing that Kant's containment definition of analyticity is satisfied. Thus, we can reconcile the Kantian definition with the fact that 'There are drakes' is an a posteriori truth without resorting to Bolzano's understanding of particular affirmative statements. Since 'Some drakes are male' does not imply 'There are drakes', there is no need to perceive the former as a posteriori and therefore synthetic. In summary, if it was accepted that propositions like 'Some drakes are male' do not have existential import, there would be no reason anymore for accusing Kant's specification of being too wide. I leave it to the reader to determine whether this is a Pyrrhic victory as well.

3 The definiens is too narrow

Bolzano's first objection can be countered, but he has not shot his bolt. Remember that he does not only criticise Kant's definition because it permits an interpretation too wide but also has an idea for improvement: a judgement is analytic if and only if its subject contains its predicate *as an essential mark*. Directly subsequent to this clarification, however, Bolzano proceeds by discrediting it for being too narrow:

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[22] See again Rosefeldt's "Kant's Begriff der Existenz" (2008) and his paper in the present volume.

> But this definition is applicable to only one kind of analytic judgements, only those of the form: A which is B is B. Should there not be others as well? Should we not count [(i)] the judgement: A which is B is A, and also [(ii)] the judgement: Every object is either B or not B, among analytic judgements? (WL II, § 148, 88; cf. NAK, 35)

Part (i) of this objection seems to say that the improved definition is too restrictive because, say, (8a) would be an analytic statement according to it whereas (8b) would be synthetic:

(8a) A ball which is red is red.
(8b) A ball which is red is a ball.

I admit unashamedly that Bolzano's meaning in this remark is not clear to me. After all, in 'ball which is red' the concept 'ball' seems to represent an essential characteristic just as much as the concept 'red' does. It thus appears that the predicates of both propositions are contained in the right manner in their common subject. I might have misinterpreted Bolzano's talk of essential marks, but I can conceive of no sustainable interpretation on which 'red' is a mark of 'ball which is red' whereas 'ball' is not. De Jong (2001, 335) comments on this part of Bolzano's objection as follows: "The first example makes clear in particular that Bolzano regards formal precision as important to a degree seldom previously encountered in traditional logic." But this mystifies me no less than Bolzano's original claim.

It has been suggested to me that Bolzano might have had in mind examples of the following type:

(9a) A sugar cube which was dissolved in water was dissolved in water.
(9b) A sugar cube which was dissolved in water is a sugar cube.

The idea is that (9a) is true and analytic whereas (9b) is false and not analytic because, although the sugar cube is not a cube anymore when dissolved in water, it is still true that it was dissolved in water. But (9a) is true only if its subject-concept is instantiated; and it is instantiated only if it reads '*former* sugar cube which was dissolved in water'. Otherwise, (9a) would be empty insofar as it was about objects which are both sugar cubes and dissolved in water and hence not cubes anymore. However, if subject and predicate of (9b) are interpreted in the same way, then (9b) is analytic no less than (9a). For then (9b) says that a *former* sugar cube which was dissolved in water is a *former* sugar cube. And this is not only true but also analytic because the predicate is contained as an essential mark in the subject. Part (i) of Bolzano's worry thus remains puzzling.

Part (ii) of the worry is that a logical truth like the following does not prove analytic:

(10) Every object is either red or not red.

Note that this example already threatens Kant's definition as given by the weak interpretation against which Bolzano's first attack was directed. The sense of 'object' contains neither the sense of 'red' nor the one of 'either or' nor the one of 'not'. In other words, even though being red or not is a necessary feature of all objects (barring category mistakes), it is not part of the definition of an object. Hence, the predicate-notion is not even in the weak sense contained in the subject-notion. Furthermore, Kant cannot resort to the excuse that (10) does not express a subject-predicate proposition but a conjunction, namely, 'Either every object is red or every object is not red'. This statement is false and thus not identical with the original one (cf. Morscher 2006, 253).

Should Kant take Bolzano's concern to heart? Wolfgang Künne (2006, 214) agrees, and he is surely right, given that Kant considers all truths of logic analytic, as some scholars assume.[23] There are then analytic judgements of subject-predicate form, such as (10), in which the subject does not contain the predicate. And there are analytic judgements which do not even have subject-predicate structure:[24]

> (11) If all fans of Werder Bremen are relationally disturbed and Tom is a fan of Werder Bremen, then Tom is relationally disturbed.

