Integrating Decision Support for Complex Systems: with Applications to Food Security

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What I plan to discuss

- With Simon French developed Bayesian decision support systems for a decision centre addressing unfolding events after an **accidental release of radiation.**
- EPSRC funds new methodological development for **generally applicable integrating decision support tools.**
- Here examine some **formal challenges**, discuss when possible to build such a system & illustrate how this might work.
- Discuss **how such technology might be applied** to complex interdependent systems e.g. for decision support for addressing UK food poverty.

Large system decision support: general idea

- Feasible large systems need to **focus on job in hand** & then elicit only evidence informing its policy decisions.
- Bayesian paradigm $\Rightarrow$ **arguments of the utility function**.
- Need to calculate **expected utility scores of each potential policy**.
- Calculations need to compose into **a distributed probabilistic model**: see below.

Example

Nuclear (Health,Public Acceptability,Cost)

Example

Food Poverty (Health, Education, Social Unrest, Cost)

Requirements for large system decision support: Dynamics

DSS usually **dynamic**: periodic changes in:

- imperatives & horizons: expressed by adaptations in **utilities.**
- **structure**: overarching qualitative framework.
- through gradual aggregation of **experimental evidence & contextual knowledge**: new **experimental data &changing environments.**

& fast movement of developing crisis:

- **unfolding process:** as **observational time series** of actual process.
- immediate impact of **enacted policy**: **effect of controls.**

Framework of the intrinsic dynamics

$$\begin{tikzcd}
 & \textit{Past} & \textbf{Current} & \textit{Future} \\
\textbf{data } (\mathbf{x},\mathbf{y}) & \boxed{\text{experiments}} \arrow[dr] & \boxed{\text{observations}} \arrow[d] & \boxed{\text{utility criteria}} \arrow[d] \\
\textbf{crisis } \theta & \boxed{\text{conditions}} \arrow[r] & \boxed{\text{evaluation now}} \arrow[r] & \boxed{\text{final outcome}} \\
\textbf{acts d} & \boxed{\text{policy}} \arrow[ur] \arrow[r] & \boxed{\text{controls}} \arrow[u] \arrow[r] & \boxed{\text{controls}} \arrow[u]
\end{tikzcd}$$

- But "evaluation now" extremely **complex** involving **diverse domain experts.**
- Can't build single stand alone probabilistic system.
- Needs overarching structure to **integrate** diverse judgments in distributed systems!

A Challenge: Integrating Systems Together

Diverse components needs an **Integrating decision support system (IDSS)** to inform & then evaluate policies with:

- Agreed **overarching qualitative structure** to express common knowledge: Dynamic Bayesian Networks, MDMs, trees, dynamic emulators,...
- **Panels** of experts communicating **quantitative local domain knowledge**. Outputs then need to be knitted together to quantify IDSS.
- Quantifications **distributed** - so right panels donate appropriate inputs **autonomously**.
- the IDSS must be **transparent**, **make sense** & give integrated **evaluations**.

Two Running Examples: (informal schemata)

Established: Leonelli [RODOS IDSS real time countermeasure team]

$$\begin{tikzcd}
\boxed{\text{plant}} \arrow[r] & \boxed{\text{dispersion}} \arrow[r] \arrow[d] \arrow[dr] & \boxed{\text{food chain}} \arrow[d] & \boxed{\text{demography}} \arrow[d] \arrow[dl] \\
\textbf{Radiation Risk} & \boxed{\text{inhalation}} \arrow[r] & \boxed{\text{exposure}} \arrow[r] & \boxed{\text{health}}
\end{tikzcd}$$

Beginning: Barons [Poverty, IDSS for UK government]

$$\begin{tikzcd}
\boxed{\text{production}} \arrow[r] & \boxed{\text{supply chain}} \arrow[r] \arrow[dr] & \boxed{\text{shop?}} \arrow[d] \arrow[dr] & \boxed{\text{demography}} \arrow[d] \\
\textbf{Food Security} & & \boxed{\text{price}} \arrow[r] & \boxed{\text{health + educ.}}
\end{tikzcd}$$

Combining Expert judgments.

- (Nuclear experiences) Modules individual expert systems
- Panel delivers inputs needed by next panel.

