Information Aggregation and Optimal Structure of the Executive

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We model two aspects of executives in parliamentary democracies: Decision-making authority is assigned to individuals, and private information is aggregated through communication. When information is relevant to all policies and communication is private, all decisions should be centralized to a single politician. A government that holds cabinet meetings, where information is made available to all decision makers, outperforms one where communication is private: A multimember cabinet can be optimal; it need not be single peaked around the most moderate politician or ideologically connected. Centralization is nonmonotonic in the degree of ideological divergence. In a large cabinet, all power should be given to the most moderate politician. Even when uncertainty is policy specific and a single politician is informed on each policy, power should never be fully decentralized. Our model provides a justification for centralized authority and cabinet meetings that enhance the quality of policy.

he relationship between the president, Congress, and its committees has been the subject of numerous theoretical studies, but the allocation of decision-making authority in a parliamentary democracy is less well understood. This might reflect the fact that such authority is not clearly codified in these systems. For example, central governance in the United Kingdom is underpinned by several piecemeal and unwritten conventions: The role of the prime minister and cabinet have no constitutional status, and both arose due to historical circumstance, practice, and convention. Even when the role of those with executive authority is codified—as in Germany, where Article 65 of the Basic Law defines the status of the chancellor, cabinet, and ministers—the constitution is typically silent about how the executive should be structured and the relations between its decision makers.

In the absence of constitutional guidelines that define a clear game form, we propose a new model that incorporates common and distinguishing features of executive governance in parliamentary democracies. The first is that decision-making authority over different policies is assigned to individual ministers who head state departments. The second is the existence of a cabinet: The set of ministers who hold executive office meet at a designated time and place where policy-relevant information, obtained through discussion and debate both outside and inside the chamber, is shared. Though we restrict our model to the set of institutions that allow for assignment of authority to individuals with collective deliberation over outcomes, we nevertheless address two core aspects of optimal executive structure in parliamentary democracies.

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First, and central to our investigation, is the role of the cabinet. It has been studied historically as an efficient means of rationalizing parliamentary activity (Cox 1987), and more formally as a system of incentives operated by a prime minister (Dewan and Myatt 2007, 2010; Indridason and Kam 2008). To our knowledge, however, no model exists that assesses the role of cabinet meetings. There are several practical justifications for such meetings: They are necessary for a collective decision-making body that votes, though few real-world cabinets have explicit voting procedures; they are one of several bodies that help coordinate executive activities;1 and they enable collective responsibility, by which the policies implemented by ministers are those of the government and have the support of all ministers. We assess the impact of cabinet meetings on the quality of policies that are implemented by the executive. A key principle of cabinet government is that significant policy issues that fall within the remit of a minister's portfolio should be discussed in cabinet. We contrast the quality of policy outcomes obtained when this principle is upheld with that when cabinet meetings are replaced by informal modes of communication, such as private conversations between policy makers and politicians.

Second, we address the extent to which power in cabinet should be centralized. In the United Kingdom, for example, a long-standing controversy exists "about whether monocratic control is exercised by the premier," reflecting "normative anxieties about Britain's unbalanced constitution" (Dunleavy and Rhodes 1990, 4). Similarly, in Germany, key constitutional principles that guide the allocation of authority often conflict with one another, so the "question of whether an issue falls under a departmental minister's competence or the Chancellor's right to determine the government's policy is sometimes ambiguous" (Saalfeld 2000, 51). In the absence of clear constitutional guidelines, we study how the allocation of power affects the quality of the final decisions implemented.

We relate this quality to the amount of policy-relevant information that decision makers have at their disposal. As in classic models of information going back to Condorcet, and more recently Feddersen and Pesendorfer (1996), in our model, there is uncertainty over the best policy to pursue. This is related to corresponding uncertainty about underlying fundamentals; different prognoses about economic growth might affect the optimal policies on public spending, defense, and immigration, for example. We assume that information about such fundamentals is dispersed among the set of politicians who are the players in our game. Differences in a politician's private information might reflect

differences in his information sources: A politician forms a viewpoint based on his local understanding, research, or the views of interest groups. We impose a coarse information structure that arguably provides a more accurate depiction of the information obtained from such sources. For example, a politician may observe whether a national recession has had an impact on his constituency, which impacts his preferences over economic and social policy.

Specifically, we develop a game-theoretic model where there are a set of politicians and issues to be decided. The game has three stages: First, politicians receive private information that is relevant either to all policies or to a subset of them; next, politicians use simultaneous cheap talk messages to convey their information to ministers, who then, finally, implement policies. Politicians—we might think of these as members of a governing majority—share a common goal of implementing well-informed policies. But they hold different beliefs as to the best policy to pursue. These differences stem from the variation in their information sources as well as idiosyncratic biases that reflect their worldview, or those of their constituents, and can prohibit the truthful revelation of private information to a decision maker. We ask whether variation in the executive structure how centralized it is, who has authority, and whether cabinet meetings are held-impacts on the strategic communication of this policy-relevant information.

Communication may be affected by the way authority is allocated; a politician may be truthful when the issue is decided by one minister, while not doing so when it is decided by another. It may also be that a politician is truthful when a minister has control over one issue but not when she has more authority. To explore this issue, and with the restriction that the power to finally determine each policy is in the hands of a single minister, we allow for all allocations: At one extreme, all politicians could share authority; at the other, all power could be centralized to a single minister; alternatively, power might be shared, either equally or unequally, between a cabinet of ministers.²

The structure of communication can also have consequences. We assume that at cabinet meetings, any information available to one minister, and upon which he consequently bases his decision, is made available to all ministers. This contrasts with a situation where communication with ministers is private and creates different incentives for strategic communication. Notably,

¹(See, for example, Saalfeld 2000, 60–63).

²In the United Kingdom, for example, decision-making authority has been centralized in a cabinet dominated by the prime minister since the late 19th century, whereas during Parliament's previous "golden age," power was more equally dispersed among individual members of Parliament (see Cox 1987 for a discussion of the emergence of centralized authority in Victorian England).

without cabinet meetings, a politician could convey a different message to each decision maker, whereas this is not possible with them. A politician may then choose to be truthful only when talking in confidence with a minister. Or by the same token, truthful communication may be made possible by the fact that information is shared. We contrast a world with cabinet meetings to one where communication between a minister and a politician takes place only in private.

We first show that under all circumstances of our model (i.e., any allocation of authority, communication structure, and whether a minister's private information is specific to all policies or a subset of them), ex ante welfare depends on two features: the moderation of decision makers that minimizes aggregate ideological loss, and their information that reduces the aggregate residual variance of decisions implemented. Our first main result shows that in the absence of cabinet meetings and when politicians private information is relevant to the entire set of policies to be implemented, the optimal assignment involves fully centralized power exercised by a unique individual. With cabinet meetings, it may instead prove optimal for decision-making authority to be shared between ministers. A politician may be unwilling to communicate truthfully to a single leader who is ideologically distant, whereas she is truthful when power is shared with another cabinet member whose ideology is intermediate. This apparently innocuous observation leads to a powerful normative result: Cabinet meetings outperform private communication as a form of information aggregation and so lead to more informed policies being implemented. This central result holds whether private conversations can be held outside of cabinet or not.

Next, we characterize the optimal degree of centralization in a cabinet and show, surprisingly, that it is non-monotonic in the ideological divergence between politicians. When it is small, ideological disagreement does not prohibit information aggregation and so it is better that a moderate executive leader emerges. When it is large, politicians are unable to communicate truthfully with one another, and so full centralization to a moderate leader is desirable. For intermediate values, however, information aggregation is sensitive to the allocation of authority. Then it is better that power is shared in a multimember cabinet.

Analyzing the allocation of authority further, we find that such a cabinet need not be ideologically connected: When two politicians with different ideology share power, there may be a politician with intermediate ideology who has none. The finding that the optimal allocation of authority may involve such "holes" contrasts with the connected-winning coalitions in Axelrod (1970). A similar prediction arises in models

of complete information where bargaining takes place over policies and portfolio allocation (Austen-Smith and Banks 1988) and so our results can be seen as providing a corresponding and novel information aggregation rationale for nonconnected executive coalitions.

We then consider the limit case of a large cabinet and show that all decision-making authority should be concentrated to politicians who are ideologically close to the most moderate one. In the online supporting information, we perform numerical simulations—randomly drawing ideology profiles and calculating the optimal policy assignment—of an intermediately sized parliament. We find that fully centralized authority is fairly frequent and that, when it is optimal for authority to be shared, a single minister (perhaps a prime minister) should be assigned a large share (on average, at least 80%) of decisions.

Our results where information is relevant to all policies and widely dispersed provide a novel account for the stylized fact that in parliamentary democracy, the diverse preferences of an assembly sit alongside fairly centralized decision-making authority. A challenging case is where uncertainty is specific to each policy and expertise is widely dispersed. Does this lead to decentralized decision-making authority assignments? Surprisingly not. We find that full decentralization is never the optimal decision-making authority assignment. In fact, all policy decisions should be granted to the most moderate politician, unless the policy expert has "intermediate" ideology (i.e., she is neither too moderate nor too extreme). Numerical simulations reveal that the optimal decision-making authority assignment is no less centralized than in the common state case.

Literature Review

Our article relates to a broad literature on the politics of information aggregation that builds on the contributions by Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1996, 1997, 1998), who study situations where, as in our model, players share common interests. Following the seminal work by Crawford and Sobel (1982) and Gilligan and Krehbiel (1987), communication games are now a canonical framework to study the politics of information aggregation. Most of this literature has focused on the aggregation of information in committees where a single outcome is determined by voting. In our model, information is aggregated through communication. While Coughlan (2000), Doraszelski, Gerardi, and Squintani (2003), and Austen-Smith and Feddersen (2006) explore the consequences of allowing

committee members to communicate before they vote,³ we instead consider communication when a set of policy outcomes is allocated to individuals with decision-making rights which better fits the cabinet setting. Our study of optimal executive structure from an information aggregation perspective relates to work by Dragu and Board (2013), who show that the imposition of judicial review can lead to more informed outcomes.

We analyze our game using the multiplayer communication model by Galeotti, Ghiglino, and Squintani (2013), who build on Morgan and Stocken (2008). Its key feature is a coarse information structure with the implication that a message sent to a decision maker is either truthful or not. This contrasts with information aggregation with a continuous signal space and a single policy dimension where information can be conveyed only when the message space is partitioned, as in the seminal article by Crawford and Sobel (1982). Extensions of that article have shown, however, that in multiple dimensions, the decision maker can extract all information from an informed agent (Battaglini 2002). Dewan and Hortala-Vallve (2011) extend that framework to provide insights into a prime minister's control over ministers who are perfectly informed. In our model with coarse information, by contrast, no player can be perfectly informed, so the quality of final policies depends on the number of politicians communicating truthfully. Such multiplayer communication is, we believe, the relevant aspect of information aggregation in parliaments and cabinets.

Imposing this structure on the message space provides tractability and substantive new political insights, as witnessed by recent papers by Patty and Penn (2013) on small networks, Dewan and Squintani (2012) on factions in political parties, and Gailmard and Patty (2009) on delegation and transparency with sequential decision making. Our article significantly extends the multiplayer communication model in considering the possibility that players have specific information about some decisions but not others. Moreover, in studying the question of the optimal assignment of decision-making rights, we derive an entirely new set of theoretical results.