However, to my knowledge Kant nowhere explicitly says that all truths of logic are analytic. Hence, there remains a loophole that is simply clinging to the original definition. "Should we not count (10) among analytic judgements?", Bolzano asks. "No, at least not in my sense of 'analytic'", Kant could answer, "because (10) and the like add to the subject a predicate that was not thought in it at all, and could not have been extracted from it through any analysis. (10) is thus a further synthetic truth a priori."[25]

Thus the examples by which Bolzano tried to show that Kant's explication is too close are weakened. They would have bite if Kant considered all truths of logic to be analytic, or if there was some inevitable sense of 'analytic' in which logical truths are of that type; but both of these claims are vulnerable. Nonetheless, by indicting Kant's definition for being too narrow, Bolzano was on the right track even if he used the wrong examples. Initially, his worry seems to be applicable to two of Kant's own examples:

> To be sure, a few principles that the geometer presupposes are actually analytic and rest on the principle of contradiction, e.g., a = a, the whole is equal to itself, or (a + b) > a, i.e., the whole is greater than its part. (KrV, B 16f.)

According to this passage, Kant perceives the following propositions as analytic:

> (12a) Every whole is equal to itself.
> (13a) Every whole is greater than a proper part of it.

Moreover, one would expect Kant to count geometrical instances of 'a = a' and '(a + b) > a' among the analytic statements, such as:

> (12b) The length of the straight line AB is equal to the length of the straight line BA.
> (13b) The area of the triangle ABC plus the area of the triangle DEF is greater than the area of the triangle ABC.

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[23] Cf. Hanna 2001, 140; Morscher 2006, 250, 261; Pap 1958, 29; and WL III, § 315, 240.

[24] (11) would have subject-predicate structure if it could be read as being composed of the subject 'Tom' and the predicate 'is relationally disturbed if he is a fan of Werder Bremen and all fans of Werder Bremen are relationally disturbed'. But then the subject does not enclose the predicate.

[25] Kant has to grant then that some synthetic judgements are not "judgements of amplification" because (10) can hardly be said to increase knowledge in any substantial sense.

Taken at face value, however, in all of these cases the subject-concept does not even contain the predicate-concept in Bolzano's broad sense. For the subject in (12a) and (13a), viz. the sense of 'whole', does neither include the sense of the relational expression 'equal to' nor the one of 'greater than'. And the same holds for the sense of the subjects in (12b) and (13b), that is, 'the length of the straight line AB' and 'the area of the triangle ABC plus the area of the triangle DEF'. It thus seems that Kant's definition of analyticity is too parsimonious even from his own perspective because it does not allow (12a)–(13b) to be analytic.

Prima facie, there is an avenue. I have assumed that the predicate-concepts of these propositions are expressed by the entire phrases following the copula 'is'. In this spirit, the predicate-concepts of (12a)–(13b) are 'equal to itself', 'greater than a proper part of it', 'equal to the length of the straight line between BA' and 'greater than the area of the triangle ABC', respectively. But Kant could reply that the purely relational parts of these notions, viz. 'equal to' and 'greater than', do not belong to the predicate. Rather, the predicate-notions are nothing more than 'it(self)', 'the length of the straight line between BA' and 'the area of the triangle ABC'; they are thus contained in the given subject-notions. Regarding the statements (12a) and (12b), at least, this interpretation is confirmed by Kant's rationale for the syntheticity of arithmetical equations, such as '7 + 5 = 12' (KrV, B 15; my emph.): "The concept of *twelve* is by no means already thought merely by my thinking of that unification of seven and five; and no matter how long I analyse my concept of such a possible sum, I will still not find *twelve* in it." Kant's reason for deeming '7 + 5 = 12' synthetic is not that the subject '7 + 5' does not contain the concept '*identical with* 12'. His reason is rather that '7 + 5' does not contain the concept '12', which thus seems to be the constituent of the given proposition Kant treats as the predicate-notion.

However, such a treatment of equations amounts to lumping together the 'is' of identity and the 'is' of predication. Whereas the 'is' in '7 + 5 is even' might be thought of as providing a kind of glue sticking together the subject and the predicate, the 'is' in '7 + 5 is 12' goes beyond that. It is an abbreviation for 'is equal to' (or 'is identical with'), expressing not only the glue given by the predicative 'is' but also what is given by 'equal to' (or 'identical with'). The 'is' of identity thus appears to make a contribution to the predicate in '7 + 5 is 12', this predicate being not only '12' but 'is equal to 12'.

Furthermore, this problem is nothing compared to the difficulties involved in the analogous handling of (13a) and (13b). If the concept expressed by 'greater than' was not a part of the given predicate-notions, then the same would hold for the concept expressed by 'smaller than' in the following variants of (13a) and (13b):

(14a) Every whole is smaller than a proper part of it.

(14b) The area of the triangle ABC plus the area of the triangle DEF is smaller than the area of the triangle ABC.

