Monetary Risk and Prudence in Pension Fund Valuation

Saul Jacka and Elena Hernandez

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1. Introduction

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Prudence (Latin: *prudentia* meaning "seeing ahead, sagacity")

- Ability to govern and discipline oneself by reason; one of the four Cardinal virtues;
- The virtue is ability to judge between virtuous and vicious actions,. . . *with regard to appropriate actions at a given time and place*;
- In modern English, virtually synonymous with cautiousness, reluctance to take risks, which *can become the vice of cowardice*;
- Prudence has a directive capacity with regard to the other virtues-“right reason applied to action”. [Pieper 1966]
- Requires you to respect reality– utterly rejects the “I meant well” excuse’;
- Conversely commends taking a good bet even when outcome is uncertain.

Pension fund valuation

- 'Valuation' is shorthand (now) for liability valuation.
- Restrict attention to Defined Benefit (DB) Pension Schemes.
- The introduction of FRS17 (now consolidated and amended in FRS102 s28) mandated valuation of assets at market rates[1]. The Projected Unit Credit method is also mandated.

[1] Prior to that, actuarial valuation of assets (based on projected income) was commonly practised.

Mortality

• Systematic study of mortality (and epidemiology) due to Graunt's collected *Observations on ...the Bills of Mortality* (1662).

• (1762) Equitable Life founded.

- Idea due to James Dodson (1705–1757), who had been refused life assurance by Amicable Life because he was over 45.
- Equitable Life went on to "pioneer age and sex-based premiums based on mortality rates" although the idea goes back to Jan de Witt[2] for pricing life annuities! [3]

[2] *The Worth of Life Annuities Compared to Redemption Bonds (1671)*.

[3] In the mid-seventeenth century, the standard practice dictated selling annuities at one price regardless of the age of the nominee.

Interest rates

In 1719 base rate/ bank rate rose to 5%. Stayed at that level for *103 years*. From 1757, yields on Consols were 3% for 130 years.

- believe this led valuation methods to focus entirely on mortality as key ingredient.
- “actuarial principle of prudence” was implemented by taking a mortality rate/table which allowed for some negative experience.

Valuation of Equitable Life

- Expected value of discounted payments with respect to conservative mortality rates.
- Non-random discount rate.
- Investment made in bonds/gilts so *valuation method matched the investment policy*.

Pension funds

- The first defined benefit pension *funds* in UK seem to have started around 1850. These were invested in gilts and used a 4% discount rate for valuation.
- The Imperial Tobacco Pension Fund pioneered the shift of pension fund investment into Equities in the UK:

  "*...one factor over which we can exercise some influence, and that is the rate of interest earned...one can do something to ensure that the actuary's estimates are fulfilled, or more than fulfilled*" [Ross Goobey 1956]
- Pension funds are valued using some pessimistic ("prudent") assumptions about mortality and investment returns.

Remark

Pension funds often insure death benefits. Thus they value liabilities by being pessimistic about how long people will live and then insure against their death (being pessimistic about how short a time they will live).

- Conflict about pessimism: annuities and assured lives have opposite definitions of prudent/pessimistic assumptions for mortality.
- DB scheme usually has death benefits as well as pensions. Prudence might dictate a pessimistic assumption about mortality-but what is it?

Remark

Most finance academics believe in the absence of arbitrage opportunities.

• In strict theoretical terms: an arbitrage is an opportunity for riskless profit from trading.

• Fundamental theorem states that absence of arbitrage is equivalent to existence of a risk-neutral measure.

- Dynamic and static equilibria are what nearly all economics is about and these don't apply in the short term and may only apply to risk neutral measures.
- Absence of arbitrage in a market in dynamic equilibrium is necessary but this ignores shorter term but significant dynamics.
- In addition, the argument only really tells us about the existence of risk-neutral measures and gives us little to no information about “real-world” measures.

Example

- 30 November 2018 Kier Group launch a £264m, 33 for 50 rights issue, priced at 409p.
- Kier shares drop almost 33%, from 752p to 482p.
- 3 December: Kier shares close at 455p.
- 5 December: share price dropped below rights price: cheaper to buy shares in open market than to buy rights issue.

• 10 December: shares subject to renewed 'shorting' (two hedge funds, BlackRock and Marshall Wace, were short 6.9% of Kier's shares). Shares close at 376.4p.

• 19 December: only 38% of rights taken up, leaving the lenders facing losses (even after fees).