A related paper by Dewan and Squintani (2012) adapts the multiplayer communication model to analyze the formation of party factions. In their model, each party politician is endowed with some say over the party manifesto. They can choose voluntarily to join a faction. Doing so involves delegating authority to the faction leader, who makes the final choice. After factions have formed,

the party politicians communicate their information to faction leaders in private. The authors study a situation where communication can be made only within factions and one where the factional structure does not prohibit intraparty communication. They analyze the welfare consequences of factionalism in each case. The key features of that model—private communication and voluntary delegation of authority—are relevant to party structures. Here, in analyzing the optimal structure of the executive in a parliamentary democracy, our main focus is public communication. The optimal allocation of authority is that which would be chosen by a welfare-maximizing prime minister and it does not arise through a process of voluntary delegation.

Recent and related work by Patty (2013) complements ours in looking at how the exclusion and inclusion of cabinet members affects strategic communication. While sharing the same broad motivation and some modeling choices, the two studies answer distinct questions. Our focus is on the optimal assignment of executive authority and on the comparison between public and private communication. We show that cabinet meetings are optimal and present novel insights as to the degree to which power should be centralized in a cabinet and to whom it should be allocated. Patty (2013), by contrast, considers the optimal size and composition of cabinet meetings with an exogenous allocation of decision-making authority. He does not make the restriction that cabinet meetings are limited to those who hold executive authority; indeed, in an interesting finding, he shows that it may be optimal for some individuals with executive authority to be excluded, whereas others without such authority should be included.

The result that cabinet meetings yield higher welfare than private conversations can be related to Farrell and Gibbons (1989) and Goltsman and Pavlov (2011), who compare public communication and private communication in a much simpler game with a single expert and two decision makers, but critically do not consider the possibility of reassignment of decision-making authority.

More broadly, our results on the deliberative value of collective meetings provide a new angle on the study of cabinet governance that typically has focused on the cabinet as a system of incentives, managed strategically by a prime minister as in citations mentioned earlier. And we provide an information aggregation justification for the centralization of decision-making rights in an assembly (either to a single leader or a cabinet with public information) that contributes to a broad rational choice literature on why majorities adopt restrictive procedures, which looks at the role of committees (Gilligan and Krehbiel 1987), political parties (Cox and McCubbins 1993)

³A different, normative approach consists in devising optimal mechanisms for optimal decisions in committees (see, e.g., Gerardi, McLean, and Postlewaite 2009 or Gershkov and Szentes 2009).

and cabinets (Cox 1987). Recent contributions to this debate include Diermeier and Vlaicu (2011) and Diermeier, Prato, and Vlaicu (2013).

Model

We consider the following information aggregation and collective decision problem. Suppose that a set $\mathcal{I}=\{1,\ldots,I\}$ of politicians form a parliamentary majority that provides consent for a governing executive. They are faced with the collective task of choosing an assignment $a:\mathcal{K}\to\mathcal{I}$ of policy decisions that grants decisionmaking authority over a set of policies $\mathcal{K}=\{1,\ldots,K\}$. For each $k\in\mathcal{K}$, the decision \hat{y}_k is a policy on the left-right spectrum \Re . For simplicity, we think of the assignment as granting complete jurisdiction over policy k, though of course other interpretations, such as the assignment of agenda-setting rights, could also be incorporated. The important element is that decision-making authority over each policy is granted to a unique individual.

In a fully decentralized executive, each policy decision is assigned to a different politician so that $a(k) \neq a(k')$ for all k, k' in \mathcal{K} . At the opposite end of the spectrum, all decisions are centralized to a single leader so that a(k) = a(k') for all k, k' in \mathcal{K} . We let the range of a be denoted by $a(\mathcal{K}) \subseteq \mathcal{I}$, which we term as the set of politicians with decision-making authority. We refer to such politicians as *ministers*. We let a_j denote the number of policies that minister j takes under assignment a. Our specification thus allows us to capture important elements of the executive body: its size (beyond the extremes of full decentralization and the leadership of one, there are a range of possibilities); and its balance (among the set of ministers, some may have more authority than others).

Politicians are ideologically differentiated and care about all policy choices made. For any policy decision \hat{y}_k , their preferences also depend on unknown states of the world θ_k , uniformly distributed on [0, 1]. Specifically, were she to know the vector of states $\theta = (\theta_k)_{k \in \mathcal{K}}$, politician i's payoff would be

$$u_i(\hat{\mathbf{y}}, \theta) = -\sum_{k=1}^K (\hat{y}_k - \theta_k - b_i)^2.$$

Hence, each politician i's ideal policy is $\theta_k + b_i$, where the bias b_i captures ideological differentiation, and we assume without loss of generality that $b_1 \le b_2 \le \ldots \le b_I$. The vector of ideologies $\mathbf{b} = \{b_1, \ldots, b_I\}$ is common knowledge.

Each politician i has some private information on the vector θ . Specifically, we make two opposite assumptions

on politicians' information. First, for some of our analysis, we assume that uncertainty over all policies is captured by a single *common* state that represents the underlying economic and social fundamentals. For example, an underlying economic recession will influence policy choices of all ministries, from the home office immigration policy to the fiscal policy of the chancellor of the exchequer. We represent these fundamentals by a single uniformly distributed state of the world θ , so that $\theta_k = \theta$ for all k and each politician i's signal s_i is informative about θ . Conditional on θ , s_i takes the value equal to 1 with probability θ and to 0 with probability $1 - \theta$. Second, and in an alternative specification, we say that the politician's information is policy specific. Each policy has its own underlying set of circumstances over which politicians may be informed. Thus, the random variables θ_k are identically and independently distributed across $k \in \mathcal{K}$, and each politician k receives a signal $s_k \in \{0, 1\}$ about θ_k only, again with $Pr(s_k = 1 | \theta_k) = \theta_k$. In the case of policy-specific information we take $\mathcal{K}=\mathcal{I}$ so that each politician is informed on a single issue. This specification allows us to explore a situation where expertise on policies varies and is widely dispersed among the set of politicians.

In our setup, politicians can communicate their signals to each other before policies are executed. We allow for such communication to take the form of either private conversations or general meetings. We might think of the former as taking place over dinner or via a secure communication network, with no leakage of information transmitted. Hence, each politician i may send a different message $\hat{m}_{ij} \in \{0, 1\}$ to any politician j. In a meeting, by contrast, a politician is unable to communicate privately with a decision maker as all communication is available to those who exercise authority. Hence, each politician i sends the same message \hat{m}_i to all decision makers. A pure communication strategy of player i is a function $m_i(s_i)$.

Communication between politicians allows information to be transferred. Up to relabeling of messages, each communication strategy from i to j may be either *truthful*, in that a politician reveals her signal to j so that $m_{ij}(s_i) = s_i$ for $s_i \in \{0, 1\}$, or *babbling*, and in this case $m_{ij}(s_i)$ does not depend on s_i . Hence, the communication strategy profile \mathbf{m} defines the truthful communication network $\mathbf{c}(\mathbf{m})$ according to the rule: $c_{ij}(\mathbf{m}) = 1$ if and only if $m_{ij}(s_i) = s_i$ for every $s_i \in \{0, 1\}$, which provides us with the communication structure within the set of politicians $\mathcal{I} = \{1, \ldots, I\}$.

The second strategic element of our model involves the final policies implemented. Conditional on her information, and after communication has taken place, each assigned decision maker implements her preferred policy. We denote a policy strategy by i as $y_{i,k} : \{0, 1\}^{\mathcal{I}} \to \Re$

for all policies k such that i = a(k). Given the received messages $\hat{\mathbf{m}}_{-i,i}$, by sequential rationality, politician i chooses $\hat{y}_{i,k}$ to maximize expected utility for all k such that i = a(k). So

$$y_{i,k}(s_i, \hat{\mathbf{m}}_{i,-i}) = b_i + E[\theta_k | s_i, \hat{\mathbf{m}}_{-i,i}],$$
 (1)

and this is due to the quadratic loss specification of players payoffs.

Given an assignment a, an equilibrium then consists of the strategy pair (m, y) and a set of beliefs that are consistent with equilibrium play. We use the further restriction that an equilibrium must be consistent with some beliefs held by politicians off the equilibrium path of play. Thus our equilibrium concept is pure-strategy perfect Bayesian equilibrium. Fixing policy assignment a, then regardless of the communication mode adopted, there may be multiple equilibria (m, y). For example, the strategy profile where all players "babble" is always an equilibrium. Equilibrium multiplicity makes the ranking of decision-making authority assignments a not well defined: Given the same assignment a, different equilibria may yield different payoffs to the politicians, so that the politicians' initial collective decision over assignments a is impossible. To avoid this issue we assume that for any assignment a, politicians coordinate on the equilibria (m, y) that give them the highest payoffs. This equilibrium selection is standard in games of communication.

We focus on the *optimal assignment of authority*, defined as the assignments a that induce the equilibria (\mathbf{m}, \mathbf{y}) with the largest joint payoffs:

$$W(\mathbf{m}, \mathbf{y}; a) = -\sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}} E[(y_{a(k),k} - \theta_k - b_i)^2].$$

Our notion of welfare is ex ante utilitarian: we assume that the collective decision on the optimal assignment a by politicians in \mathcal{I} maximizes the sum of their expected payoffs. But for some of our results we can invoke the weaker principle of Pareto optimality.

We first provide a useful derivation for our main results. We first say that a politician j's moderation is $|b_j - \sum_{i \in \mathcal{I}} b_i/I|$, the distance between b_j and the average ideology $\sum_{i \in \mathcal{I}} b_i/I$. We note that politicians' moderation does not depend on the assignment a, nor on the equilibrium (\mathbf{m}, \mathbf{y}) . Second, we let $d_{j,k}(\mathbf{m})$ denote politician j's information on the state θ_k given the equilibrium (\mathbf{m}, \mathbf{y}) . Specifically, $d_{j,k}(\mathbf{m})$ consists of the number of signals on θ_k held by j, including her own, after communication has taken place and before she makes her choice.

With common value information, each politician's information coincides with the number of politicians communicating truthfully with her plus her own signal. Later on in our analysis, we will adopt a specification with policy-specific knowledge, in which each politician j may hold at most one signal on each θ_k , either because s_k is her own signal (j = k) or because s_k was communicated by k to j given the equilibrium communication structure $\mathbf{c}(\mathbf{m})$.