The subjects of these propositions would then contain their predicates, which would be again just 'it(self)' and 'the area of the triangle ABC'. But Kant would definitely not regard (14a) and (14b) as analytic. In addition, 'the area of the triangle ABC' is contained in 'the area of the triangle ABC plus the area of the triangle DEF' in Bolzano's weak sense of the word. But there fails to be containment in Kant's strong sense. After all, it is not true that being (equal to) the area of the triangle ABC is one of the properties of the whole composed of the area of the triangle ABC and the area of the triangle DEF. Hence, 'the area of the triangle ABC' is not among the essential marks of 'the area of the triangle ABC plus the area of the triangle DEF'.

In short, the proposed avenue is an impasse. No matter how one looks at them, what Kant presents as examples of analytic judgements does not conform to his own definition. Kant is well advised to either rework his definition or exclude the recalcitrant judgements from the class of analyticities. De Jong (2010) comes to the aid of Kant by recommending the second option. Judgements like (12a)–(13b), so de Jong suggests, are relational and thus not of subject-predicate form, entailing that Kant's definition is not applicable. However, if affirmative subject-predicate judgements are defined as judgements in which a property is assigned to some object(s), then every judgement which relates something to something is of subject-predicate form. For if a and b stand in relation R, then a has the (relational) property of standing in relation R to b.

Nonetheless, de Jong is right in describing it as far from obvious that Kant accepts propositions like (12a)–(13b) as analytic. Although Kant opens the crucial paragraph by claiming that 'a = a' and '(a + b) > a' are analytic, he continues in a surprising way:

> And yet even these, although they are valid in accordance with mere concepts, are admitted in mathematics only because they can be exhibited in intuition. What usually makes us believe here that the predicate of such apodictic judgements already lies in our concept, and that the judgement is therefore analytic, is merely the ambiguity of the expression. We *should*, namely, add a certain predicate to a given concept in thought, and this necessity already attaches to the concepts. But the question is not what we *should think* in addition to the given concept, but what we *actually think* in it, though only obscurely, and there it is manifest that the predicate certainly adheres to those concepts necessarily, though not as thought in the concept itself, but by means of an intuition that must be added to the concept. (KrV, B 17)

Kant appears to be asserting here that the judgements in question are *not* analytic because their predicates are *not* contained in their subjects. He could substantiate this claim with reference to the arguments above. The concept 'equal to' is included in 'whole' and 'the length of the straight line AB' just as little as the concept 'greater than' is included in 'whole' and 'the area of the triangle ABC plus the area of the triangle DEF'. (12a)–(13b) are thus synthetic according to the containment-or-negation conception. This gives Kant the possibility of adhering to this characterisation of analyticity. But it has the serious drawback that it cannot be accommodated with the first sentence of the paragraph, where it is explicitly said that (12a) and (13a) are analytic.

There is a way to solve this discrepancy. Have a further look at the first sentence and especially its mention of the principle of contradiction:

> To be sure, a few principles that the geometer presupposes are actually analytic *and rest on the principle of contradiction*, e.g., a = a, the whole is equal to itself, or (a + b) > a, i.e., the whole is greater than its part. (KrV, B 16f.; my emph.)

Perhaps, the 'and' in 'and rest on the principle of contradiction' is to be interpreted as prefacing a *rationale* for the analyticity of the propositions 'a = a' and '(a + b) > a': they are analytic *because* they rest on the principle of contradiction. However, Kant would then claim in the paragraph in question that these propositions are *analytic* even though their predicates are *not* contained in their subjects. That is, he would straightforwardly concede that the containment definition, which he presented only six pages earlier, is too narrow.

This kind of criticism can easily be extended to a wide range of examples. The problematic paragraph from the Introduction anticipates Kant's famous reference to the law of contradiction in a later part of the *Kritik*: "if the judgment is analytic, whether it be negative or affirmative, its truth must always be able to be cognised sufficiently in accordance with the principle of contradiction" (KrV, B 190). As I said in section 1, this is presumably not meant to provide an alternative definition of analyticity, but is to be understood as an epistemological remark. Nevertheless, it can be used to shed light on the extension of Kant's concept of analyticity.

Taken literally, Kant offers a *necessary* condition. However, in the *Prolegomena* he says that synthetic judgements "can never originate according to the principle of analysis alone, namely the principle of contradiction" (AA 4, 267). Similarly, we read in the *Kritik* that in "the synthetic part of our cognition we will, to be sure, always be careful not to act contrary to this inviolable principle, but we cannot expect any advice from it in regard to the truth of this sort of cognition" (KrV, B 191). Kant thus maintains that, if a judgement is synthetic, then it is not cognisable solely on the basis of the rule of contradiction. By implication, this means that, if a judgement *is* cognisable in this way, then it is *not* synthetic and thus, given that it is of subject-predicate form, *analytic*. Hence, Kant appears to take knowability on the basis of the law of contradiction to be a *sufficient* condition for the analyticity of subject-predicate judgements (cf. de Jong 1995, 619f.). But this provides grist to Bolzano's mill: Kant's definiens is then too parsimonious because there is a plethora of judgements which meet the principle-of-contradiction criterion and should therefore be accepted as analytic by Kant, even though their predicate-notion is neither contained in nor negates a constituent of the subject-notion.