Panels for accident

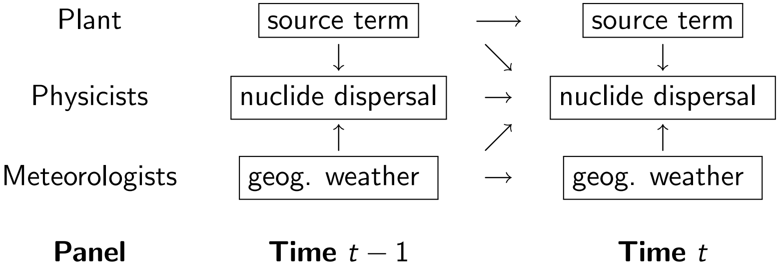
$$
\begin{tikzcd}
 & \boxed{\text{Demography panel}} \arrow[dr] & & \\
\boxed{\text{Plant panel}} \arrow[r] & \boxed{\text{Dispersal panel}} \arrow[r] \arrow[dr] & \boxed{\text{Health panel}} & \cdots \\
 & \boxed{\text{Weather panel}} \arrow[u] & \boxed{\text{Food panel}} \arrow[u] & \cdots
\end{tikzcd}
$$

So panels:

1. agree common structure - e.g. what might influence what.
2. deliver initial vector of predictions based on their best science - modifying judgments as crisis unfolds.

BN: template how to formally combine expert judgments.

Part of Dynamic Bayesian Network for Nuclear



- A **valid** dynamic BN: (high dim) nodes outputs -inputs parents!
- For BNs "global independence" across components $\Rightarrow$ **distributive**! So panels can stay autonomous!
- Implicitly also **causal**: see Pearl (2008)!

Structural Consensus: Common Knowledge

Assumption 1 Can achieve consensus on qualitative relationships between different components of problem.
Observation Easier to find *consensus* on **nature & structure of relationships** than on numerical **probs/utilities**.

- Must identify **domain expert panels** e.g. panels for forensic-scientists & police, court recorders, jurors.
- Ideally structure **causal** - invariant under control & experimentation.
- **Variety of possible structure**s e.g. trees (forensic), DBNs (ecology, nuclear), MDMs, (nuclear, food).
- Domains have varying complexity & quality of information.

Suprabayesian, consensus, common knowledge & trust.

- For decision centre to be rational must act like a single expected utility maximising decision maker - the *SupraBayesian (SB).*
- So must deliver (sufficients features of) a probability distribution for SB to calculate expected utility associated with each possible policy & choose highest scoring one.
- SB adopts *relevant panel's* probability judgements as her own & pastes these together.

Assumption 2 All agree to trust relevant panel to deliver appropriate **sample distribution family** of unfolding potential crisis.
Assumption 3 All agree to trust relevant panel to deliver appropriate **priors over their domain parameters** and adopt as their own.
Assumption 4 IDSS *adequate* to **identify expected utility maximising strategy** from delivered judgments.

Trusting Quantitative Judgments of other Expert Panels?

Panel G_i, $i = 1.2, \ldots, m$ delivers beliefs $\{\Pi_i(d) : d \in D\}$.to SB about distribution of their outputs given their inputs & each policy $d \in D$ .SB constructs evaluations needed to calculate expected utility for each $d \in D$.

Question Could SB ever legitimately achieve this consensus evaluation?

Let $I_0(d)$:information *common knowledge* to all panels, $I_{ij}(d)$ be information panel i brings to $\boldsymbol{\theta}_j$ $i, j = 1, 2, \ldots, m$ &

$$I^+(d) \triangleq \{I_{ij}(d) : 1 \leq i, j \leq m\}, I(d) \triangleq \{I_{jj}(d) : 1 \leq j \leq m\}$$

Definition

An IDSS is *delegable* if for any $d \in D$ there is a consensus that $\boldsymbol{\theta} \amalg I^+(d) | I_0(d), I(d)$

All useful information about parameters union of common knowledge + each individual panel's *specialist* info.

A necessary separation assumption?

[$I_0(d)$:information shared by all panels, $I_{ij}(d)$ information panel i brings to $\boldsymbol{\theta}_j$ $i, j = 1, 2, \ldots, m$]

Definition

An IDSS is *separately informed* if $\coprod_{j=1}^{m} (\boldsymbol{\theta}_j, I_{jj}(d)) | I_0(d)$

So for panel j parameters $\boldsymbol{\theta}_j$ & supporting info. $I_{jj}(d)$ from relevant panel j mutually independent given background info. everyone shares. ($\Rightarrow$ panel independence: $\amalg_{i=1}^{m} \boldsymbol{\theta}_i | I_0(d)$ under any policy $d \in D$: for BNs implied by almost universally assumed global independence of prior.