Armed with these definitions, and given an assignment a and an equilibrium (\mathbf{m}, \mathbf{y}) , we show in the appendix that the equilibrium ex ante welfare $\mathcal{W}(\mathbf{m}, \mathbf{y}; a)$ can be rewritten as:

$$\mathcal{W}\left(\mathbf{m}, \mathbf{y}; a\right) = \underbrace{-\sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{I}} (b_i - b_{a(k)})^2}_{\text{aggregate ideological loss}} - \underbrace{\sum_{k \in \mathcal{K}} \frac{I}{6[d_{a(k),k}(\mathbf{m}) + 2]}}_{\text{aggregate residual variance}}.$$
 (2)

This decomposes the welfare function into two elements—aggregate ideological loss and the aggregate residual variance of the politicians' decisions—and proves useful in the results that follow.⁵

Private Conversations in the Common State Model

We begin our study of the optimal assignment of decision-making in an environment where underlying fundamentals are common to all policies so that politicians' information is relevant to all decisions. Initially, we explore the situation where politicians communicate only in private with decision makers. We first describe the equilibrium communication structure given any policy assignment a. The characterization extends Corollary 1 of Galeotti, Ghiglino, and Squintani (2013) to the case of arbitrary policy assignments. For future reference, for any assignment a, we write $d_j^*(a)$ as the information $d_j(\mathbf{m})$ associated with any welfare-maximizing equilibrium (\mathbf{m}, \mathbf{y}) . When the state θ is common across policies, and the communication is private, we prove in the

⁴Indeed, it can be easily shown that for any given assignment a, each politician's ranking among the possible equilibria (\mathbf{m}, \mathbf{y}) is the same (see Galeotti, Ghiglino, and Squintani 2013, Theorem 2).

⁵Note that, statistically, the residual variance may be interpreted as the inverse of the precision of the politicians' decisions.

⁶Because there is a single policy-relevant state θ , we drop the subscript k from the notation $d_{j,k}(\mathbf{m})$, the number of informative signals held by politician j on state θ_k in equilibrium.

appendix that the profile \mathbf{m} is an equilibrium if and only if, whenever i is truthful to j,

$$\left|b_i - b_j\right| \le \frac{1}{2\left[d_j(\mathbf{m}) + 2\right]}.\tag{3}$$

The fact that truthful communication can arise in our model is a consequence of the binary signal structure. When the direction of policy under truthful revelation is opposite to the bias of the politician relative to the minister (e.g., a politician with $b_i > b_j$ and $s_i = 0$), he is concerned that by misrepresenting his signal, the final policy will overshoot his preferred one. An important consequence of equilibrium Condition (3) is that truthful communication from politician i to minister j is independent of the specific policy decisions assigned to j and of the possibility of communicating with any other politician j'. Furthermore, truthful communication from politician i to minister j becomes less likely with an increase in the difference between their ideological positions.

Having characterized equilibrium communication between politicians and ministers, we explore the implication of Condition (3) with respect to the assignment of executive authority.

Proposition 1. Suppose that θ is common across policies, and that communication is private. For generic ideologies **b**, any Pareto optimal assignment involves decision-making authority being centralized to a single leader j, that is, a(k) = j for all k.

The finding that all decisions should be assigned to a single leader and, hence, executive authority should be fully centralized follows from two different facts. First, truthful communication from politician i to minister j in equilibrium is independent of the specific policy decisions assigned to j (or to any other politician j'). Second, the stipulation that every politician's information is relevant for all policies implies that politicians and policies are

 7 A perhaps more surprising effect is that the possibility for *i* to communicate truthfully with j decreases with the information held by j in equilibrium. To see why communication from i to jis less likely to be truthful when j is well informed in equilibrium, suppose that $b_i > b_j$, so that i's ideology is to the right of j's bliss point. Suppose j is well informed and that politician i deviates from the truthful communication strategy—she reports $\hat{m}_{ij} = 1$ when $s_i = 0$ —then she will induce a small shift of j's action to the right. Such a small shift in j 's action is always beneficial in expectation to i, as it brings j's action closer to i's (expected) bliss point. Hence, politician i will not be able to truthfully communicate the signal $s_i = 0$. By contrast, when j has a small number of players communicating with her, i's report $\hat{m}_{ij} = 1$ moves j's action to the right significantly, possibly beyond i's bliss point. In this case, biasing rightward j's action may result in a loss for politician i, so she would prefer to report truthfully—that is, she will not deviate from the truthful communication strategy.

"interchangeable." As a consequence of these two facts, whoever is the optimal politician to make one policy decision will also be the optimal politician to make all of them. This result holds with our utilitarian welfare criterion and under the weak welfare concept of Pareto optimality. In sum, with the restriction to private conversation between a politician and a minister, the optimal size of the executive is one: Leadership by a dominant prime minister emerges.

Cabinet Meetings versus Private Conversations in the Common State Model

Having considered communication via private meetings, we now study optimal assignment of decision-making authority when there is a cabinet providing a forum for information to be conveyed to the set of ministers. This change to the communication environment affects the strategic calculus of information transmission: It is possible that politician i would not wish to communicate with minister j on a policy if that information is shared with minister j'; conversely, politician i might share information with j because minister j' also has access to that information.

The next result characterizes communication equilibria under any policy assignment. It extends Theorem 1 of Galeotti, Ghiglino, and Squintani (2013) to the case of arbitrary policy assignments.

Lemma 1. Suppose that the state θ is common across policies k and that communication takes place in cabinet meetings. The strategy profile \mathbf{m} is an equilibrium if and only if, whenever i is truthful,

$$\left| b_i - \sum_{j \neq i} b_j \gamma_j(\mathbf{m}) \right| \leq \sum_{j \neq i} \frac{\gamma_j(\mathbf{m})}{2[d_j(\mathbf{m}) + 2]}, \quad (4)$$

where for every $j \neq i$,

$$\gamma_j(\mathbf{m}) \equiv \frac{a_j/[d_j(\mathbf{m})+2]}{\sum_{j'\neq i} a_{j'}/[d_{j'}(\mathbf{m})+2]}.$$

Intuitively, each politician i's willingness to communicate truthfully depends on a weighted average of all the ministers' ideologies. The specific weights are inversely related to the equilibrium information of each politician. Analyzing them reveals that, in contrast to the earlier case, truthful communication from politician i to minister j in equilibrium depends upon the policy assignment. Thus, the characterization of the communication structure given by Lemma 1 implies that our earlier result in

Proposition 1—namely, that private conversation leads to fully centralized authority—can be reversed once we allow for public meetings. So formal power-sharing agreements in a multimember executive that meets in cabinet may be optimal. We illustrate this possibility with a simple example with four politicians and a generic set of biases.

Example 1. Suppose that I = K = 4. Biases are $b_1 = -\beta$, $b_2 = \varepsilon$, $b_3 = \beta$, and $b_4 = 2\beta$, where ε is a positive quantity smaller than β . We compare four assignments: full decentralization, leadership by politician 2 (the most moderate politician), and two forms of power-sharing agreements between politicians 2 and 3. In the symmetric power-sharing agreement politicians 2 and 3 make two decisions each; in the asymmetric power-sharing agreement politician 2 makes three choices and politician 3 makes one choice.

The analysis requires calculating the welfaremaximizing equilibria for each of the four assignments and comparing welfare across them. Details are relegated to the appendix. Here, we note that taking the limit for vanishing $\varepsilon > 0$, the following observations obtain. First, for β < 1/24, all players are fully informed under any of the four considered assignments; at the same time, for $\beta > 1/18$, there is no truthful communication regardless of the assignment; in both cases, the optimal assignment entails selecting the most moderate politician 2 as the unique leader. Second, for $\beta \in (1/24, 1/21)$, politicians 1 and 4 are willing to communicate truthfully under any power-sharing agreement, but politician 4 is not willing to share information if politician 2 is the single leader. Third, for $\beta \in (1/21, 1/18)$, players 1 and 4 are both willing to talk publicly only when the symmetric power-sharing agreement is in place. Finally, for $\beta \in (1/24, 1/18)$, there is no advantage from assigning any choice to player 3 instead of player 2. Our result is summarized as follows.

Result 1. Suppose that I = K = 4, with $b_1 = -\beta$, $b_2 = \epsilon$, $b_3 = \beta$, and $b_4 = 2\beta$, and compare leadership by 2, full decentralization, and power-sharing agreements between 2 and 3, under public communication of information with common state. As ϵ goes to 0, the following holds: For $\beta < 1/24$ or $\beta > 1/18$, it is optimal to select 2 as the leader. For $\beta \in (1/24, 1/21)$, the optimal assignment is the symmetric power-sharing agreement of 2 and 3. For $\beta \in (1/21, 1/18)$, the optimal assignment is the asymmetric power sharing agreement where 2 makes three choices and 3 makes one choice.

The fact that full-authority centralization is always optimal when conversations are private, though not necessarily when there are public meetings, together with the observation that private and public communication equilibria coincide when all authority is granted to a single leader, provides a striking result: The possibility of cabinet meetings induces a Pareto improvement.

This result, one of the main findings of our article, holds independently of whether private conversations take place alongside cabinet deliberations. The above argument is, evidently, conclusive when private conversations are ruled out. To assess the opposite case, note that private conversation may always involve babbling in equilibrium. Then, because we always select the Pareto optimal equilibrium of any communication game, it immediately follows that the argument developed above holds also when cabinet discussion may be supplemented with a private exchange of views between policy makers. We state our finding formally:

Proposition 2. Suppose that the state θ is common across policies k. For generic ideologies \mathbf{b} , the optimal assignment of decision-making authority when information is exchanged in cabinet meetings Pareto dominates any authority assignment when information is exchanged only privately.

Proposition 2 bears important consequences for optimal executive structure. It shows that if politicians in \mathcal{I} can assign authority optimally, then imposing a cabinet structure to the executive—a public meeting at a designated time and place where ministers provide the information relevant to their decisions—induces a Pareto improvement over other forms of executive governance. In particular, cabinet government Pareto dominates an executive where individual ministers implement policy but are not bound to share policy-relevant information.

Optimal Cabinet Design in the Common State Model

We have seen that when conversations with ministers take place in private, the optimal executive is centralized with a single decision maker. A multimember executive that meets in cabinet can, however, outperform a single-member executive. This provides a normative justification for cabinet governance. What is the optimal assignment of authority within a cabinet? An analysis of Result 1 suggests that it is nonmonotonic in the level of disagreement between politicians and that centralization is optimal in the two polar cases, those with very high or very low agreement. The next result formalizes this intuition for our general setting.

 $^{^{8}}$ When $\epsilon=0$, there are a multiplicity of optimal allocations, which is not generic.

Proposition 3. Let i^* be the most moderate politician, that is, $i^* = \arg\min_{j \in I} |b_j - \sum_{i \in \mathcal{I}} b_i / I|$

- 1. There exists $\Delta > 0$ such that if $\max_{j \in I} |b_j b_{i^*}| \leq \Delta$, then the optimal allocation of authority is centralized to politician i^* .
- 2. For every profile of ideologies $\{b_1, \ldots, b_I\}$, there exists $\delta < \infty$ such that, under profile of ideologies $\{\beta b_1, \ldots, \beta b_I\}$ and for all $\beta \geq \delta$, the optimal allocation of authority is centralized to politician i^* .

When politicians have similar ideological preferences, there are few strategic constraints on communication in the parliament. In particular, even a relatively extreme politician can communicate with the most moderate one. Since information can then be aggregated regardless of the specific allocation of authority, it is better that power is centralized to a single moderate politician. When disparity in the ideological views of the politicians is large, communication with ministers is not possible and so, from a welfare perspective, it is desirable that the decision maker respects the diversity of viewpoints. Again, the most moderate politician is best placed to do so, so power should be centralized. The fact that a multimember cabinet can sometimes be optimal, as a direct consequence of Proposition 2 and Example 1, completes the proof of the claim.