Among other things, this applies to the following judgement:

(15) There is no married bachelor.

The principle of contradiction, as conceived by Kant, says that no object both possesses and lacks a property at the same time.[26] To prove the truth of a judgement by recourse to this principle means to derive from the opposite of the judgement an explicit contradiction to the effect that some object both has and lacks a certain attribute (cf. *Metaphysik Arboldt (K 3)*, AA 29, 964f.). This can be done with (15). Its opposite 'There is a married bachelor' implies that there is a man who is both married and not married. Since this is incompatible with the principle of contradiction, we can, vice versa, infer from this principle that (15) must be true. Its truth can thus be recognised solely with the help of the princi-

[26] Cf. KrV, B 190; and *Metaphysik Mrongovius*, AA 29, 789.

ple of contradiction, so that it is analytic according to the corresponding criterion. But it is not analytic according to the containment-or-negation definition, given that (15) means 'The concept of a married bachelor is uninstantiated'. After all, the corresponding subject 'concept of a married bachelor' neither contains 'instantiated' nor a notion which negates 'instantiated', and the same holds for 'uninstantiated'. Hence, the definition seems to be too narrow.[27]

Or remember (10), 'Every object is either red or not'. If (10) expressed a falsehood, there would be an object for which it does not hold that it is red or non-red: $(\exists x)\neg(Rx \vee \neg Rx)$. Due to a close relative of De Morgan's laws, this entails that there exists an object which is non-red and not non-red: $(\exists x)(\neg Rx \,\&\, \neg\neg Rx)$, implying straightforwardly that there is something which is both not red and red: $(\exists x)(\neg Rx \,\&\, Rx)$. Since this is in conflict with the law of contradiction, this law entails that (10) is a truth, even though, again, the predicate-concept is not contained in the subject-concept.[28]

Thirdly, take 'All A are A or B'. If its opposite 'Some A are not A or B' were true, there would be an A which is neither A nor B because '$(\exists x)(Ax \,\&\, \neg(Ax \vee Bx))$' implies '$(\exists x)(Ax \,\&\, (\neg Ax \,\&\, \neg Bx))$'. Contrary to the rule of contradiction, there would thus be something which is A and not A. Nonetheless, if the concept of a B is not contained in the concept of an A, the predicate 'A or B' is a fortiori not contained in the subject 'A'. To present a concrete example, the concept 'red or green apple' is not included in 'red apple'. Hence,

(16) All red apples are red or green apples.

is analytic on the rule-of-contradiction criterion while it is synthetic on the containment-or-negation definition.

Finally, consider judgements of the form 'No non-A is a B which is A', for example, a close relative of Kant's 'No unlearned person is learned':

(17) Nothing learned is a person who is unlearned.

If (17) were false, i.e., if something learned were an unlearned person, there would be an object which is both learned and unlearned. So, again, we arrive at a claim which is ruled out by the principle of contradiction. But (17) does not satisfy Kant's official specification. Since the judgement is a negative truth, it would be awarded as analytic by (KA2) if the predicate 'unlearned person' negated the subject 'learned (thing)' or a constituent of it. But remember the associated definition of negation: a notion negates another notion if and only if one of them is composed of the other and the notion expressed by 'not', the prefix 'un-' or the like. According to this explanation, only a *part* of the predicate,

[27] There is a further reason for foisting the analyticity of (15) on Kant. In the next section, it will be pointed out that he seems to see no obstacle to incorporating analytic *falsehoods*, such as 'All bachelors are married'. But it is quite natural to suggest that, if 'All bachelors are married' is an analytic falsehood, then 'There are no married bachelors' is an analytic truth:

[28] Kant could reply that the derivation of (10) does not only assume the principle of contradiction but also the given De Morgan-like law, so that the truth of (10) is not cognisable *solely* on the basis of the principle of contradiction. But, firstly, we do not need anything like that law for most of the other examples. And, secondly, Kant would have a hard time explaining why the use of other purely logical principles does not have the same effect. Just think of the first step in such an indirect inference: the formation of the opposite proposition.

namely ‘unlearned’, but not the predicate ‘unlearned person’ as a whole, negates the subject ‘learned’. (17) is therefore not analytic in the sense of the Kantian containment-or-negation regulations.