Note Assumptions certainly not automatic - but can check!.

Now able to use the mathematical property of conditional independence to prove the following.

A recent theorem (after Goldstein & O'Hagan)

Definition

An IDSS is *sound* if adequate & by adopting the structural consensus & panel independence, SB (& so all panel members) can faithfully adopt expected utility scores calculated from probs communicated by relevant panels of domain experts as their own.

Theorem

Barons, Leonelli & Smith(2014) An adequate,delegable & separately informed IDSS it is sound.

Note

1. Panels might only need to deliver a few conditional expectations of identified functions (not *full* distributions).
2. Even when conditions do not hold SB's probs are interpretable "Were there not this other information then.." Hold back other info as supplements.

Separable Likelihoods: key to distributivity

Panels able to **update their beliefs autonomously**:SB can input revised judgements $\{\Pi_i. : 1 \leq i \leq m\}$.after new data using same framework.

Definition

Data $\mathbf{x}$ with likelihood $l(\boldsymbol{\theta}|\mathbf{x},d)$,$d \in D$, is *panel separable* over $\boldsymbol{\theta}_i$, $i = 1, \ldots, m$ when

$$l(\boldsymbol{\theta}|\mathbf{x},d) = \prod_{i=1}^{m} l_i(\boldsymbol{\theta}_i|\mathbf{t}_i(\mathbf{x}), d)$$

where $l_i(\boldsymbol{\theta}_i|\mathbf{t}_i(\mathbf{x}))$ is fn. of $\boldsymbol{\theta}$ only through $\boldsymbol{\theta}_i$ and $\mathbf{t}_i(\mathbf{x})$ is a function of the data $\mathbf{x}$, $i = 1, 2, 3, \ldots, m$, for each $d \in D$.

Theorem

If all information conditional on common knowledge $I_0(d)$ is data giving rise to panel separable likelihoods then prior panel independence implies IDSS always separately informed & delegatable.

Examples of different structures & their Panel Independence

- BNs: Panel independence $\backsim$ global independence.
- Context specific or OOBNs. Single panel responsible for shared cpts.
- Chain graphs: One panel for each variable box conditional on parents.
- MDM structures (Queen & Smith,1993, Leonelli & Smith 2014a,2015): Panels donate distributions on dynamic regression states.
- CEG Smith(2010) SB believes panels probs for their parts of tree independent.

Examples of Unambiguous Priors.

Example

Forensic event tree. Panels allocated provision of distributions on uncertain edge probs out of particular situations in tree.

Example

Causal DBNs / MDMs. Single panels give beliefs on conditional probability table of allocated node in graph conditional on parents.

Example

Undirected graph & panels deliver a clique probability table. Not necessarily consistent since the distribution on probabilities in shared separator margins may not agree.

Example: Combining two panels' beliefs

Educational hrs. Y lost by vulnerable child. X nutritional balance.
Model:Panel 1 $(m_X \triangleq E(X), \sigma_X^2 \triangleq Var(X))$ Panel 2 $(\mu \triangleq E(\theta), \sigma^2 \triangleq Var(\theta), \tau^2 \triangleq Var(\varepsilon))$

$$Y|X,\theta = \theta X + \varepsilon$$

$\left(m_Y \triangleq E(Y), \sigma_Y^2 \triangleq Var(Y)\right)$. Under panel independence SB uses **tower rule**

$$\begin{aligned} m_Y &= \mu m_x \\ \sigma_Y^2 &= \left(\sigma^2 m_x^2 + \tau^2\right) + \sigma_X^2 \left(\mu^2 + \sigma^2\right) \end{aligned}$$

Now calculate utility (e.g. no. of children losing $\geq h$ hrs. of education)
Note Simple arithmetic to score different policies. Scales up!

General combination of several panels' beliefs

- With panel independence & likelihood separability **similar high dim. tensor algebraic relationships** apply.
- In **dynamic** settings typically **each panel donates** a finite number of judgements in terms of **moments** for each candidate policy.
- **Messages** needing to be donated depends on topology of overarching structure & form of utilities.
- **CK structure & utilities determine scoring formula** for each policy.
- Like **propagation algorithms** for BNs. Often quicker than calculations made by individual panels to deliver their domain information..

General theorems for message passing in big systems now completed - **see Leonelli and Smith (2015)**

Separable Likelihoods: Do these really apply??