Combining Propositions 2 and 3 provides an answer to the question posed in our introduction: In the absence or ambiguity of constitutional guidelines, to what extent should power be centralized? Our results reveal that when information aggregation is not an important consideration—even relative extremists can communicate to the center, or politicians do not communicate at all–full centralization is desirable. But when information aggregation is more sensitive to the allocation of authority, it is better that power is shared in a multimember cabinet.

Beyond this insight, what does the optimal allocation of authority within a cabinet look like? There are subtle considerations that make it difficult to provide a precise answer. However, note that in the example described in Result 1, the optimal allocation is single peaked around the moderate politician. One might conjecture then that this is a general property of the optimal authority allocation in cabinets. This conjecture, as our next example shows, is incorrect.

⁹To further explore optimal decision-making authority assignments in cabinet governments, we have also run simulations for a seven-member parliament. The results show that in most cases, centralization of authority in a multimember cabinet is Pareto superior to other executive forms. The details and the results of this exercise are reported in the online supporting information.

Example 2. Suppose I = |K| = 7 and assume that $b_{i+1} - b_i = 0.116$ for all $i \in I$. In this case, one can easily show that the optimal allocation of authority a_i^* is $a_3^* = 1$, $a_4^* = 5$, $a_6^* = 1$. That is, one policy is allocated to politician 3, five are allocated to politician 4, and the remaining one is allocated to politician 6. Optimal equilibrium communication involves only politicians 4 and 5 communicating truthfully; all other politicians babble.

Here the optimal allocation of authority is not single peaked; whereas the most moderate politician 4 has the largest share of authority, her immediate neighbor 5 has none, and politician 6 has some. To provide intuition for this surprising result, consider an alternative single-peaked allocation $a_3 = 1$, $a_4 = 5$, $a_5 = 1$. This closely resembles a^* except that the policy allocated to politician 6 under a^* is now given to politician 5. This transfer of authority makes the allocation more concentrated toward the moderate politician 4 and so better reflects the diversity of politicians' views. However, under this single-peaked allocation, less information is aggregated.

To understand why, focus on the incentive for politician 5 to communicate truthfully. Under the alternative and suboptimal allocation a^* , politician 5 has an incentive to misreport a low signal to other cabinet members as doing so would bias their decisions toward her ideal point. Under the optimal allocation, by contrast, her incentive to misreport a low signal to 3 and 4 is offset by its effect on politician 6, who is now included in the cabinet. Misreporting a low signal could bias politician 6's action far away from 5's bliss point. So we see that whereas 5 communicates truthfully in the optimal allocation, she does not do so in the single-peaked allocation.

An intriguing implication of our finding is that the set of ministers included in the cabinet is not ideologically connected. The prediction that they will be connected arises in classic models of coalition formation (Axelrod 1970), though this result can be overturned in noncooperative bargaining models (Austen-Smith and Banks 1990). From our information aggregation creating "holes" in the cabinet—so that moderate politicians are bypassed for more extreme ones—provides better incentives for communication and disciplines politicians who otherwise would misrepresent their views.

Till now, we have considered a small group of politicians. Since parliament is a large representative body, and those on the government payroll can represent a significant fraction (in the United Kingdom, of around 650 members of Parliament, roughly 20% play some role in government), it is interesting to observe the optimal allocation of authority in the limit case as the number of politicians becomes large. In doing so, we provide a

strong characterization result: All decision-making authority should be concentrated to politicians who are ideologically close to the most moderate one.

Proposition 4. Suppose that biases b_i , $i=1,\ldots,I$ are i.i.d. and drawn from a distribution with connected support with mean \bar{b} . For every small $\delta > 0$, there exists a possibly large $I_{\delta} > 0$ so that for all $I > I_{\delta}$, with at least $1 - \delta$ probability, the fraction of decisions in the optimal assignment concentrated to politicians with biases b such that $|b - \bar{b}| < \delta$ is larger than $1 - \delta$.

The proof of Proposition 4 consists of two parts. First, we show that when all decisions are allocated to a single politician *i* and parliament grows large, the decision maker becomes fully informed. Second, we compare the case where all decisions are allocated to the most moderate politician to that where some decisions are allocated to a less moderate one. As the parliament becomes large, the aggregate residual variance obtained in each of these assignments vanishes. The difference in the aggregate ideological loss of these assignments is, however, bounded below.

It is important to stress that Proposition 4 does not imply that welfare is equivalent in large parliaments that adopt cabinet meetings and ones where communication with decision makers is private. In fact, our normative justification for cabinet governance holds for any size majority, including large ones.

Policy-Specific Information

This section studies optimal assignment of decision-making authority when each politician's information is policy specific, so that only politician k receives a signal about θ_k , for each policy k. We begin by characterizing equilibrium communication.

Lemma 2. Suppose that information is policy specific. The profile (\mathbf{m}, \mathbf{y}) is an equilibrium if and only if, whenever politician k is truthful to $a(k) \neq k$,

$$|b_k - b_{a(k)}| \le 1/6$$
.

Since each politician has only one signal and that signal is informative of only one policy decision, the amount of information held by politician $a(k) \neq k$ depends only on whether k is truthful. Hence, whether k is truthful (or not) does not depend on the communication strategy of any other politician. Further, because each politician is informed on one policy only, and this policy may be assigned to a single policy maker, there is no difference between private conversations and cabinet meetings.

This characterization of information transmission bears the following implication. The possibility that a politician k truthfully communicates her signal to the minister a(k) to whom decision k is assigned is independent of any other assignment. Hence, for all choices k, the optimal assignment a(k) can be selected independently of other assignments. The optimal assignment is to allocate decision k to the politician k who maximizes:

$$-\sum_{i=1}^{I} \frac{(b_j - b_i)^2}{I} - \frac{1}{6(d_{j,k}(\mathbf{m}) + 2)},$$

where $d_{j,k}(\mathbf{m}) = 1$ if $|b_k - b_j| \le 1/6$ and $d_{j,k}(\mathbf{m}) = 0$, otherwise.

Simplifying the above expression, and using Lemma 2, we see that the optimal selection of a(k) takes a simple form when information is policy specific; policy decision k should be assigned to either the most moderate politician $m^* = \arg\min_m |b_m - \sum_{i=1}^I b_i/I|$ or to the most moderate politician m(k) informed of k, that is, to $m(k) = \arg\min_{m:|b_m - b_k| \le 1/6} |b_m - \sum_{i=1}^I b_i/I|$, depending on whether

$$\sum_{i=1}^{I} \frac{\left(b_i - b_{m(k)}\right)^2}{I} - \sum_{i=1}^{I} \frac{\left(b_i - b_{m^*}\right)^2}{I} > (<) \frac{1}{36}.$$
 (5)

Because for any j, the quantity $\sum_{i=1}^{I} (b_i - b_j)^2 / I$ is the average ideological loss, whereas the information gain is 1/36, we may summarize our analysis as follows.

Lemma 3. When information is policy specific, each decision k is optimally assigned to either the most moderate informed politician m(k) or to the most moderate one m^* , depending on whether the difference in average ideological loss is smaller or greater than the informational gain.

With the above characterization, we now show that although policy-specific information might lead one to believe that full decentralization is optimal, this is, in fact, never the case.

Proposition 5. Despite policy-specific information, full decentralization is never optimal for generic ideologies **b**. The most moderate politician m^* is assigned the policies of sufficiently moderate and of sufficiently extreme-bias expert politicians, but not necessarily the policies of intermediate-bias politicians.

The complete proof of this proposition is provided in the appendix; here we convey the main intuition behind the result. Because moderate policy experts are willing to inform the most moderate politician m^* , it is optimal that she is given authority on these policies. Since extreme policy experts are willing to communicate only with extreme politicians, it is better to let the (uninformed) most

moderate politician decide. Only for intermediate policies k, it is suboptimal that the most moderate politician decides.

Our result relates to Dessein (2002), who shows, in a different environment, that it is optimal for a decision maker to delegate authority to an expert with a small bias and a signal from a continuum. We show that decentralization is not optimal with policy-specific information; instead, it is better that the most moderate politician is assigned the decisions of moderate experts. The difference arises because our model has binary signals instead of continuous ones. Nevertheless, our result that full decentralization is never optimal still holds if signals are continuous: Then, the decision of extreme experts should be optimally assigned to the most moderate politician.

Conclusion

We have analyzed the optimal structure of the executive restricting our model to the set of institutions that allow for assignment of decision-making authority to individuals with collective deliberation over outcomes, two of the core features of cabinet governance found in parliamentary democracies. Our main results focus on the case where uncertainty over policy outcomes is generated by a shock that affects all policies. Then we show that cabinet meetings, which provide a mechanism for information made available to one minister to be available to all decision makers, can improve the quality of policies implemented: They allow for more policy-relevant information to be aggregated than would be the case in their absence. We then analyzed the optimal degree of centralization of executive power and showed that it is nonmonotonic in the ideological divergence in the majority party: For intermediate levels of divergence, it is optimal for power to be shared between cabinet ministers; otherwise, it is optimal for power to be concentrated to the most moderate politician. Surprisingly, when power is shared in a cabinet of ministers, in the optimal allocation, the set of ministers with decision-making authority need not be ideologically connected. In large cabinets, however, authority should be concentrated to the most moderate politicians. Overall, our results highlight the desirability of a centralized governing executive that holds cabinet meetings. We ask whether this depends upon the dispersion of policyrelevant expertise. Surprisingly, in the case where shocks are policy specific and a single politician is imperfectly informed about its effects, the degree of centralization is no less centralized.

The two informational environments we have considered—perfect correlation and independence—are

the classic ones most studied in game-theoretical applications, for example, with respect to auctions. We thus take them as the natural starting point for our investigation into optimal executive structure. An obvious question is whether our core insights are robust to the imposition of a mixed case. Our preliminary investigations reveal that "this is in fact so" the case: The results developed here are not confined to the two cases we study, and the tradeoffs we have highlighted are indeed relevant to the mixed case. ¹⁰

Finally, we highlight several areas for further research. Our work establishes a normative benchmark for evaluating the assignment of decision-making authority to heads of executive departments in parliamentary democracies. In practice, the assignment of decision-making rights is carried out by a prime minister who is answerable to the parliamentary majority. As noted by Strøm (2000), this creates a "singular chain" of delegation, from the parliamentary majority to a prime minister and the heads of departments, that distinguishes parliamentary democracies from presidential ones. 11 Thus, our analysis reveals the delegation patterns of a prime minister who maximizes the welfare of a parliamentary majority. Since the class of parliamentary democracies is large, there are, of course, several variations from this ideal type that could be considered. For example, in some parliamentary democracies, the assignment is to a senior executive head shadowed by a junior minister within the same department.¹² In other parliamentary democracies, such as Israel, the cabinet votes over policy rather than delegating the decision to a single minister. And, of course,

¹⁰Due to space limitations, we omit the analysis but provide a brief summary here. The full analysis is available upon request. To investigate the case of correlation among policies, we considered the simple environment where, before making a choice, each politician knows that with probability p all policies are perfectly correlated and with the remaining probability they are perfectly independent. The model degenerates to the two environments we have considered for $p \in \{0, 1\}$. Characterizing the incentive compatibility conditions for truthful communication for arbitrary allocations and modes of communication reveals that, perhaps unsurprisingly, it is continuous in p. This, in turn, implies that our results derived under the assumption of perfect correlation and of independence hold for p sufficiently high and p sufficiently low, respectively. Moreover, the analysis reveals that optimal allocation in the case of intermediate correlation involves a trade-off between politicians' moderation and the information that they hold in equilibrium that is the driving force behind our results.