• 20 December: shares fell by further 13% in early trading to 335p (underwriters sold at a loss).

• 11 January 2019: *shares trading at 529p* when some shareholders were reportedly seeking changes in Kier's leadership team.

This looks very close to an arbitrage opportunity to me. Certainly not clear it arises from a dynamic equilibrium.

Assertions

1. *Lower volatility assets such as gilts are less risky.*
2. *Gilts are more highly correlated with pension fund liabilities, so as assets they do a better job of asset-liability matching.*
3. *The yield curve is the market expectation of future short rates of interest.*
4. *Value at Risk (V@R) is a good monetary measure of risk.*
5. *Average Value at Risk is a coherent risk measure so it addresses the shortcomings of V@R.*

Valuation and prudence

Assume that for puproses of following examples:

- We value a liability by taking expectation with mean asset returns adjusted to the 10-year 33rd percentile of the asset return distribution.
- I is the investment in year t.
- R_t^I is the annual log-return of investment I in year t.
- R^I is the total return over a relevant period.
- F^I denotes the initial valuation of the liability, using the discount corresponding to I.

1. Lower volatility assets such as gilts are less risky

'Gilts have lower returns so funding levels are higher and have lower volatility so they are more likely to fund the liabilities. What's not to like?'

Example

Suppose:

- Pension liabilities. Lump sum of £10,000,000 after 10 years.
- 33rd percentile estimate is that
  - a fund invested in gilts will earn (each year) independent log-returns R^G normally distributed, N(.02,.004), and
  - a fund invested in equities will have independent returns R^E normally distributed, N(.04,.01).

So prudent 10-year log-return assumption will be R^I, which is N(0.2,0.04) for gilts and N(0.4, 0.1) for equities.

- Liability valuations (and initial funds). Using that $F^I = 10,000,000\mathbb{E}[e^{-R^I}]$ gives

$$F^G = 8,352,702$$

and

$$F^E = 7,046,881.$$

The gilt fund requires an additional 18.5% funding.

- Probability that the funds fail to meet the final liability:

$$\mathbb{P}(F^G e^{R^G} < 10,000,000) = 29.7\%$$

whereas

$$\mathbb{P}(F^E e^{R^E} < 10,000,000) = 27.6\%.$$

Even with a significantly smaller fund, the *more volatile* equity investment has a higher chance of meeting the liability.

2. Gilts are more highly correlated with pension fund liabilities so as assets they do a better job of asset-liability matching.

'Gilts have a structure of regular, predictable payments similar to pension fund liabilities so they should be better correlated with them.'

Example

• Same liability: £10,000,000 at the end of 10 years.
• The fund will earn independent log-returns each year of R^I_t (if invested in asset I).

- The liability valuation at time t, L_t^l, is given by

$$L_t^l = \mathbb{E}[10,000,000e^{-(R_{t+1}^l + \ldots + R_{10}^l)}|\mathcal{F}_t],$$

where $\mathcal{F}_t$ stands for the information accrued by time t.

- Since returns are independent, we obtain

$$L_t^l = \mathbb{E}[10,000,000e^{-(R_{t+1}^l + \ldots + R_{10}^l)}].$$

Consequently, since L_t^l is deterministic, it is *uncorrelated with anything*, and in particular with the fund value at any time or the investment returns.

'OK, so that version was "obviously silly" and we need to have more realistic liabilities.'

Example

- Liabilities: £1,000,000 p.a. for 10 years.
- Asset returns are independent (over time).

The (residual) liability valuation in year t is

$$1,000,000\mathbb{E}[e^{-R^l_{t+1}} + e^{-(R^l_{t+1}+R^l_{t+2})} + \ldots + e^{-(R^l_{t+1}+\ldots+R^l_{10})}|\mathcal{F}_t]$$

Since returns are independent, the (residual) liability valuation is deterministic and so uncorrelated with anything else.

‘The problem is that we are assuming that asset returns in successive years are independent which is *clearly unrealistic*’.

• FTSE All Share, FTSE 100 and long-dated Gilt returns, all show negative auto-correlation (successive returns are negatively correlated).

Example

- Assume log-returns are N(.04,.0009).
- Negative correlation between successive asset returns[4], say

$$Corr(R_t^I, R_s^I) = (-\alpha)^{|t-s|}.$$

- Take $T = 10$ $\alpha = 0.6$ and $t = 5$.