More generally, those regulations embody the idea that a judgement is analytic only if an analysis of the *subject* unearths the predicate or something negating it. Judgements of the type ‘No non-A is a B which is A’ suggest that even Kant himself might condemn this idea as too narrow-minded upon closer inspection. For it does not leave room for an analysis of the *predicate* leading to a similar result, such as in the case of (17) where decomposing the predicate results in a concept negating the subject. Moreover, propositions of the form ‘No A which is B is a C which is not B’ can also be recognised as true just on the basis of the rule of contradiction. As an example, consider

> (18) No German-speaking person who is blonde is an English-speaking person who is not blonde.

Its opposite ‘Some German-speaking persons who are blonde are English-speaking persons who are not blonde’ entails the existence of someone who is blonde and not blonde. It thus appears that Kant should also provide for statements whose analyticity is based on the fact that an analysis of *both subject and predicate* reveals constituents negating each other.

To be sure, opening Kant’s account for such statements is not too difficult. We just need a specification of the following type:

> A true negative subject-predicate proposition x is analytic $=_{\text{df.}}$
> The predicate-concept of x or a constituent of it negates its subject-concept or a constituent of it.

However, this update merely results in judgements of the form ‘No non-A is a B which is A’ and ‘No A which is B is a C which is not B’ emerging as analytic. It does *not* yield a definition capturing (10), (15) and (16); and the same holds true for what Kant considers as analytic “principles that the geometer presupposes”, i.e. (12a) and (13a). Hence, even though the examples presented by Bolzano are debatable, his criticism goes in the right direction. It seems that Kant’s containment-or-negation definition of analyticity does not do justice to all of the propositions it should.

4 The definiendum is too limited

It was, and still is, quite popular to accuse Kant’s definition of being too restricted because it has nothing to say about judgements which are not of subject-predicate form. Frege (1884, § 88) and Quine (1951, 20f.) are among those who addressed this problem; and Ayer (1936, 72) complained even more rigorously that Kant’s definition is based on “the unwarranted assumption that every judgement, as well as every German or English sentence, can be said to have a subject and a predicate”. Bolzano, however, never expressed his thoughts on this issue.

He most likely does not make this point because, in his eyes, the confinement to subject-predicate propositions does not amount to any limitation at all. After all, like Leibniz, he thinks that each and every proposition has subject-predicate structure even though the corresponding sentences do not always exhibit it.[29] For example, Bolzano takes a disjunction 'Either P or Q' to express a statement of the form 'The collection of the proposition that P and the proposition that Q contains exactly one true proposition' (cf. WL II, § 160.3). Note, however, that this reading does not allow disjunctions to express analytic propositions in Kant's sense. For even in the case of 'Either it is raining or it is not raining', the predicate-concept – 'contains exactly one true proposition' – is not included in the subject-concept – 'the collection of the proposition that it is raining and the proposition that it is not raining'.

Bolzano's reason for treating Kant's definiendum as too limited is not that it is restricted to subject-predicate propositions but that it is meant to comprise only analytic *truths* and thus does not account for analytic *falsehoods*:

> I thought it useful to interpret both notions, of analytic as well as synthetic propositions, so broadly that not only true but only false propositions could be included under them.(WL II, § 148, 88)

Of all Bolzano's attacks, this one can be parried most easily. Since Kant was primarily interested in knowledge, his division concerns only truths. That is why his definition of analyticity does not provide for the fact that there is a respect in which the truth 'All bachelors are unmarried' is more similar to the falsehood 'All bachelors are married' than to the truth 'All bachelors are younger than 1000 years'. Nonetheless, as Proops (2005, 590f.) worked out, Kant could be quite happy with extending the analytic-synthetic division. In his *Reflexions on Metaphysics*, he wrote: "If it is said: a resting body is moved, then this means: insofar as I conceive it as resting, it is moved, and the judgement would be analytic and false" (AA 18, 648). Elsewhere he suggests that 'God is mortal' expresses an analytic falsehood because the predicate-concept negates a constituent of the subject-concept.[30] Moreover, it seems that Kant has the resources to capture analytic falsehoods with a close variant of his definition (KA2):

> (KA3) A subject-predicate proposition x is analytic $=_{\text{df.}}$
> either the predicate-concept of x is contained in its subject-concept (this holds for true affirmative and false negative analyticities), or the predicate-concept of x negates a constituent of its subject-concept (this holds for true negative and false affirmative analyticities).

In light of this definition, the falsehoods 'All bachelors are married' and 'No bachelor is unmarried' are as much analytic as the truths 'All bachelors are unmarried' and 'No bachelor is married' are. For the predicate-notion of the affirmative judgement 'All bachelors are married' negates a constituent of the subject-notion, namely 'unmarried'; and the predicate-notion of the negative judgement 'No bachelor is unmarried' is included in the subject-notion.

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[29] See WL II, § 127; and Morscher 2006, 253.

[30] Cf. *Versuch den Begriff der negativen Größen in die Weltweisheit einzuführen*, AA 2, 203; and *Metaphysik Mrongovius*, AA 29, 810.