 $^{11}\mathrm{Though},$ as argued by Patty (2013), the distinction is perhaps not so clear.

¹²In Thies (2001), such arrangements allow ministers with different preferences to enforce the implementation of a compromise policy. Here, and as in the models of Austen-Smith and Banks (1996) and Laver and Shepsle (1996), the minister in charge of an issue always implements his preferred policy.

the assignment of authority might be part of a prime minister's strategic plan, and so her objectives may conflict with those of members of her government and the majority in parliament. A further extension might consider how centralized authority affects the interaction between party elites and voters, whose actions jointly determine the ideological composition of the assembly, as well as how the degree of centralization of decision-making authority responds to party control over nomination of the members of parliament. All of these substantive applications could be approached within the current modeling framework, though we have not done so here.

Further lines of inquiry can be addressed within our framework, but they would involve more extensive modifications of the model. Here we have assumed that the parliamentary majority assigns decision-making authority to ministers, having in mind the fusion of legislative and executive powers found in many parliamentary democracies. Our model could be modified so that the parliamentary majority nominates agenda setters whose proposal needs then to be formally approved by the parliament. More generally, the parliamentary majority could assign authority to committees rather than to individuals. As these lines of inquiry require significant changes to the model, we defer them to future research.

Appendix

Equilibrium Beliefs

In our model, a politician's equilibrium updating is based on the standard beta-binomial model. Suppose that a politician i holds n bits of information, that is, she holds the private signal s_i and n-1 politicians truthfully reveal their signal to her. The probability that l out of such n signals equals 1, conditional on θ , is

$$f(l|\theta, n) = \frac{n!}{l!(n-l)!} \theta^l (1-\theta)^{(n-l)}.$$

Hence, politician i's posterior is

$$f(\theta|l, n) = \frac{(n+1)!}{l!(n-l)!} \theta^{l} (1-\theta)^{(n-l)},$$

the expected value is

$$E\left(\theta\mid l,\,n\right)=\frac{l+1}{n+2},$$

and the variance is

$$V(\theta \mid l, n) = \frac{(l+1)(n-l+1)}{(n+2)^2(n+3)}.$$

Derivation of Equilibrium Welfare, Expression 2. Assume (m, y) is an equilibrium. The ex ante expected utility

of each player i is

$$Eu_{i}(\mathbf{m}, \mathbf{y}) = -E\left[\sum_{k=1}^{K} (\hat{y}_{k} - \theta - b_{i})^{2}; (\mathbf{m}, \mathbf{y})\right]$$

$$= -\sum_{k=1}^{K} E\left[\left(y_{a(k), k} - \theta - b_{i}\right)^{2}; (\mathbf{m}, \mathbf{y})\right]$$

$$= -\sum_{k=1}^{K} E\left[\left(b_{a(k)} + E\left[\theta \mid \Omega_{a(k)}\right] - \theta - b_{i}\right)^{2}; \mathbf{m}\right]$$

where $\Omega_{a(k)}$ denotes the equilibrium information of player a(k), and we have dropped the reference to y in the last equality, as, after substituting $y^{a(k)}$ with $b_{a(k)} + E[\theta|\Omega_{a(k)}]$, the reported expression no longer depends on y. Hence,

$$E u_{i}(\mathbf{m}, \mathbf{y})$$

$$= -\sum_{k=1}^{K} E \left[(b_{a(k)} - b_{i})^{2} + (E[\theta \mid \Omega_{a(k)}] - \theta)^{2} - 2(b_{a(k)} - b_{i})(E[\theta \mid \Omega_{a(k)}] - \theta); \mathbf{m} \right]$$

$$= -\sum_{k=1}^{K} \left[(b_{a(k)} - b_{i})^{2} + E \left[(E[\theta \mid \Omega_{a(k)}] - \theta)^{2}; \mathbf{m} \right] - 2(b_{a(k)} - b_{i})(E[E[\theta \mid \Omega_{a(k)}]; \mathbf{m}] - E[\theta; \mathbf{m}]) \right],$$

by the law of iterated expectations, $E[E[\theta|\Omega_{a(k)}]; \mathbf{m}] = E[\theta; \mathbf{m}]$, and by definition $E[(E[\theta|\Omega_{a(k)}] - \theta)^2; \mathbf{m}] = \sigma_k^2(\mathbf{m})$.

Further, note that the equilibrium information $\Omega_{a(k)}$ of player a(k) may be represented as any vector in $\{0, 1\}^{d_j(c)}$. Letting l be the number of digits equal to 1 in any such vector, we obtain

$$\begin{split} &E\left[\left(E\left[\theta\mid\Omega_{a(k)}\right]-\theta\right)^{2};\mathbf{m}\right]\\ &=\int_{0}^{1}\sum_{l=0}^{d_{a(k)}(\mathbf{c})}\left(E\left[\theta\mid l,\,d_{a(k)}(\mathbf{c})\right]-\theta\right)^{2}f\left(l\mid d_{a(k)}(\mathbf{c}),\,\theta\right)\,d\theta\\ &=\int_{0}^{1}\sum_{l=0}^{d_{a(k)}(\mathbf{c})}\left(E\left[\theta\mid l,\,d_{a(k)}(\mathbf{c})\right]-\theta\right)^{2}\frac{f\left(\theta\mid l,\,d_{a(k)}(\mathbf{c})\right)}{d_{a(k)}(\mathbf{c})+1}\,d\theta, \end{split}$$

where the second equality follows from $f(l \mid d_{a(k)}(\mathbf{c}), \theta) = f(\theta \mid l, d_{a(k)}(\mathbf{c}))/(d_{a(k)}(\mathbf{c}) + 1)$.

Because the variance of a beta distribution of parameters l and d is

$$V(\theta \mid l, d) = \frac{(l+1)(d-l+1)}{(d+2)^2(d+3)},$$

we obtain

$$E\left[\left(E\left[\theta\mid\Omega_{a(k)}\right]-\theta\right)^{2};\mathbf{m}\right]$$

$$= \frac{1}{d_{a(k)}(\mathbf{c}) + 1} \left[\sum_{l=0}^{d_{a(k)}(\mathbf{c})} V\left(\theta \mid l, d_{a(k)}(\mathbf{c})\right) \right]$$

$$= \sum_{l=0}^{d_{a(k)}(\mathbf{c})} \frac{(l+1) \left(d_{a(k)}(\mathbf{c}) - l + 1\right)}{\left(d_{a(k)}(\mathbf{c}) + 1\right) \left(d_{a(k)}(\mathbf{c}) + 2\right)^2 \left(d_{a(k)}(\mathbf{c}) + 3\right)}$$

$$= \frac{1}{6(d_{a(k)}(\mathbf{c}) + 2)}.$$

Proof of Proposition 1. Fix any assignment a. Any Pareto optimal equilibrium (\mathbf{m}, \mathbf{y}) maximizes the welfare

$$\mathcal{W}(\mathbf{m}, \mathbf{y}; \boldsymbol{\gamma}; a) = -\sum_{i \in \mathcal{I}} \gamma_i \sum_{k \in \mathcal{K}} E[(\hat{y}_k - \theta_k - b_i)^2 | s_i, \mathbf{m}_{\tilde{N}_i, i}],$$

for some Pareto weights γ . Following the same steps in the derivation of Expression 2, we obtain that

$$W(\mathbf{m}, \mathbf{y}; \gamma; a) = -\sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{I}} \gamma_i (b_i - b_{a(k)})^2$$
$$-\sum_{k \in \mathcal{K}} \frac{1}{6[d_{a(k),k}(\mathbf{m}) + 2]}.$$

This decomposition, together with equilibrium Condition (3), imply that as long as j is active under a, the equilibrium information $d_j^*(a)$ associated with any Pareto optimal equilibrium (\mathbf{m}, \mathbf{y}) is independent of the set of policy choices $a^{-1}(i)$ assigned to any player i, including j. Hence, choosing the Pareto optimal assignment is equivalent to finding the index j that maximizes

$$-\sum_{i=1}^{I} \gamma_i \left(b_j - b_i\right)^2 - \frac{1}{6(d_j(\mathbf{m}) + 2)},\tag{6}$$

and to assigning all policy choices k to such optimal j. For generic vectors of biases \mathbf{b} , Expression (6) has a unique maximizer.

Proof of Lemma 1 and Derivation of Expression 3. We first prove Lemma 1 and then derive Expression 3 as a corollary. Consider any $j \in a(\mathcal{K})$, and let $C_j(\mathbf{c})$ be the set of players truthfully communicating with j in equilibrium, that is, the equilibrium network neighbors of j. The equilibrium information of j is thus $d_j = |C_j(\mathbf{c})| + 1$, the cardinality of $C_j(\mathbf{c})$ plus j's signal. Consider any player $i \in C_j(\mathbf{c})$. Let s_R be the vector containing s_j and the (truthful) messages of all players in $C_j(\mathbf{c})$ except i. Let also $y_{s_R,s}^j$ be the action that j would take if he has information s_R and player i has sent signal s; analogously, $y_{s_R,1-s}^j$ is the action that j would take if he has information s_R and player i has sent signal 1-s. Agent i reports

truthfully signal s to a collection of agents J if and only if

$$-\sum_{j\in J}\sum_{k:a(k)=j}\int_{0}^{1}\sum_{s_{R}\in\{0,1\}^{d_{j}-1}}\left[\left(y_{s_{R},s}^{j}-\theta-b_{i}\right)^{2}\right.\\ \left.-\left(y_{s_{R},1-s}^{j}-\theta-b_{i}\right)^{2}\right]f(\theta,s_{R}|s)d\theta\geq0.$$

Using the identity $a^2 - b^2 = (a - b)(a + b)$ and simplifying, we obtain:

$$- \sum_{j \in J} \int_{0}^{1} a_{j} \sum_{s_{R} \in \{0,1\}^{d_{j}-1}} \left[\left(y_{s_{R},s}^{j} - y_{s_{R},1-s}^{j} \right) \right. \\ \left. \times \left(\frac{y_{s_{R},s}^{j} + y_{s_{R},1-s}^{j}}{2} - (\theta + b_{i}) \right) \right] f(\theta, s_{R}|s) d\theta \ge 0.$$

Next, observing that

$$y_{s_R,s}^j = b_j + E\left[\theta \mid s_R, s\right],\,$$

we obtain

$$- \sum_{j \in J} \int_{0}^{1} a_{j} \sum_{s_{R} \in \{0,1\}^{d_{j}-1}} \times \left[\left(E[\theta + b_{j} \mid s_{R}, s] - E[\theta + b_{j} \mid s_{R}, 1 - s] \right) \times \left(\frac{E[\theta + b_{j} \mid s_{R}, s] + E[\theta + b_{j} \mid s_{R}, 1 - s]}{2} - (\theta + b_{i}) \right) \right] f(\theta, s_{R} \mid s) d\theta > 0.$$