Net fund value at time t is

$$F_t^I = F_0^I e^{R_1^I + \ldots + R_t^I} - (1,000,000 e^{R_2^I + \ldots + R_t^I} + \ldots + 1,000,000).$$

and

$$Corr(F_5^I, L_5^I) = 18.3\%$$

[4] the log returns form an AR1 process

*N*otice, however, that actual liabilities are entirely independent of asset returns.

- Correlation is an *artifact of the valuation method*.
- So, for example if we valued using equities but funded using gilts then (assuming independence between gilt and equity returns) the correlation would disappear.

'OK, but 'actually what we care about is that the funding deficit (or surplus) is correlated with the investment returns'.

Example

D_t^l, the calculated deficit at time t is:

$$D_t^l = L_t^l - F_t^l.$$

It follows that the co-variance is

$$Cov(D_t^l, F_t^l) = Cov(L_t^l - F_t^l, F_t^l) = Cov(L_t^l, F_t^l) - Var(F_t^l).$$

• So liabilities and fund value uncorrelated $\Rightarrow$ valuation deficit and fund value *are* negatively correlated. It tells us nothing about what the best matching investments are.

'Right. Lets assume that (some) assets and liabilities are correlated.'

Example

• Liabilities: £1bn p.a. *index linked to CPI* for 50 years.

• Assets are either index linked gilts (ILGs), which are well-correlated with CPI, or equities.

• Real returns (i.e. discounted by CPI) are assumed to be as follows:

- ILGs: R_t^G assumed iid with risk-adjusted mean[5] $\tilde{\mu}^G = -1\%$ and variance $(\sigma^G)^2 = .00001$ (s.d. 0.316%).
- Equities: R_t^E assumed i.i.d. with risk-adjusted mean $\tilde{\mu}^G = 0.5\%$ and variance $(\sigma^G)^2 = .0009$ (s.d of 3% or roughly 10 times that of the ILGs)

[5] adjusted to the 10-year 33rd percentile for prudence

Equity valuation is £45.21 bn; Index-linked Gilt valuation is £64.89bn;

Equity shortfall probability is 19.2%; ILG shortfall probability is 19.6% [6]

- The two valuations/investment methods have very similar shortfall probabilities but wildly differing costs.
- To reduce to *a 0.1% chance of a shortfall* need to increase equity investment/fund to £55.86bn (still *£9bn/14% less than the ILG valuation*).

[6] valuations and shortfall probabilities are based on a continuous time approximation

3. The yield curve is the market expectation of future rates of interest

• Not clear why current yield curve should make predictions about future interest rates.

• In [Jacka et al. 2005] we described *all* risk-neutral models for the term-structure of interest rates. The only ones where forward rates are unbiased predictors of future rates are non-random models.

• Current supply and future uncertainty are relevant economic factors. The Pensions Regulator encouraging "low-risk investment" gives strong push to invest in gilts whatever their perceived characteristics.

Schroeders (2017) analysed demand for ILGs:

• “Over 15-year index-linked gilt market was valued at . . . £338 billion in June 2016.

• Pension scheme holdings amounted to over 80% of the total.
– they dominate the market.

• As the motivation for buying is grounded in risk mitigation, long-dated index-linked gilt yields are unlikely to settle at a level consistent with normal economic fundamentals.

• Prices are more likely to be set by the trade-off between the supply of index-linked gilts and the demand created by pension funds seeking hedging assets.”

• Economists might anticipate a reduction in demand due to substitution of assets. Demand, however, seems driven by actuarial fund-matching recommendations and doesn't seem to be going anywhere. *So prices/yields are not being driven at all by views of future (real) interest rates*.

Turning to conventional gilts/bonds:

• "Situation is not quite as stark in the fixed-interest gilt market, but here too pension funds are a key investor, wielding significant influence alongside insurance companies."

• Under QE, Bank of England (BoE) so far purchased c £480bn of long-dated gilts and AAA-bonds. In same period the Fed has purchased c $2tn of Treasury Bonds.

- Stated purpose is to lower long-term interest rates and to increase investment in other assets.

- There is a strong, although contested, argument that this has strongly distorted the market's 'statement' of the time value of money.

- Arguing otherwise is to discount the idea that monetary policy has any effect.

- Taper Tantrum indicated even when no intervention, rolling back QE would lead to sharp rise in interest rates.

- Additionally low long-term yields in 'safe' assets are not an argument that more volatile securities should necessarily have lower mean returns. Price of risk (Sharpe ratio) not constant and can argue that current assumptions about low equity-type yields ignore this effect.