Question: But are likelihoods typically separable?
Answer: Not always but surprisingly often!
Thus for example we have

Theorem

Barons, Leonelli & Smith (2015) When consensus that the quantitative causal structure is a (dynamic) causal BN or casual CEG or a causal MDM & parameters of different variables in an IDSS sound at any time t: then the IDSS remains sound under a likelihood composed of ancestral sampling experiments as well as observational sampling.

Note The key demands are that:

1. all agree common **causal structural framework**.
2. data is collected **ancestrally** (no hidden shared causes).

What we can do when likelihoods do not separate

In cases when all the available data is not of the right form we can either

1. Approximate as well as we can - using maths to examine the robustness of decisions under uncertainty.
2. Apply an *admissibility protocol* to determine what is let into IDDS ensuring it remains consensual & sound.

- e.g. *admissibility protocol* allows in only types of sample surveys, observational studies experiment, etc.
- Note that such protocols are used for admittance of evidence for medical and public health inference.

Note Information not admitted still useful e.g. for diagnostics.

Conclusions, Reflections & Future Research

- Benchmark *subjectivity* - best we can do with unambiguous information - often more helpful than "*objectivity*" for Bayesian Decision Support: directing science.
- Consensual *structural & causal* hypotheses central to IDSS! Are causal hypotheses so critical because they are natural building blocks of agreed rational evaluations?
- New issue : soundness. When can IDSS demonstrate this (at least approximately)? Panels composed appropriately, right quality of information?
- New issue: data admissibility - introducing information only when data not open to diverse interpretation (Cochraine) But now also links to feasibility of appropriate calculation even if interpretations cohere!
- Often need only small dim inputs - (extensions of BLEs -Goldstein and Wooff 2007). So support of big systems feasible!

What are specific challenges for Food Poverty Decision Support?

Thank you Thank you Thank you

THANK YOU FOR YOUR ATTENTION!!!

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Food Security Needs: How support is delivered now.

- *Current* situation/past situations presented to Government/ Local Government through graphs & maps.
- But *no annotated predictions* of impact on poor of future events.
- Or *impact of central government changes* in legislation or evaluations of *effectiveness of different implementations* of changes.

Plan Use their standard gui & Bayesian methods to support them.

$$
\begin{tikzcd}
\boxed{\text{production}} \arrow[r] & \boxed{\text{supply chain}} \arrow[r] \arrow[dr] & \boxed{\text{shop?}} \arrow[d] \arrow[dr] & \boxed{\text{demography}} \arrow[d] \\
 & & \boxed{\text{price}} \arrow[r] & \boxed{\text{health + educ.}}
\end{tikzcd}
$$

Beliefs and Facts: What goes into/is excluded from a system?

- Shared *beliefs* collective agrees reflect best (generally acceptable) available judgments about the global domain. Examples ci / causal/ functional relationships hardwired into system.
- Accepted *facts* Published data from well conducted experiments and sample surveys/events.

BUT most analyses implicitly or explicitly exclude certain *data*
Typical selection criteria in other contexts:

- *Compellingness* of the evidence (e.g.to user ÷ auditor/Cochraine).*Defensibility* of assumptions, *Wealth* of less ambiguous/less costly evidence.

SB updates *only* in the light of *admitted* experiments/surveys/observational studies. Cannot necessarily use *all* relevant information.

External Bayesianity

External Bayesianity (EB) if all *individually* update priors using experiment (common knowledge) - giving likelihood $l(\boldsymbol{\theta}|\mathbf{x})$ - this same as if all first combined beliefs into single panel density to accommodate their new information and then updated.
EB property characterises the *logarithmic pool*

$$\overline{\pi}(\boldsymbol{\theta}|\mathbf{w}) \propto \prod_{i=1}^{k} \pi_i^{w_i}(\boldsymbol{\theta})$$

where $\mathbf{w} = (w_1, \ldots, w_k)$ weights, reflecting credibility of different experts, sum to unity.
Collective appears Bayesian from outside irrespective of sampling and order of information. Consistent with the Strong Likelihood Principle. Preserves integrity of panel independence over time.

Recapping the Problem

- Collective agrees set of qualitative (e.g. conditional independence) assumptions about $\{Y_i : 1 \leq i \leq n\}$ conditional on $\boldsymbol{\theta} = (\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots \boldsymbol{\theta}_m)$ whatever $d \in D$.
- Let $\Pi = f(\Pi_1, \Pi_2, \ldots, \Pi_m)$ be the distributional statements about $\boldsymbol{\theta}$ available to the user. Panel beliefs $\{\Pi_j(d) : 1 \leq j \leq m, d \in D\}$ the *only* quantitative inputs to the collective beliefs $\Pi(d)$ about $\boldsymbol{\theta}$.