Denote

$$\Delta(s_R, s) = E[\theta \mid s_R, s] - E[\theta \mid s_R, 1 - s].$$

Observing that

$$f(\theta, s_R \mid s) = f(\theta \mid s_R, s) P(s_R \mid s),$$

and simplifying, we get

$$-\sum_{j\in J} a_j \sum_{s_R\in\{0,1\}^{d_j-1}} \int_0^1 \left[\Delta(s_R, s) + \left(\frac{E[\theta \mid s_R, s] + E[\theta \mid s_R, 1-s]}{2} + b_j - b_i - \theta \right) \right] \times f(\theta \mid s_R, s) P(s_R \mid s) d\theta \ge 0.$$

Furthermore,

$$\int_0^1 \theta f(\theta \mid s_R, s) d\theta = E[\theta \mid s_R, s],$$

and

$$\int_0^1 P(\theta \mid s_R, s) E[\theta \mid s_R, s] d\theta = E[\theta \mid s_R, s],$$

because $E[\theta|s_R, s]$ does not depend on θ . Therefore, we obtain

$$\begin{split} & -\sum_{j \in J} a_{j} \sum_{s_{R} \in \{0,1\}^{d_{j}-1}} \\ & \left[\Delta(s_{R}, s) \left(\frac{E[\theta \mid s_{R}, s] + E[\theta \mid s_{R}, 1-s]}{2} \right. \right. \\ & + b_{j} - b_{i} - E[\theta \mid s_{R}, s] \right) \right] P(s_{R} \mid s) \\ & = -\sum_{j \in J} a_{j} \sum_{s_{R} \in \{0,1\}^{d_{j}-1}} \left[\Delta(s_{R}, s) \right. \\ & \times \left(-\frac{E[\theta \mid s_{R}, s] - E[\theta \mid s_{R}, 1-s]}{2} + b_{j} - b_{i} \right) \right] \\ & \times P(s_{R} \mid s) \geq 0. \end{split}$$

Now note that.

$$\begin{split} \Delta\left(s_{R},s\right) &= E\left[\theta \mid s_{R},s\right] - E\left[\theta \mid s_{R},1-s\right] \\ &= E\left[\theta \mid l+s,d_{j}+1\right] - E\left[\theta \mid l+1-s,d_{j}+1\right] \\ &= \left(l+1+s\right) / \left(d_{j}+2\right) - \left(l+2-s\right) / \left(d_{j}+2\right) \\ &= \begin{cases} -1/\left(d_{j}+2\right) & \text{if } s=0 \\ 1/\left(d_{j}+2\right) & \text{if } s=1, \end{cases} \end{split}$$

where l is the number of digits equal to 1 in s_R . Hence, we obtain that agent i is willing to communicate to agent j the signal s = 0 if and only if

$$-\sum_{j \in J} a_j \left(\frac{-1}{d_j + 2} \right) \left(-\frac{-1}{2(d_j + 2)} + b_j - b_i \right) \ge 0$$

or

$$\sum_{j \in J} a_j \frac{b_j - b_i}{d_j + 2} \ge -\sum_{j \in J} a_j \frac{1}{2(d_j + 2)^2}$$

Note that this condition is redundant if $\sum_{j \in J} a_j(b_j - b_i) > 0$. On the other hand, she is willing to communicate to agent j the signal s = 1 if and only if

$$-\sum_{j\in J} a_j \left(\frac{1}{d_j+2}\right) \left(-\frac{1}{2(d_j+2)} + b_j - b_i\right) \ge 0$$

or

$$\sum_{j \in J} a_j \frac{b_j - b_i}{d_j + 2} \le \sum_{j \in J} a_j \frac{1}{2(d_j + 2)^2}.$$

Note that this condition is redundant if $\sum_{j \in J} a_j(b_j - b_i) < 0$. Collecting the two conditions yields.

$$\left| \sum_{j \in J} a_j \frac{b_j - b_i}{d_j + 2} \right| \le \sum_{j \in J} \frac{a_j}{2 (d_j + 2)^2}.$$
 (7)

Rearranging Condition (7) completes the proof of Lemma 1. \Box

Proof of Result 1. Consider a cabinet of four politicians, with biases $b_1 = -\beta$, $b_2 = \varepsilon$, $b_3 = \beta$, and $b_4 = 2\beta$. We suppose that $\varepsilon > 0$ is small, so that politician 2 is the most moderate. We compare four assignments: full decentralization, leadership by politician 2, a symmetric power-sharing agreement where politicians 2 and 3 make two decisions each, and an asymmetric power-sharing agreement where politician 2 makes three choices and politician 3 makes one choice.

First, consider leadership by politician 2. We calculate $d_2=4$ if $2\beta-\epsilon \le 1/12$, that is, $\beta \le \epsilon/2+1/24$, whereas $d_2=3$ if $\beta+\epsilon \le 1/10$, that is, $\beta \le 1/10-\epsilon$, as well as $d_2=2$ if $\beta-\epsilon \le 1/8$, that is, $\beta \le 1/8+\epsilon$, and $d_2=1$ if $\beta>1/8+\epsilon$.

Consider the symmetric power-sharing rule. First note that if 1 is willing to talk, then so are all the other players. Hence, for $2 (\beta + \varepsilon) + 2 \cdot 2\beta \le \frac{4}{2(3+3)}$, that is, $\beta \le 1/18 - \varepsilon/3$, both $d_2 = 4$ and $d_3 = 4$. Further, for $2 (2\beta - \varepsilon) + 2\beta \le \frac{4}{2(2+3)}$, that is, $\beta \le 1/15 + \varepsilon/3$, 1 does not talk, but 4 does, and so $d_2 = 3$ and $d_3 = 3$. Finally, for $\beta - \varepsilon \le 1/8$, that is, $\beta \le 1/8 + \varepsilon$, both 2 and 3 talk to each other: $d_2 = 2$ and $d_3 = 2$. Of course, $d_2 = 1$ and $d_3 = 1$ if $\beta > 1/8 + \varepsilon$.

Hence, the symmetric power-sharing rule dominates the single leader 2 on $(\epsilon/2 + 1/24, 1/18 - \epsilon/3]$ in terms of information transmission. It will dominate on a subset because of the moderation effect, but as $\epsilon \to 0$, the subset converges to (1/24, 1/18).

Consider now the asymmetric power-sharing rule. In this case, the condition for 1 to talk (if 4 is talking) becomes $\frac{3}{4}$ ($\beta + \epsilon$) + $\frac{1}{4}2\beta \le \frac{1}{2(3+3)}$, that is, $\beta \le 1/15 - 3\epsilon/5$. The condition for 4 to talk if 1 is talking becomes $\frac{3}{4}$ ($2\beta - \epsilon$) + $\frac{1}{4}\beta \le \frac{1}{2(3+3)}$, that is, $\beta \le 1/21 + 3\epsilon/7$. Hence, for $\beta \le 1/21 + 3\epsilon/7$, both $d_2 = 4$ and $d_3 = 4$. Instead, the condition for 1 to talk if 4 does not talk is $\frac{3}{4}$ ($\beta + \epsilon$) + $\frac{1}{4}2\beta \le \frac{1}{2(2+3)}$, that is, $\beta \le 2/25 - 3\epsilon/5$. The condition for 4 to talk if 1 does not talk is $\frac{3}{4}$ ($2\beta - \epsilon$) + $\frac{1}{4}\beta \le \frac{1}{2(2+3)}$, that is, $\beta \le \frac{3}{7}\epsilon + \frac{2}{35}$. Hence, for $\beta \le 2/25 - 3\epsilon/5$, both $d_2 = 3$ and $d_3 = 3$. The condition for 2 and 3 to talk to each other is $\beta \le 1/8 + \epsilon$; in this case, $d_2 = 2$ and $d_3 = 2$. Again, $d_2 = 1$ and $d_3 = 1$ if $\beta > 1/8 + \epsilon$.

Hence, the asymmetric power-sharing agreement dominates the single leader 2 informationally on $(\varepsilon/2 + 1/24, 1/21 - 3\varepsilon/7]$. Due to the moderation effect, it also dominates the symmetric power-sharing agreement. For $\varepsilon \to 0$, the asymmetric power-sharing agreement dominates on (1/24, 1/21).

Finally, consider full decentralization. The player who is least likely to speak publicly is 1. Given that all other players speak, he speaks if and only if $(\beta + \varepsilon) + 2\beta +$ $3\beta \le \frac{3}{2[3+3]}$ or $\beta \le \frac{1}{24} - \varepsilon/6$. In this case, all players receive three signals, $\frac{1}{24} = 0.041667$. Then, if 1 does not speak, the least likely to speak is 4. This occurs if and only if $\frac{3\beta}{3+3} + \frac{2\beta - \epsilon + \beta}{2+3} \le \frac{1}{2(3+3)^2} + \frac{2}{2(2+3)^2}$, that is, if $\beta \le \frac{2}{11}\epsilon + \frac{97}{1980}$ for $\epsilon \to 0$; this is close to $\frac{97}{1980} \approx 0.04899$. When 1 does not speak publicly, whereas 4 does, the ddistribution is 3, 2, 2, 2, which is informationally better than the private communication to 2. But, of course, it is worse in terms of moderation. Further, decentralization is dominated by the symmetric power-sharing agreements, for the range $\beta \le 1/18 - \varepsilon/3$, as $1/18 \approx$ 0.055556, because in this range, $d_2 = 3$ and $d_3 = 3$ for the asymmetric power-sharing agreements. Then, if 1 and 4 do not speak, the least likely to speak is 3 because 2 is more central. This occurs if and only if $\frac{2\beta}{2+3} + \frac{\beta-\epsilon}{1+3} + \frac{\beta}{2+3} \le \frac{1}{2(1+3)^2} + \frac{2}{2(2+3)^2}$, that is, $\beta \le \frac{5}{17}\epsilon + \frac{57}{680} \approx 0.083824$, with the distribution 2, 1, 1, 2. This is dominated by the asymmetric power-sharing agreements because for $\beta \le 1/10 - 3\epsilon/5$, that is, essentially, $\beta \le 1/10$, we have $d_2 = 2$ and $d_3 = 2$. Finally, 2 's condition to speak if nobody else speaks under decentralization is $2\beta - \varepsilon + \beta - \varepsilon + \beta + \varepsilon \le \frac{3}{2(1+3)}$, that is, $\beta \leq \frac{1}{4}\epsilon + \frac{3}{32} \approx 0.09375$. Because this yields the distribution 1, 0, 1, 1, we obtain that it is dominated by the asymmetric power-sharing agreements.

Proof of Proposition 2. From Proposition 1, we know that all Pareto optimal assignments a under private communication of common value information entail a single leader, that is, there is j such that a(k) = j for all k. Suppose now that communication is public, and suppose that an assignment a with a unique leader j is selected. Then, because $\gamma_i(\mathbf{m}) = 1$ and $\gamma_{i'}(\mathbf{m}) = 0$ for all $j' \neq j$, Condition (4) in Lemma 1 reduces to Condition (3). Hence, the set of equilibria under private and public communication coincide under a. But because the optimal assignment under public communication a* need not entail a single leader, the statement of the result immediately follows in the case that private conversations are ruled out under public communication. Allowing for private conversations does not change the argument because babbling all private conversations is always possibly part of an equilibrium, and we select the optimal equilibrium in any communication game that follows the assignment and the choice of communication rule.