Coherent risk measures

A monetary measure of risk, ρ, is a measure of how much money would want to reserve/be paid to take on a (suitably discounted) liability L.

Minimal desirable properties for a coherent measure:

- (real) *cash-invariance*: $\rho(L + c) = \rho(L) + c$ for c a constant.
- *positive homogeneity*: $\rho(\lambda L) = \lambda\rho(L)$.
- *sub-additivity*: $\rho(L + L') \leq \rho(L) + \rho(L')$.

4. Value at Risk (V@R) is a good monetary measure of risk

- Define $\text{V@R}_p(L)$ (L_p =for short) as p^{th} upper percentile for L so $\mathbb{P}(L > L_p) = p$.
- V@R is not sub-additive.
- If risk measure is used for valuation or reserving but not sub-additive then would be tempted to subdivide in order to reduce liability valuation.

Example

• Can trade 3-month digital option on a stock which pays £10,000 if and only if stock price S_3 is in range $[a, b]$.
• Sell 25 of these, $D_1, \ldots, D_{25}$,with ranges $[a_i, a_{i+1}]$ $i = 1, \ldots, 25$ chosen so that $\mathbb{P}(S_3 \text{ lies in } [a_i, a_{i+1}]) = 4\%$.
• $\text{V@R}_{5\%}(D_i) = 0$ for each i.

Examples

• Consider a life assurance company providing annuities and life assurance; pessimistic assumptions require different mortality tables.

Two (or more) probability measures for beneficiaries' mortality and we calculate costs using both and take the worst answer.

• Continuous Mortality Investigation of the IFoA produces several life tables and gives different rates for annuitants and for assured lives.

• For large companies Strong Law of Large Numbers means essentially no risk due to the random nature of mortality –uncertainty is about the rates (the corresponding probability measures).

Coherent risk measures (CRMs)

• Introduced in [Atzner, Delbaen, Eber and Heath (1999)].

• Key example based on Chicago Mercantile Exchange's margin requirements.

• Update to Basel III Accords[7] mandates use of Average Value at Risk (a coherent risk measure unlike V@R) for reserving risk-capital for certain derivatives-based liabilities [Basel Committee on Banking Supervision, 2013].

[7] Commonly referred to as Basel 3.5

• One can define CRMs in terms of probabilistic scenario analysis: every CRM ρ can be written as

$$\rho(L) = \max_{\mathbb{Q} \in \mathcal{Q}} \mathbb{E}_{\mathbb{Q}}[L]$$

for some collection of probability measures $\mathcal{Q}$.

• We can summarise ρ as 'take the worst expectation of L under the p.m.s $\mathcal{Q}$'. Pricing life assurance products (with a fixed discount rate) exactly corresponds to this.

5. Average Value at Risk (AV@R) is a coherent risk measure so it addresses the shortcomings of V@R

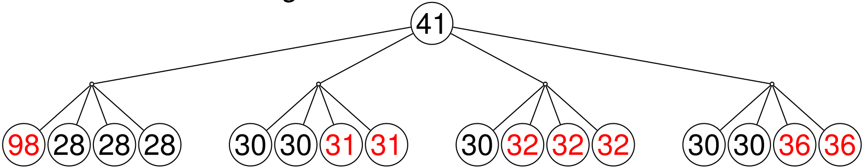
AV@R works very like V@R but then takes expectations so

$$\text{AV@R}_{50}(L) = \mathbb{E}[L|L > L_{50\%}]$$

Example

Consider successive triennial valuations over (for simplicity) a 6 year period and suppose we are using a 50% threshold for AV@R.
We subdivide each successive outcome into 4 so we get 16 possibilities (see Fig 1) which are equally likely:

Fig 1: Valuation outcomes



• AV@R$_{50}$ takes the worst 50% of outcomes and averages them giving $\frac{98+2*31+3*32+2*36}{8} = 41$.

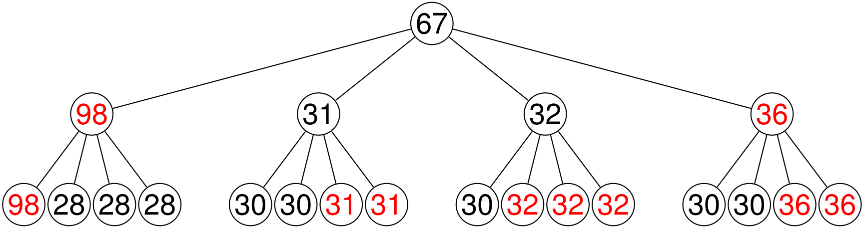
• Obtained by putting weights/probabilities of $\frac{1}{8}$ on 8 of 16 paths in the tree (including the one with outcome 98).