Note: not trivial that $\Pi(d)$ is function of $\Pi_j(d) : 1 \leq j \leq m$.
e.g distribution of parameters of $\mathbf{Y} = (Y_1, Y_2)$ is not fully recoverable from the two marginal densities $\pi_i(\boldsymbol{\theta}_i)$,provided by G_i, $i = 1, 2$ e.g. no covariance between Y_1 and Y_2 .

Example: decision support after a nuclear accident

Many panels of experts/statistical models in the system:

- Power station described by a Bayesian Network - **Panel** nuclear physicists, engineers and managers.
- Accidental release into the atmosphere or water supply the dangerous radiation will be distributed into the environment, **Panel** atmospheric physicists, hydrologist, local weather forecasters....
- Taking outputs of dispersion models and data on demography and implemented countermeasures predict exposure of humans animal and plants of the contaminant. **Panel** biologists Food scientists, local adminstrators, ..
- Taking outputs giving type and extent of exposure predict health consequences: **Panel** epidemiologists, medics, genetic researchers
- And so on ...

Big demands of the 21st Century more generally

- Complex *domain specific probabilistic expert systems* inform different parts of process.
- Cannot single probabilistic *composite*: too big! ever changing modules, only interested in certain outputs of these modules.
- So *Integrating Decision Support System (IDSS)* essential: pasting together the pertinent outputs of autonomous dynamic expert judgments to deliver benchmark numerical evaluation (with justification) of each candidate policy.
- Panels deliver *updated* judgments autonomously as a function of much in depth analysis.

Coherence in a decision centre over many systems

Principle 1

- An IDSS should be *coherent*.
- Coherence requires virtual responsible *SupraBayesian(SB)* to represent the centre.
- IDSS evaluate SB's *expected utility function*, for candidate unfoldings and policies.

Note Bet caller - regulators, stakeholders, users, other experts *actually there* to test out integrity."*coherent*" = no-one without domain knowledge can exploit SB's implied preferences over specific types of gambles.

Faithfulness for a decision centre over many systems

Principle 2 An IDSS should be *faithful:*

- IDSS to express broad qualitative consensus over qualitative features of problem.
- SB's *single probability* distribution over space needed to calculate expected utilities. *"best" most consensual/faithful/defensible probabilities*: (e.g. Smets,2005).
- SB should adopt beliefs of *relevant panel* of domain experts (coded with probs). IDSS *justifiable*: relevant domain experts to field regulator queries about faithfulness/plausibility.

Principle 3 An IDSS must be *feasible*, *transparent* & *fast*.

Example: Observables a pair of binary variables

- $\mathbf{R} = \mathbf{Y} \triangleq (Y_1, Y_2)$. Panel G_1 inputs about $\theta_1 \triangleq P(Y_1 = 1)$.
- Panel G_2, $\theta_{2,0} \triangleq P(Y_2 = 1 | Y_1 = 0)$ and $\theta_{2,1} \triangleq P(Y_2 = 0 | Y_1 = 1)$.
- Distribution of $\mathbf{R}$, $\overline{\boldsymbol{\theta}} \triangleq \left(\overline{\theta}_{00}, \overline{\theta}_{01}, \overline{\theta}_{10}, \overline{\theta}_{11}\right)$ given by the polynomials

$$\begin{aligned} \overline{\theta}_{00} &= (1-\theta_1)(1-\theta_{2,0}), \overline{\theta}_{01} = (1-\theta_1)\theta_{2,0}, \\ \overline{\theta}_{10} &= \theta_1(1-\theta_{2,1}), \overline{\theta}_{11} = \theta_1\theta_{2,1} \end{aligned}$$

- G_1 donates densities $\Pi_1 = \{\pi_1(\theta_1, d) : d \in D\}$.
- G_2 gives densities $\Pi_2 = \{(\pi_2(\theta_{2,0}, d), \pi_2(\theta_{2,1}, d)) : d \in D\}$.

Example: The Queen in Danger!!