Proof of Proposition 3. Part 1. Define Δ so that $\Delta \leq \frac{1}{2(I+2)}$. Since $\max_{j \in I} |b_j - b_{i^*}| \leq \Delta$ and $\Delta \leq \frac{1}{2(I+2)}$, then for all $i \in I$,

$$|b_i - b_{i^*}| \le \frac{1}{2(I+2)}.$$

Next, consider allocation a^* where all policies are given to i^* , that is, $a^*(k) = i^*$ for all k. Since $|b_i - b_{i^*}| \le \frac{1}{2(I+2)}$, there exists an equilibrium where all politicians communicate truthfully to player i^* .

The welfare of any assignment a is determined by Expression 2. Consider the assignment a^* in which all decisions are given to player i^* . Because all players are truthful in equilibrium to i^* , the second term of Expression 2 is maximized by a^* . The first term is also maximized by a^* by definition of i^* . In fact, supposing that any decision k is allocated to any player a(k), we can write the sum $\sum_{i \in I} (b_i - b_{a(k)})^2 \text{ as } \sum_{i \in I} (b_i - b_{i^*} + b_{i^*} - b_{a(k)})^2 = \sum_{i \in I} [(b_i - b_{i^*})^2 + (b_{i^*} - b_{a(k)})^2 + 2(b_i - b_{i^*})(b_{i^*} - b_{a(k)})^2] = I(b_{i^*} - b_{a(k)})^2 + \sum_{i \in I} (b_i - b_{i^*})^2 + 2(b_{i^*} - b_{a(k)}) \sum_{i \in I} (b_i - b_{i^*}),$ and the last term is 0 if and only if $a(k) = i^*$

Part 2. Fix $\{b_1, \ldots, b_I\}$ and consider the game with $\{\beta b_1, \ldots, \beta b_I\}$. Consider allocation a^* where the moderate politician i^* takes all the decisions, that is, $a^*(k) = i^*$ for all k, and let m^* be the associated best communication equilibrium. The welfare associated with (a^*, m^*) is

$$W(a^*, m^*) = -\beta^2 |\mathcal{K}| \sum_{i \in I} (b_i - b_{i^*})^2 - |\mathcal{K}| RV_{i^*}(a^*, m^*)$$

$$\geq -\beta^2 |\mathcal{K}| \sum_{i \in I} (b_i - b_{i^*})^2 - |\mathcal{K}| \frac{I}{18},$$

where $RV_{i^*}(a^*, m^*)$ is the residual variance of decision maker i^* under communication equilibrium m^* , and the inequality follows because aggregate residual variance can be bound above by the aggregate residual variance obtained in the babbling equilibrium, which is $|\mathcal{K}|I/18$.

Next, consider an arbitrary allocation $a \neq a^*$ and let m be the associated best communication equilibrium. We have that

$$W(a, m) = -\beta^{2} \sum_{k \in \mathcal{K}} \sum_{i \in I} (b_{i} - b_{a(k)})^{2} - \sum_{k \in \mathcal{K}} RV_{a(k)}(a, m)$$

$$\leq -\beta^{2} \sum_{k \in \mathcal{K}} \sum_{i \in I} (b_{i} - b_{a(k)})^{2},$$

where the inequality follows because, for every a(k), $RV_{a(k)}(a, m)$ can be bounded below by 0. Hence, to prove that for β sufficiently high $W(a^*, m^*) > W(a, m)$ for all $a \neq a^*$, it is sufficient to prove that

$$-\beta^{2}|\mathcal{K}|\sum_{i\in I}(b_{i}-b_{i^{*}})^{2}+\beta^{2}\sum_{k\in\mathcal{K}}\sum_{i\in I}(b_{i}-b_{j})^{2}$$

can be made arbitrarily large by increasing β . To see that this holds, note that

$$-|\mathcal{K}| \sum_{i \in I} (b_i - b_{i^*})^2 + \sum_{k \in \mathcal{K}} \sum_{i \in I} (b_i - b_{a(k)})^2$$

$$= \sum_{k \in \mathcal{K}} \left[\sum_{i \in I} (b_i - b_{a(k)})^2 - (b_i - b_{i^*})^2 \right],$$

which is a positive constant, and, since i^* is independent of β , such a positive constant is independent of β .

Proof of Proposition 4. The proof of Proposition 4 proceeds in two steps. The first step shows that if all decisions are allocated to a single agent, the information of this agent approaches infinity as the number of agents I goes to infinity. This is formalized in the following lemma. \square

Lemma 4. Suppose that biases b_j , j = 1, 2, ..., I are i.i.d. and drawn from a distribution of connected support. If all decisions are assigned to the same politician, i, then the optimal equilibrium information d_i^I of politician i grows to infinity in probability as I becomes infinite.

Proof of Lemma 4. Recall that for any I, the optimal equilibrium information d_i^I solves the condition

$$\left| \left\{ j = 1, 2, \dots, N : |b_i - b_j| \le \frac{1}{2(d_i^I + 2)} \right\} \right| = d_i^I.$$

We now show that, for d > 0,

$$\lim_{I \to \infty} \Pr\left(d_i^I \le d\right) = 0.$$

Note, in fact, that,

$$\Pr\left(d_i^I \le d\right) = \Pr\left(\left|\left\{j = 1, 2, \dots, N : \left|b_i - b_j\right|\right\}\right| \le \frac{1}{2(d+2)}\right\} \right| \le d\right)$$

$$= \Pr\left(\times_{j=1}^{I-d} \left\{b_j : \left|b_i - b_j\right| > \frac{1}{2(d+2)}\right\}\right)$$

$$= \left(\Pr\left\{b_j < b_i - \frac{1}{2(d+2)}\right\}\right)^{I-d},$$

$$+ \Pr\left\{b_j > b_i + \frac{1}{2(d+2)}\right\}\right)^{I-d},$$

and it is now immediate to see that

$$\lim_{I \to \infty} \Pr\left(d_i^I \le d\right) = \lim_{I \to \infty} \left(\Pr\left\{b_j < b_i - \frac{1}{2(d+2)}\right\}\right) + \Pr\left\{b_j > b_i + \frac{1}{2(d+2)}\right\} = 0.$$

This concludes the proof of Lemma 4.

We now turn to the second step. We compare the expected per-person per-action payoff $W_I^{m_I}$ for assigning all decisions K to the most moderate politician $m_I = \arg\min_i \left(b_i - \sum_{j=1}^I \frac{b_j}{I}\right)^2$, to the payoff $W_N^{\alpha\epsilon}$ for assigning a fraction $\alpha_I \geq \alpha > 0$ of the K actions, such that $\alpha_I K$ is an integer, to a different politician j_I such that $b_{j_I} - E\left[b_j\right] > \delta > 0$, for all I. The remaining fraction $1 - \alpha_I$ of actions is assigned to m_I . Hence,

$$W_{I}^{m_{I}} - W_{I}^{\alpha \varepsilon}$$

$$= E \left[\alpha_{I} \left(\sum_{i=1}^{I} \frac{(b_{i} - b_{j})^{2}}{I} - \sum_{i=1}^{I} \frac{(b_{i} - b_{m_{I}})^{2}}{I} + \frac{1}{2(d_{j} + 2)} - \frac{1}{2(d_{m_{I}} + 2)} \right) \right]$$

$$= \alpha_{I} E \left[\sum_{i=1}^{I} \frac{(b_{i} - b_{m_{I}} - (b_{j} - b_{m_{I}}))^{2}}{I} + \frac{1}{2(d_{j} + 2)} - \frac{1}{2(d_{m_{I}} + 2)} \right]$$

$$= \alpha_{I} E \left[\sum_{i=1}^{I} \frac{1}{2(d_{j} + 2)} - \frac{1}{2(d_{m_{I}} + 2)} \right]$$

$$\times \frac{(b_{i} - b_{m_{I}})^{2} + (b_{j} - b_{m_{I}})^{2} - 2(b_{i} - b_{m_{I}})(b_{j} - b_{m_{I}})}{I}$$

$$+ \alpha_{I} E \left[\frac{1}{2(d_{j} + 2)} - \frac{1}{2(d_{m_{I}} + 2)} \right].$$

Since, by Lemma 4, $\lim_{I\to\infty}\Pr(d^I_{m_I}\leq d)=0$ for all d>0, it follows that $\lim_{I\to\infty}E\left[\frac{1}{2(d_{m_I}+2)}\right]=0$. Further, $\lim_{I\to\infty}m_I=E\left[b_i\right]=\lim_{I\to\infty}E\left[\sum_{i=1}^I\frac{b_i}{I}\right]$. Using these facts, we have that

$$\lim_{I\to\infty}W_I^{m_I}-W_I^{\alpha\varepsilon}\geq \lim_{I\to\infty}\alpha\sum_{i=1}^I\frac{\left(b_j-b_{m_I}\right)^2}{I}\geq \alpha\delta^2>0.$$

This result implies that as I approaches infinity, all decisions are optimally concentrated to politicians sufficiently close to the most moderate agent m_I . This concludes the proof of Proposition 4.

Proof of Proposition 5. We first prove that full decentralization is never optimal. Note that if there is i > 1 such that $b_i - b_{i-1} \le 1/6$, then i is informed of

i-1's message and vice versa. For generic assignments of \mathbf{b} , it cannot be the case that $|\sum_{j=1}^{I} \gamma_j b_j - b_i| = |\sum_{j=1}^{I} \gamma_j b_j - b_{i-1}|$. Supposing without loss of generality that $|\sum_{j=1}^{I} \gamma_j b_j - b_i| < |\sum_{j=1}^{I} \gamma_j b_j - b_{i-1}|$, it is therefore welfare superior to assign a(i-1)=i rather than a(i-1)=i-1.

So suppose that $b_i - b_{i-1} > 1/6$ for all i, so that for all $j \neq i$, $d_{i,j}(\mathbf{m}) = 0$ in any equilibrium (\mathbf{m}, \mathbf{y}) . Hence, assigning $a(1) = \lfloor (I+1)/2 \rfloor \equiv m^*$ yields higher welfare than a(1) = 1 if and only if.

$$\sum_{i=1}^{I} \frac{(b_i - b_1)^2}{I} - \sum_{i=1}^{I} \frac{(b_i - b_{m^*})^2}{I} > \frac{1}{36}.$$

The left-hand side can be rewritten as

$$D(\Delta) = \sum_{i=2}^{I} \left[(i-1)\frac{1}{6} + \sum_{j=2}^{i} \left(\Delta_{j} - \frac{1}{6} \right) \right]^{2}$$
$$- \sum_{i=m+1}^{I} \left[(i-m)\frac{1}{6} + \sum_{j=m+1}^{i} \left(\Delta_{j} - \frac{1}{6} \right) \right]^{2}$$
$$- \sum_{i=1}^{m-1} \left[(m-i)\frac{1}{6} + \sum_{j=i+1}^{m} \left(\Delta_{j} - \frac{1}{6} \right) \right]^{2},$$

where $\Delta_2 = b_2 - b_1, \dots, \Delta_I = b_I - b_{I-1}$.