• This pricing measure $\mathbb{Q}$ puts mass $\frac{1}{8}$ on the first path and zero on next three.

• Under $\mathbb{Q}$ if we take the first branch by the first valuation we are “certain” to reach a final liability of 98!

Using corresponding conditional probabilities from AV@R_{50} for each branch we get Figure 2:

Fig 2: Iterated valuation/AV@R_{50} outcomes



AV@R has a serious consistency problem.

Example

- Liability is £10,000,000 after 15 years
- Asset returns are cyclical: 9 good (i.e. high mean) years followed by 9 bad in every cycle.
- Not sure where we are in the cycle (since asset returns are variable).
- It's clear we should value assuming 9 bad years first.
- Same will be true at next valuation... Thus each valuation will be performed assuming that the "worst is yet to come".

Remark

Average Value at Risk is not, in general, time-consistent*.*

- Potential severe problems attendant on the failure of time-consistency: insurance companies will release capital not required for reserves in the form of dividends- if this is too much then there is an attendant insolvency risk.
- Pension funds will have recourse to unanticipated additional contributions from the sponsor to fund a shortfall while banks will face regulatory sanctions if they fail to meet their capital adequacy requirements.
- Conversely the adoption of a time-inconsistent measure will usually *over-reserve on repeated application* and may have unexpected properties.

What risk measures?

As we pointed out in [Jacka at al. (2019)], requirement of time-consistency overemphasises the unit of account. We showed how to check whether a CRM could be reserved for *using multiple assets (rather than just the unit of account)* in a time-consistent fashion (called this **V**-time consistency).

Have a result which shows how to find the smallest risk measure σ which dominates a given CRM ρ and which is **V**-time consistent i.e. which can be reserved for using the assets whose terminal prices are given by vector **V**.

- Turns out that a CRM is **V**-time-consistent if there is a sequence of (random, convex) sets K_t such that the pricing measures $\mathcal{Q}$ consist of all probability measures $\mathbb{Q}$ for which $\mathbb{E}_{\mathbb{Q}}[\mathbf{V}|\mathcal{F}_t] \in K_t$ for each t.
- Tempting to start with some well-established CRM, ρ, (such as AV@R), a list of assets **V** and use theorem above to find a CRM which is **V**-time-consistent.
- Likely to produce a very pessimistic valuation mechanism and hard to implement[8].

[8] Consider for example, the superhedging price for a contingent claim in an incomplete frictionless market. In general, this is unacceptably large

It may be much better to specify asset classes and generic liabilities and then include them all in **V** (we can include some product terms to allow for fixed ranges of covariance values) then specify the sets K_t based on other considerations (such as those we have seen earlier).

Example. Take as unit of account £1 inflated by some measure of inflation. Suppose there is a range of assets to invest in-generically I. Then the I-based valuation is given by

$$F^I = \rho^I(L/I_T) \stackrel{def}{=} min\{c : \; \rho(L - cI_T) \leq 0\},$$

so that F^I is the minimum units of I (assumed to have initial price £1) such that the risk of our liability less our assets is acceptable (negative). Of course ρ is the risk measure specified by the K_t.

Theorem (Change of numéraire)

ρ^I is also a risk measure and it is specified by a new collection of convex sets K_t^I and the discounted asset class $\mathbf{V}^I = \frac{1}{I_T}\mathbf{V}$, so that our asset-dependent valuation is $\mathbf{V}^I$-time consistent.

- Procedure relatively easy to operationalise. For example, we could follow the approach using cautious percentile adjustments to mean returns to get a class of sets K_t for the vector $\mathbf{V} = (1, I_T^1, \ldots, I_T^n, L)$.
- Each such valuation is a 'standard actuarial valuation': it is the expected discounted cost of the liabilities under some prudent probability measure $\mathbb{Q}$.

1. As the examples in section 3 show, prudence can become imprudent: caution is often incautious and risk-aversion can increase risk and destroy DB pension schemes.
2. We have briefly sketched how to generate a time-consistent valuation process within the traditional valuation approach.
3. We have not modelled uncertainty about mortality (or other demographic factors) but in a coherent valuation this needs to be modelled in the same way as financial uncertainty.
4. It behoves everyone involved to respect: reality, economic and probability models, and their limitations.

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