5 The definiens does not capture the proper essence of analyticity

Bolzano saved his most severe criticism for the conclusion. In the *Wissenschaftslehre* he demurs that the Kantian definitions "fail to place enough emphasis on what makes this sort of judgement really important" (WL II, § 148, 88), and in the *Neuer Anti-Kant* we are told in a sterner voice:

> Kant's explication [...] keeps entirely unaffected the proper essence, the difference philosophers should be most after when establishing this division, namely, that the truth or falsity of certain propositions (and these are only the analytic ones) in no way depends upon each of the notions of which these propositions are composed, but that they remain true or false whatever variation some of those notions are subjected to [...]. (NAK, 35)

Unsurprisingly, the distinctive feature given here is the very same one that is highlighted by Bolzano's own definition of analyticity (cf. WL II, § 148, 83):

(BA1) The proposition x is analytic $=_{df.}$
x contains at least one notion whose uniform substitution leads only to propositions with the same truth-value as x.

According to this specification, both the true 'All drakes are male' and the false 'No drake is male' are analytic. For the uniform substitution of the concepts 'male' and 'duck' in the former invariably results in true propositions, such as 'All green cars are green'; and substituting 'male' and 'duck' in the latter always leads to a falsehood, such as 'No three-year-old girl is three years old'. In other words, these propositions contain elements which are insignificant to their truth-value.

As customary for Bolzano's method of variation, the substitution of the variable notions must not result in a variant of the original proposition whose subject-concept is *uninstantiated* (cf. WL II, § 147, 80). Bolzano takes propositions with uninstantiated subject-concepts to be false (cf. WL II, § 127, 16). Hence, if such variants were permitted, the judgement 'All drakes are male' would not be analytic because, by replacing 'male' with 'eight-legged' or 'duck' with 'lioness', one would get a false variant of this judgement. Note also that Bolzano's explanation of analyticity in § 148 of the *Wissenschaftslehre* allows for a conception slightly differing from (BA1):[31]

(BA2) The proposition x is analytic $=_{df.}$
x contains at least one notion whose uniform substitution leads only to true or only to false propositions.

[31] Bolzano's explication reads as follows: "If there is even a single notion in a proposition which can be arbitrarily varied without disturbing its truth or falsity, i.e., if all propositions which result from replacing this notion by arbitrary others are either all true or all false, provided that they have objectuality [= their subject-notions are instantiated], then this is a property noteworthy enough to distinguish such propositions from all others for which this is not the case." In Siebel 1996, ch. 4.4, it is pointed out that what precedes the 'i.e.' amounts to definition (BA1) whereas what follows it suggests (BA2). Cf. also Künne 2006, 192-194 and fns. 21, 36, 39. De Jong (2001, 337; 2010) and Lapointe (2007, 230; 2010) just offer (BA2) as Bolzano's definition without noting that Bolzano's words suggest also (BA1).

On (BA2), 'Every green-haired redhead is green-haired' is analytic because substituting 'redhead' results only in true variants (provided that we do not tolerate uninstantiated subject-concepts). On (BA1), however, the given statement is *not* analytic because it is *false* and thus differs in truth-value from its admissible variants.

Back to Bolzano's fourth objection, stating that Kant does not capture the proper essence of analyticity as it is given by Bolzano's explication. In a similar way, Morscher (2006, 256) avers that Bolzano's specification (i) "catches Kant's concept of analyticity more appropriately than his own definition" and (ii) "blocks all the objections raised against Kant's definition by Bolzano". Part (ii) is true and does not pose a problem for Kant. Consider just one of the examples discussed. 'Every son of a bachelor is a bachelor' does not express an analytic falsity in Bolzano's sense because neither 'son' nor 'bachelor' nor one of its other constituents is irrelevant for its falsity. For example, if you replace 'bachelor' with 'son', you end up with a truth; and the same holds for substituting 'son' with 'unmarried father'.

However, if part (i) of Morscher's claim were true, this would be extremely embarrassing for Kant. Let us suppose that, when introducing his containment account of analyticity, there actually was analyticity in *Bolzano's* sense at the back of his mind. As to the Kantian paradigms, such as 'All bodies are extended', this is unproblematic because they are also Bolzano-analytic. Since the notion in predicate-position is also contained in the subject as an essential mark, this notion is insignificant for their truth. Rather, it is embarrassing because, if Morscher were right, Kant would have accidentally centred on a special case of insignificance, thereby carelessly neglecting that there are other cases en masse. Just consider the following examples in which the irrelevant constituents are italicised:

(10) Every object is either *red* or not *red*.
(13b) The area of *the triangle ABC* plus the area of *the triangle DEF* is greater than the area of *the triangle ABC*.
(15) There is no married bachelor (= unmarried *man*).
(18) No *German-speaking person* who is *blonde* is an *English-speaking person* who is not *blonde*.
(19) 1 + 2 = 3, when read in Leibniz' way: *1* + (*1* + *1*) = (*1* + *1* + *1*).[32]
(20) Every *equilateral* triangle has the same angular sum as two right angles.
(21) All *green* bodies are heavy.
(22) Every *past* event has its cause.