We now show that $D(\Delta)$ increases in all its terms Δ_k .

When k > m, we obtain

$$\begin{split} \frac{\partial}{\partial \Delta_k} D\left(\Delta\right) &= 2 \sum_{i=k}^{I} \left[(i-1) \frac{1}{6} + \sum_{j=2}^{i} \left(\Delta_j - \frac{1}{6} \right) \right] \\ &- 2 \sum_{i=k}^{I} \left[(i-m) \frac{1}{6} + \sum_{j=m+1}^{i} \left(\Delta_j - \frac{1}{6} \right) \right], \end{split}$$

which is clearly positive because m > 1 and $m + 1 \ge 2$. When k = m, we have

$$\frac{\partial}{\partial \Delta_k} D(\Delta) = 2 \sum_{i=k}^{I} \left[(i-1) \frac{1}{6} + \sum_{j=2}^{i} \left(\Delta_j - \frac{1}{6} \right) \right] > 0.$$

Suppose, finally, that k < m:

$$\frac{\partial}{\partial \Delta_k} D(\Delta) = 2 \sum_{i=k}^{I} \left[(i-1) \frac{1}{6} + \sum_{j=2}^{i} \left(\Delta_j - \frac{1}{6} \right) \right]$$
$$-2 \sum_{i=1}^{k-1} \left[(m-i) \frac{1}{6} + \sum_{j=i+1}^{m} \left(\Delta_j - \frac{1}{6} \right) \right]$$

$$=2\sum_{i=k}^{I}(i-1)\frac{1}{6}-2\sum_{i=1}^{k-1}(m-i)\frac{1}{6} \qquad (8)$$

$$+2\sum_{i=k}^{I}\sum_{j=2}^{i}\left(\Delta_{j}-\frac{1}{6}\right)-2\sum_{i=1}^{k-1}\sum_{j=i+1}^{m}\left(\Delta_{j}-\frac{1}{6}\right). (9)$$

Because k < m, evidently

$$2\sum_{i=k}^{I}(i-1)\frac{1}{6} > 2\sum_{i=m+1}^{I}(i-1)\frac{1}{6} > 2\sum_{i=m+1}^{I}(i-m)\frac{1}{6},$$

and
$$2\sum_{i=1}^{k-1}(m-i)\frac{1}{6} < 2\sum_{i=1}^{m-1}(m-i)\frac{1}{6}$$
.

and hence Expression (8) is strictly positive. Further,

$$2\sum_{i=k}^{1}\sum_{j=2}^{i}\left(\Delta_{j}-\frac{1}{6}\right) > 2\sum_{i=k}^{m}\sum_{j=2}^{i}\left(\Delta_{j}-\frac{1}{6}\right)$$

$$=2\sum_{i=2}^{k}\sum_{j=i}^{m}\left(\Delta_{j}-\frac{1}{6}\right)=2\sum_{i=1}^{k-1}\sum_{j=i+1}^{m}\left(\Delta_{j}-\frac{1}{6}\right),$$

and hence Expression (9) is strictly positive, concluding that $\partial D(\Delta)/\partial \Delta_k$ is strictly positive.

Therefore, we may take $\Delta = 1/6$, so that

$$D(1/6) = \sum_{i=2}^{I} \left[(i-1)\frac{1}{6} \right]^{2} - \sum_{i=m+1}^{I} \left[(i-m)\frac{1}{6} \right]^{2}$$
$$- \sum_{i=1}^{m-1} \left[(m-i)\frac{1}{6} \right]^{2}.$$

Noting that for *I* odd,

$$D(1/6) = \sum_{i=2}^{I} \left[(i-1)\frac{1}{6} \right]^{2} - 2\sum_{i=m+1}^{I} \left[(i-m)\frac{1}{6} \right]^{2}$$
$$= \frac{1}{4}I(I-1)^{2}\frac{1}{36} \ge \frac{1}{4} \cdot 3 \cdot 4\frac{1}{36} > \frac{1}{36},$$

and for I even,

$$D(1/6) = \sum_{i=2}^{I} (i-1)^2 - \sum_{i=I/2+1}^{I} (i-I/2)^2$$
$$- \sum_{i=1}^{I/2-1} (I/2-i)^2 = \frac{1}{4} I^2 (I-2) \frac{1}{36}$$
$$\ge \frac{1}{4} \cdot 16 \cdot 2 \frac{1}{36} > \frac{1}{36},$$

we conclude that $a(1) = \lfloor (I+1)/2 \rfloor \equiv m^*$ yields higher welfare than a(1) = 1.

Having proved that full decentralization is never optimal, we now show the most moderate politician should

be assigned the decision of sufficiently moderate and sufficiently extreme-bias politicians.

Indeed, first note that for any k such that $|b_k - b_{m*}| < 1/6$, the most moderate politician m^* is equally informed as k in equilibrium, and hence it is optimal that m^* is assigned policy k.

Second, recall that whenever $m(k) \neq m^*$, policy k should be assigned to m^* if and only if the inequality is satisfied. Expand the left-hand side of this inequality as follows:

$$\sum_{i=1}^{I} \frac{b_i^2 + b_{m(k)}^2 - 2b_i b_{m(k)} - b_i^2 - b_{m^*}^2 + 2b_i b_{m^*}}{I}$$

$$= \sum_{i=1}^{I} \frac{(b_{m(k)} + b_{m^*} - 2b_i)(b_{m(k)} - b_{m^*})}{I}$$

$$=(b_{m(k)}+b_{m^*}-2\bar{b})(b_{m(k)}-b_{m^*}),$$

where \bar{b} is the average of the bias vector b. Take k such that $b_k - b_{m^*} > 0$, the case when $b_k - b_{m^*} < 0$ is analogous. Note that, by construction, $0 < b_k - b_{m(k)} < 1/6$. Hence, if $b_k - b_{m^*}$ is sufficiently large, this is also the case for $b_{m(k)} - b_{m^*}$, so that the above expression is larger than 1/36 and the inequality 5 is satisfied.

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Supplementary Material for Online Appendix: Cabinet Simulations

In this appendix we explore further optimal decisionmaking authority assignments in cabinet governments by running simulations for a 7 member parliament in which players' biases are independent and identically distributed according to a skew normal distribution, a distribution chosen for tractability. Skew normal distributions depend on three parameters which are related with the three usual moments; mean μ , variance σ^2 and skewness γ , where γ controls the asymmetry of the sampled distributions of ideology draws and σ determines the concentration of such sampled distributions draws. The normal distribution is obtained as a special case when $\gamma = 0$, whereas the most extreme skewness is for $\gamma = 1$. Because only difference in ideologies matter for our characterization, we can normalize μ to zero, without loss of generality.

We first analyze cabinets where the θ is the same for all policies. We calculate two statistics that capture the degree of centralization of authority: (i) the average number of decisions allocated to the executive leader—the individual who makes the most decisions; and (ii) the frequency of draws for which a single leader makes all decisions in a cabinet environment. The results shown in

TABLE 1 The Average Number of Decisions made by the Executive Leader

	$\gamma = 0$	$\gamma = 1/4$	$\gamma = 1/2$	$\gamma = 3/4$	$\gamma = 1$
$\sigma^2 = 10$	7.00	7.00	7.00	6.99	7.00
$\sigma^2 = 1$	6.91	6.93	6.89	6.91	6.88
$\sigma^2 = 0.1$	6.17	6.21	6.18	6.16	6.21
$\sigma^2 = 0.01$	5.58	5.51	5.53	5.68	5.67
$\sigma^2 = 0.001$	6.35	6.35	6.37	6.49	6.35
$\sigma^2 = 0.0001$	7.00	7.00	7.00	7.00	7.00

TABLE 2 Frequency with which the Executive Leader makes all Decisions

	$\gamma = 0$	$\gamma = 1/4$	$\gamma = 1/2$	$\gamma = 3/4$	$\gamma = 1$
$\sigma^2 = 10$	1.00	1.00	1.00	0.99	1.00
$\sigma^2 = 1$	0.95	0.96	0.93	0.95	0.92
$\sigma^2 = 0.1$	0.57	0.60	0.58	0.61	0.62
$\sigma^2 = 0.01$	0.40	0.36	0.36	0.41	0.45
$\sigma^2 = 0.001$	0.70	0.70	0.74	0.78	0.69
$\sigma^2=0.0001$	1.00	1.00	1.00	1.00	1.00

table 1 and table 2 confirm a general tendency towards centralized authority, which have been described in large legislatures by Proposition 4. In fact, the average number of decisions made by the leader ranges from 79% to 100%. Interestingly, the fraction of decisions assigned to the leader is U-shaped in the variance of the distribution, and this holds independently of the asymmetry of the distribution, or skewness. Finally, allocating all actions to a single leader is often suboptimal: the frequency with which a single leader is chosen to implement *all* policy decisions may be below 50%. An implication is that in most cases centralization of authority in a multimember cabinet is Pareto superior to other executive forms.

Next we build on the results showing that full decentralization is never optimal when information is relevant to all policies. We explore optimal government in the case of policy specific information. As in the common state case, we discuss numerical results obtained for legislatures with I=7 politicians. The simulation shown in Table 3 and 4 report the leader's average number of assigned decisions and the frequency with which the executive leader makes decisions when information is policy specific. The results show that centralization of executive authority is not smaller (in fact, usually, it is larger) than in the common-state case with evenly distributed expertise for given parameter values γ and σ of

TABLE 3 Average Number of Decisions Assigned to the Executive Leader when Information is Policy-specific

	$\gamma = 0$	$\gamma = 1/4$	$\gamma = 1/2$	$\gamma = 3/4$	$\gamma = 1$
$\overline{\sigma^2 = 10}$	6.96	6.95	6.97	6.97	6.97
$\sigma^2 = 1$	6.70	6.73	6.69	6.81	6.74
$\sigma^2 = 0.1$	5.97	5.90	5.90	5.86	5.75
$\sigma^2 = 0.01$	6.59	6.54	6.53	6.57	6.55
$\sigma^2 = 0.001$	7.00	7.00	7.00	7.00	7.00
$\sigma^2 = 0.0001$	7.00	7.00	7.00	7.00	7.00

TABLE 4 Frequency with which the Executive Leader makes all Decisions

	$\gamma = 0$	$\gamma = 1/4$	$\gamma = 1/2$	$\gamma = 3/4$	$\gamma = 1$
$\sigma^2 = 10$	0.97	0.96	0.97	0.97	0.97
$\sigma^2 = 1$	0.78	0.82	0.80	0.85	0.83
$\sigma^2 = 0.1$	0.38	0.39	0.39	0.37	0.29
$\sigma^2 = 0.01$	0.66	0.64	0.64	0.64	0.66
$\sigma^2 = 0.001$	1.00	1.00	1.00	1.00	1.00
$\sigma^2 = 0.0001$	1.00	1.00	1.00	1.00	1.00

the skew normal distribution. Thus, whilst we uncover rich equilibrium behavior allowing for both single common state leadership and cabinet arrangements, the optimal decision-making authority assignment is no more decentralized than in the common-state case. Further, the fraction of decisions made by the most moderate politician is non-monotonic in the dispersion of politicians' ideologies.