EC9D3 Advanced Microeconomics, Part I: Lecture 7

Francesco Squintani

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Back to Pareto Efficiency

Recall

Result

An allocation x^* is Pareto-efficient if there exists a vector of weights $\lambda = (\lambda^1, \ldots, \lambda^I)$ such that x^* solves the following problem:

$$\begin{aligned} \max_{x^1,\ldots,x^I} \quad & \sum_{i=1}^{I} \lambda^i \, u_i(x^i) \\ s.t \quad & \sum_{i=1}^{I} x^i \leq \bar{\omega} \end{aligned} \tag{1}$$

We now consider the *only if* statement.

Back to Pareto Efficiency (2)

Proof: *If:* If x^* is Pareto-efficient there exist λ such that x^* solves (1).

To prove this implication we need the Second Welfare Theorem and the following remark.

Remark

Let $U : \mathbb{R}_+^N \to \mathbb{R}$ be continuously differentiable, concave and monotonic. Consider the following problem:

$$\max_{x \in \mathbb{R}_+^N} U(x) \qquad s.t. \quad p\, x \leq p\, \omega$$

Then there exists a $\mu > 0$ such that

$$\frac{\partial U(x)}{\partial x_l} = \mu\, p_l \quad \forall l = 1, \ldots, N$$

Proof: By Kuhn-Tucker Theorem. □

Back to Pareto Efficiency (3)

The *if* statement above can me written as follows.

Claim

Assume that x^* is a Pareto-efficient allocation with $x^{*,i} > 0$ for all $i \leq I$, and that $u_i(\cdot)$ are monotonic, concave and continuously differentiable.

Then there exists an I-tuple $\lambda^1, \ldots, \lambda^I > 0$ such that x^* solves the planner's problem (1).

Moreover, λ^i is the inverse of the marginal utility of income.

Back to Pareto Efficiency (4)

Proof: By Second Welfare Theorem, since x^* is Pareto-efficient it is a Walrasian Equilibrium for endowments $x^{*,i} = \omega^i$ and a price vector $p^* > 0$.

Therefore for a given p^* consumers maximize their utility subject to budget constraint by choosing $x^{*,i}$.

In other words, by Remark 1 above, there exists a I-tuple: $\gamma^1, \ldots, \gamma^I > 0$ such that:

$$\frac{\partial u_i(x^{*,i})}{\partial x_l^i} = \gamma^i \; p_l^* \quad \forall i, \; \forall l$$

Back to Pareto Efficiency (5)

Consider now Problem (1).

This is a *concave problem:* concave objective function and linear constraint,

therefore x^* solves it if we can find an L-tuple $\alpha^1, \ldots, \alpha^L > 0$ such that:

$$\frac{\partial \left[\sum_{i=1}^{I} \lambda^i \; u_i(x^{*,i}) \right]}{\partial x_l^i} = \alpha_l \quad \forall i, \; \forall l$$

Back to Pareto Efficiency (6)

or

$$\lambda^i \frac{\partial u_i(x^{*,i})}{\partial x_l^i} = \alpha_l \quad \forall i, \, \forall l$$

Choosing now

$$\lambda^i = \frac{1}{\gamma^i} \qquad \alpha_l = p_l^*$$

and noticing that γ^i is the marginal utility of income concludes the proof. $\square$

Notice that α_l are the *shadow prices of the feasibility conditions*, and according to the result above correspond to the Walrasian equilibrium prices.

Production Economy

Consider an economy with I consumers and J producers characterized by their *production possibility set* Y^j.

Define a *production economy with private ownership* as follows:

$$\hat{\mathcal{E}} = \left\{(\omega^1, \ldots, \omega^I); u_i(\cdot); Y^j; \theta_i^j, \forall i \leq I, \forall j \leq J\right\}$$

where θ_i^j is *the share owned by consumer i of firm j*, of course:

$$\sum_{i=1}^{I} \theta_i^j = 1 \quad \forall j \leq J$$

Walrasian Equilibrium with Production

Definition

A *Walrasian equilibrium* $WE = \{p^*; (x^{*,1}, \ldots, x^{*,I}); (y^{*,1}, \ldots, y^{*,J})\}$ for a production economy $\hat{\mathcal{E}}$ is such that:

1. $x^{*,i}$ solves for every $i \leq I$:

$$\max_{x^i} u_i(x^i) \qquad \text{s.t.} \quad p^* x^i \leq p^* \omega^i + \sum_{j=1}^{J} \theta_i^j \left(p^* y^{*,j}\right)$$

2. $y^{*,j}$ solves for every $j \leq J$:

$$\max_{y^j} p^* y^j \qquad \text{s.t.} \quad y^j \in Y^j$$

3. and market clearing conditions are satisfied:

$$\sum_{i=1}^{I} x^{*,i} - \sum_{j=1}^{J} y^{*,j} - \sum_{i=1}^{I} \omega^i \leq 0$$

Walrasian Equilibrium with Production (2)

Define *aggregate excess demand* for this economy as:

$$Z(p) = \sum_{i=1}^{I} x^i(p) - \sum_{j=1}^{J} y^j(p) - \sum_{i=1}^{I} \omega^i$$

where $x^i(p)$ is consumer i's Marshallian demand and $y^j(p)$ is firm j's optimal production plan.

Notice that $Z(p)$ is *homogeneous of degree zero* in p.

Both $x^i(p)$ and $y^j(p)$ are homogeneous of degree zero in p.