[32] See Leibniz' derivation of '2 + 2 = 4' in the *Nouveaux essais* (1704, IV.VII.10). It is quoted by Frege in *Die Grundlagen der Arithmetik* (1884, § 6) and used by Bolzano in the *Wissenschaftslehre* (§ 305, 186). Note that '1 + 2 = 3' is not analytic in Bolzano's sense when interpreted à la Peano because in the proposition expressed by '1 + the successor of 1 = the successor of the successor of 1' the concept '1' is not irrelevant. Substituting '1' with '2' results in a false variant because 2 + 3 (the successor of 2) ≠ 4 (the successor of the successor of 2).

Kant could have been prepared to file the first half under 'analytic', but he would surely not be amused about the second half. For Kant's prime example of the syntheticity of arithmetic, viz. '7 + 5 = 12', can be handled in the same way as (19) and would thus be analytic. Secondly, even if Kant tolerates some exceptions to the rule that geometrical truths are synthetic, e.g. 'Triangles are polygons', it is hardly conceivable that (20) is among them. After all, Kant illustrates the syntheticity of geometry by (1b), 'Every triangle has the same angular sum as two right angles', and (20) is distinguished from the latter merely by virtue of being restricted to *equilateral* triangles.[33] Likewise, (21) and (22) closely resemble Kantian paradigms of synthetic judgements a posteriori and a priori, namely 'All bodies are heavy' and 'Every event has its cause' (see KrV, B 11, 13). The sole difference is that the latter concern *all* bodies and *all* events whereas the subject-concepts of (21) and (22) contain further specifications and therefore only represent *green* bodies and *past* events, respectively. Again, I do not think Kant would react to the consideration that (21) and (22) contain an element which is insignificant for their truth by conceding analyticity to them.

Additionally, Bolzano's explication allows for *a posteriori* analyticities.[34] For example, given that all fans of Werder Bremen are relationally disturbed, 'All *adult* fans of Werder Bremen are relationally disturbed' is Bolzano-analytic because substituting 'adult' invariably results in a truth. But Kant would probably have no praise for analytic judgements of this type. Since they cannot be justified without recourse to experience, they violate a condition which seems to be central to Kant's account of analytic judgements.

All in all, if Morscher's diagnosis was true, Kant's definition of analyticity and its surroundings would be light-years removed from the conception they are meant to express. I therefore doubt that Bolzano's regulations capture Kant's ideas more appropriately than his own definition.

In section 3, it was argued that the definition in terms of containment-or-negation is too narrow because there are propositions which do not satisfy it even though they are analytic according to the principle-of-contradiction criterion. Hence, if Kant's basic idea behind the introduction of the term 'analytic' is to capture the latter propositions, the containment-or-negation specification does not properly mirror his basic idea. However, this does not mean that *Bolzano's* account gets to the heart of Kant's actual intentions. It is then more probable that Kant had something like *Frege's* or *Ayer's* account in mind because the principle-of-contradiction criterion is much closer to them than to Bolzano's regulations in terms of irrelevant elements.

Frege (1884, § 3) says in *Die Grundlagen der Arithmetik* that propositions are analytic if and only if they can be proved to be true only with recourse to definitions and general logical laws. In *Language, Truth and Logic*, Ayer (1936, 73) con-

[33] See KrV, B 744f. Since (20) follows from (1b) and (1b) is synthetic on Bolzano's standards, his account has the consequence that a synthetic proposition sometimes implies an analytic one (cf. WL IV, § 447, 115f.; Künne 2006, 194). Note, however, that Kant's conception opens up the same possibility because (1b) is Kant-synthetic and implies the Kant-analytic truth 'Every triangle having the same angular sum as two right angles has the same angular sum as two right angles'.

[34] Cf. Textor 2001; and Künne 2006, 195.

siders sentences analytic if their truth depends solely on the definition of their constituents. The most striking common feature is that a judgement is perceived as analytic if its truth rests on nothing but fundamental *logico-semantic* principles, such as the law of contradiction (Kant), general logical laws and definitions (Frege) or just definitions (Ayer). Moreover, a proposition which is analytic in the sense of being (indirectly) derivable from the law of contradiction is clearly Frege-analytic; and I suppose that Ayer would also accept it as analytic. Hence, based on the assumption that Kant's original intent is expressed in the principle-of-contradiction criterion, if there really is an explanation of analyticity which is closer to this intent than Kant's own explanation, it is rather Frege's or Ayer's than Bolzano's.

