Visual Cortex: Looking into the Klein bottle (through the lens of empirical modelling)

Abstract
This project would be a “translation” of Nicholas V Swindale’s 1996 paper “Visual cortex: Looking into a Klein bottle” into an interactive form. The paper is a brief description of the arrangement of sight-receptive neurons in “cell receptive fields” in the visual cortex of primates. It describes some relatively simple biology and mathematics, and exposes a fascinating connection between the cells in primate brains and an interesting shape called a “Klein bottle”.

All of the mathematics -and all of the biology- that get described by Swindale are things that could be visualized. A few pictures are used to help people understand the subject, though more could have been done if there was the possibility of making simple animations. And even more than *that* could be done using interaction.

Bret Victor recently “translated” a paper into “sequential art” - a combination of text, pictures, and very small (less than 90x130 pixel) interactive demos. This demystifies the systems and processes being described - the reader might have control over the probability of an event in one small demo, then in the next they control what step of a process they’re looking at. To use the language of EM, the agent controls the observables, instead of having a developer-decided selection of observables shown to them.

The Klein bottle is fascinating. It is a 4 dimensional shape and therefore can help us illuminate the fascinating world of higher-dimensional mathematics. But it is also closely linked to the mobius strip, something that is very easy to understand - for this reason and others, it is one of 4D things most easily-introduced to laypersons.

This said, does the Klein bottle offer real mathematical insights? The answer might be “no” - Norman Wildberger describes it as merely a “curiosity”, a simple step that one can briefly consider while looking at the mobius strip and the projective plane, which are the more important shapes. But Swindale’s paper shows that the Klein bottle is has a manifestation in our own brains, and our understanding of it can give us insights into the workings of sight, and may one day help us cure ailments of the visual system. So to me it seems clearly worthwhile for more people to understand this paper - rendering it interactively will make that possible.

Modelling proposal
As a subject of translation, Bret Victor took a famous paper on network theory, in which many statements were ripe for a treatment of this sort. Swindale’s paper is similarly ripe. Take this comment: “Try drawing several randomly oriented lines on a piece of paper and interpolating smoothly varying orientations between them, and you will often find that the smooth pattern is unavoidably interrupted by singularities”. Here Swindale *asks* us to actually interact with a certain simple mathematical pattern/rule - by modelling this part I wouldn’t even be changing the paper, simply making it so that the player doesn’t have to go and find a pen and paper (and also removing the risk that the reader will skip the exercise). (Douglas Hofstadter suggests interaction

like this in his famous book “Godel Escher Bach”)

There are other parts of the paper that would be susceptible to being modelled. Much of it is about describing the way a “cell receptive field” responds to a line at a certain orientation, moving in a certain direction. It would not be hard to have the agent control such a rod, and directly see the depency the paper describes.

What would happen is that there would be a monkey on the screen looking at the rod our agent controls. We would be able to see inside the monkey’s brain at their receptive field. As the agent moves the rod around, parts of the field would light up briefly and take on a color. Moving it more and more, the lit-up parts of the field would accumulate and become colored. When the player has moved it into every possible position, the whole receptive field would be colored - it would look like a picture that swindale gives in his article, a picture that was put together by scientists performing this experiment. With our version, the picture would be made by the player.

I will try to translate as much as I can into interaction, but some words will stay - this is why the EMPE will be useful. The structure of the paper will probably be kept as well - although this is not definite.

Weight
I expect to put maximal weight on the modelling rather than the write-up.

References:
-”Empirical Modelling for learning to understand numbers in non-decimal bases”
-Swindale’s “Looking into the Klein Bottle” (obviously)
-Weeks, J: “Torus and Klein Bottle games”
http://www.math.ntnu.no/~dundas/75060/TorusGames/TorusGames.html and “geometry games” http://www.geometrygames.org/TorusGames/
-Victor B: “Scientific communication as sequential art”
http://worrydream.com/#!/ScientificCommunicationAsSequentialArt
-Swindale NV: Elastic nets, travelling salesmen and cortical maps. Curr Biol 1992, 2:429–431.
-Blasdel GG: Orientation selectivity, preference, and continuity in monkey striate cortex. J Neurosci 1992, 12:3139–3161.
-Tanaka S: Topological analysis of point singularities in stimulus preference maps of the primary visual cortex. Proc R Soc Lond [Biol] 1995, 261:81–88
-Marcelja S: Mathematical description of the responses of simple cortical cells. J Opt Soc Am 1980, 70:1297–1300.