Example

Panel G_1 domain is margin of binary Y_1 - $\theta_1 = P(Y_1 = 1)$ (Y_1 queen comes in contact with a particular virus). Panel G_2 domain margin of binary Y_2, $\theta_2 = P(Y_2 = 1)$. (Y_2 when queen exposed suffers an adverse reaction).G_1 says $\theta_1 \backsim Be(\alpha_1, \beta_1)$ and G_2 says $\theta_2 \backsim Be(\alpha_2, \beta_2)$. No decision will affect these distributions. Agreed structural information is $Y_1 \amalg Y_2 | (\theta_1, \theta_2)$,

Case1: User has a separable utility

$$u_1(y_1, y_2, d_1, d_2) = a + b_1(d_1)y_1 + b_2(d_2)y_2$$

G_i needs only supply $\mu_i \triangleq \mathbb{E}(\theta_i) = \alpha_i(\alpha_i + \beta_i)^{-1}$, $i = 1, 2$. No need to be concerned about dependency.

Example

Case 2

- Interest is only in $W \triangleq Y_1 Y_2$ (whether queen is infected). So

$$u_2(w, d_{12}) = a + b_{12}(d_{12})w$$

  where $\mathbb{E}(W) = \mathbb{E}(\theta_1\theta_2)$.
- If collective assumes *global independence* $\Rightarrow$ distribution $\theta_1\theta_2$ is well defined.
- Then $\mathbb{E}(\theta_1\theta_2) = \mu_1\mu_2$ - so G_i needs only supply μ_i, $i = 1, 2$.
- However Global independence not *only* choice!

An Alternative Prior

Suppose $\alpha_1 + \beta_1 = \alpha_2 + \beta_2 \triangleq \sigma$. Panels donate (μ_1, μ_2, σ), where $\sigma = \gamma_{00} + \gamma_{10} + \gamma_{10} + \gamma_{11}$, $\pi \backsim Di(\gamma_{00}, \gamma_{10}, \gamma_{01}, \gamma_{11})$,

$$\begin{aligned} \alpha_1 &= \gamma_{10} + \gamma_{11}, \beta_1 = \gamma_{00} + \gamma_{01} \\ \alpha_2 &= \gamma_{01} + \gamma_{11}, \beta_2 = \gamma_{00} + \gamma_{10} \end{aligned}$$

- This collective prior consistent with panel margins but *not* global independence.
- Collective parameters $(\mu_1, \mu_2, \sigma, \rho)$, $\rho \triangleq \sigma^{-2} (\gamma_{11}\gamma_{00} - \gamma_{10}\gamma_{011})$
- Collective's $\mathbb{E}(\theta_1\theta_2) = \gamma_{11}\sigma^{-1} = \mu_1\mu_2 + \rho \neq \mu_1\mu_2$ unless $\rho = 0$.
- So $\mathbb{E}(\theta_1\theta_2)$ is not identified from inputs.

Now assume global independence

- Panels supplement judgments by independently randomly sampling.
- Collective needs only two updated posterior means $\mu_i^*.i = 1, 2$.
- So all data of this form allows distributed inference.

Problem 1: Global independence critical for distributivity. Even in Case 1 when only individuals margins of θ_1, θ_2 needed if collective did not believe $\theta_1 \amalg \theta_2$ it learns about θ_2 - through G_2's experiments will modify distribution of θ_1.

Problem 2 :Even if global independence is justified, assuming experiments of two panels never mutually informative also critical.

Example of data set: table of counts below (Case 2)

$Y_1 \backslash Y_2$	0	1		
0	5	45	50	$n - x_1$
1	45	5	50	x_1
	50	50	100	
	$n - x_2$	x_2		

- Each panel updates using only their respective margin (with weak priors) $\Rightarrow \mu_i^* \simeq 0.5$, $i = 1, 2 \Rightarrow \mathbb{E}(\theta_1\theta_2)$ to be approximately 0.25.
- OTOH with whole info $\mathbb{E}(\theta_1\theta_2) \simeq 0.05$.i.e. five times smaller!

(Note structural independence assumption: $Y_2 \amalg Y_1 | (\theta_1, \theta_2)$ looks dubious)

Non-compatible sampling (either case)

Binomial sample 100 units like queen, *acquiring* disease, so prob $\phi \triangleq P(W = 1)$. See 5 infected.

- In either case collective easily incorporates this information directly: e.g. giving ϕ a beta prior and treating data as random sample. However, without further assumptions such data impossible for G_i to *individually* update $\pi_i(\theta_i)$.
- Ignore this information $\div$ uniform priors $\Rightarrow$ vastly overestimate the probability.
- So $\pi(\theta_1\theta_2)$ no longer decomposes into a G_1 density and a G_2 density: Sampling induces dependence.

So problems quite involved! Distributed panels need to reflect form of typical input data as well as areas of expertise.