In addition, unlike Frege, Bolzano does not contend that his account only articulates what Kant intended all along. When offering his definition of analyticity, Frege (1884, § 3, fn.) asserts his claim "only to state accurately what earlier writers, Kant in particular, have meant". Similarly, Ayer (1936, 73) thinks that his account preserves "the logical import of Kant's distinction […] while avoiding the confusions which mar his actual account of it". Bolzano, however, does not maintain that his specification of analyticity in terms of irrelevant elements catches the Kantian thoughts. For example, when he rebukes Kant for not having grasped the analytic-synthetic division with the required clarity (see NAK, 34), he does not find fault with Kant's *formulations*, but rather with the *idea* they are meant to articulate.

Accordingly, Bolzano's fourth objection does not state that Kant's definition misses *Kant's intent* but rather that it misses "*the proper essence*, the difference philosophers should be most after when establishing this division" (NAK, 35; my emph.). Evaluating this objection with all due thoroughness would go beyond the scope of this paper. But remember Wolfgang Künne's (2006, 219) remark in "Analyticity and Logical Truth" that there are no pre-theoretical intuitions concerning analyticity because 'analytic' is a term of art. It is thus not possible to delineate "the proper essence" of analyticity by invoking such intuitions. But how to establish then what "the proper essence" is? There is something along these lines only if there is some *external* (pre-theoretical or whatever) sense of 'analytic', in other words, a standard applicable to any specification of analyticity whatsoever, whether it is Kant's, Bolzano's, Frege's, Ayer's, etc. That is to say, talk of "the proper essence" of analyticity makes sense only if questions like 'What is analyticity actually?' do. But do they?

It appears to me that questions of this sort are rather a stumbling block to the substantial issues. Just consider the dispute about arithmetic and geometry. Frege (1884, § 89) thought that Kant was mistaken about arithmetic because it is in fact analytic. Ayer (1936, 79f.) went one step further and accused Kant for being also wrong about geometry. Both overlook the possibility that there is no genuine conflict at all for the simple reason that they do not share a common topic and use the term 'analytic' in a different way than Kant does. For example, even if the following geometrical truths are analytic in some *Ayerian* sense, they are not analytic according to the *Kantian* definitions because their subjects do not contain their predicates (cf. the end of section 1):

Every triangle has the same angular sum as two right angles.

Equilateral triangles are equiangular.
In every square the side is related to its diagonal as $1 : \sqrt{2}$.

In the Introduction to the *Kritik* we find Kant's famous formulation of the central task he sets out to perform (KrV, B 19): "The real problem of pure reason is now contained in the question: How are synthetic judgments a priori possible?" This is most certainly a substantial question, no matter whether arithmetical or geometrical truths are synthetic in Bolzano's, Frege's, Ayer's or even some external sense. The examples mentioned above make it clear that there exist synthetic judgements a priori in Kant's sense; and it is legitimate to ask how it is possible to recognise without recourse to experience that such a judgement is true, and even necessarily true. This question cannot be answered by alluding to conceptual analysis, or the principle of contradiction, because the subject-concept of these propositions neither contains the predicate-concept nor a constituent negating the predicate-concept or a part of it. Kant's answer is, very briefly, that such truths are cognised through "pure intuition", whereas Bolzano takes this to be the basic mistake within the *Kritik*: "What entitles the intellect to attribute to a subject A a predicate B which does not reside in the concept of A? Nothing else, I say, than the intellect's *having* and *knowing* both concepts A and B."[35]

Whatever the correct answer might be, questions of the type 'What, really, is analyticity?' only interfere with the search for it. To be sure, further conceptions of analyticity become relevant when a Kant-synthetic truth is also analytic in a further sense. For example, if a truth is reducible to logic à la Frege, the question arises whether we need "pure intuition" in order to show that it is true. However, this does not mean that there is anything along the lines of a "proper essence" of analyticity. It just means to tell apart different conceptions which have equal right to be labelled 'conceptions of analyticity'.

To conclude, three of Bolzano's four objections against Kant's definition of analyticity can be countered or at least weakened. The accusation remains that the definition appears to be too narrow on Kant's own standards because it takes into account only those cases where an analysis of the *subject* reveals a constituent identical with the predicate or negating it. It thus discounts judgements which are analytic insofar as an analysis of the *predicate*, or both subject *and* predicate, unearths conflicting constituents. Moreover, even if we clear the way for such analyticities by means of the modification proposed at the end of section 3, the resulting explanation is still not broad enough. There remain judgements which do not satisfy it although Kant would presumably call them analytic because they are derivable from the principle of contradiction. Whether this is a serious problem is another question.[36]

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[35] WL III, § 305, 180; cf. NAK, 40, 69.

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