Walrasian Equilibrium with Production (3)

Walras Law:

$$p\ Z(p) = 0.$$

This is obtained once again by summing each consumer's budget constraint:

$$p\ x^i(p) - p\ \omega^i - \sum_{j=1}^{J} \theta_i^j \left(p\ y^j(p)\right) = 0$$

Indeed:

$$\sum_{i=1}^{I} p\ x^i(p) - \sum_{i=1}^{I} p\ \omega^i - \sum_{i=1}^{I} \sum_{j=1}^{J} \theta_i^j \ \left(p\ y^j(p)\right) = 0$$

Walrasian Equilibrium with Production (4)

In other words:

$$\sum_{i=1}^{I} p\, x^i(p) - \sum_{i=1}^{I} p\, \omega^i - \sum_{j=1}^{J} \left(\sum_{i=1}^{I} \theta_i^j \right) \left(p\, y^j(p)\right) = 0$$

or

$$p \left[\sum_{i=1}^{I} x^i(p) - \sum_{i=1}^{I} \omega^i - \sum_{j=1}^{J} y^j(p) \right] = 0$$

We can now state the three main Theorems we have proved in a pure exchange economy for the production economy $\hat{\mathcal{E}}$.

Walrasian Equilibrium with Production: Existence

Theorem (Existence Theorem)

Consider a $Z(p)$ that satisfies the following conditions:

1. $Z(p)$ *is single valued;*
2. $Z(p)$ *is continuous;*
3. $Z(p)$ *is homogeneous of degree 0;*
4. $Z(p)$ *satisfies Walras Law;*
5. $Z(p)$ *is bounded;*

then there exists p^* *such that*

$$Z(p^*) \leq 0.$$

Properties of Walrasian Equilibrium with Production

Definition

An allocation is *feasible* for a production economy $\hat{\mathcal{E}}$ if and only if there exists a production plan y^j for every firm $j \leq J$ such that

$$y^j \in Y^j \quad \forall j \leq J$$

and

$$\sum_{i=1}^{I} x^i \leq \sum_{i=1}^{I} \omega^i + \sum_{j=1}^{J} y^j$$

Definition

An allocation is *Pareto-efficient* for a production economy $\hat{\mathcal{E}}$ if and only if

1. it is *feasible*
2. and there *does not exists an alternative feasible allocation $\hat{x}$ that Pareto-dominates it*.

Properties of Walrasian Equilibrium with Production (2)

Theorem (First Welfare Theorem)

Let $\hat{\mathcal{E}}$ be a production economy with consumer preferences satisfying weak monotonicity.

Let

$$WE^* = \left\{ p^*; (x^{*,1}, \ldots, x^{*,I}); (y^{*,1}, \ldots, y^{*,J}) \right\}$$

be a Walrasian equilibrium for $\hat{\mathcal{E}}$.

Then x^* is a Pareto-efficient allocation.

Properties of Walrasian Equilibrium with Production (3)

Theorem (Second Welfare Theorem)

Assume that x^*, such that $x^{*,i} > 0$, is Pareto-efficient and that

1. *preferences $u_i(\cdot)$ are strongly monotonic;*
2. *preferences are convex: $u_i(\cdot)$ are quasi-concave;*
3. *technology Y^j is convex for every $j \leq J$ and $0 \in Y^j$.*

Then there exists a redistribution of endowments ω^i for $i \leq I$ and a vector of shares $\bar{\theta}^j_i$, for $i \leq I$ and $j \leq J$ such that

$$\left\{ p^*; (x^{*,1}, \ldots, x^{*,I}); (y^{*,1}, \ldots, y^{*,J}) \right\}$$

is a Walrasian equilibrium for the economy $\hat{\mathcal{E}}$.

Properties of Walrasian Equilibrium with Production (4)

Lemma

Assume that x^*, such that $x^{*,i} > 0$, is Pareto-efficient and that:

1. *preferences are strongly monotonic;*
2. *preferences are convex;*
3. *Y^j are convex for every $j \leq J$;*
4. *$0 \in Y^j$ for every $j \leq J$.*

Then there exists a $y^{*,j}$ for every firm $j \leq J$ and a vector p^* such that:

- $\sum_{i=1}^{I} x^{*,i} = \sum_{j=1}^{J} y^{*,j} + \sum_{i=1}^{I} \omega^i;$
- $u_i(x'^i) > u_i(x^{*,i})$ *implies* $p^* x'^i > p^* x^{*,i}$*;*
- $p^* y^{*,j} \geq p^* y^j$ *for every* $y^j \in Y^j$ *and every* $j \leq J$.

Properties of Walrasian Equilibrium with Production (5)

Proof of Second Welfare Theorem: Given this Lemma we just need to find the re-distribution which at p^* gives exactly $\left(p^* x^{*,i}\right)$ to every consumer.

Let $V^* = \sum_{i=1}^{I} p^* x^{*,i}$ and

$$V_i^* = p^* x^{*,i}.$$

Let also

$$s^i = \frac{V_i^*}{V^*}$$

be i's share of the total value.

Properties of Walrasian Equilibrium with Production (6)

We prove the Theorem by setting:

$$\bar{\omega}_i = s^i \sum_{i=1}^{I} \omega^i \quad \forall i \leq I$$

and

$$\bar{\theta}_i^j = s^i \qquad \forall i \leq I \quad \forall j \leq J.$$

By construction this choice of $\bar{\omega}_i$ and $\bar{\theta}_i^j$ implies that the budget constraint of each agent is satisfied at p^*:

$$p^* \; x^{*,i} = p^* \; \bar{\omega}^i - \sum_{j=1}^{J} \bar{\theta}_i^j \left(p^* \; y^{*,j} \right) =$$

$$= s^i \; p^* \left(\sum_{i=1}^{I} \omega^i + \sum_{j=1}^{J} y^{*,j} \right) = s^i \; V^* = V_i^* \quad \square$$

Externalities

Definition

An externality is any indirect effect that either a production or a consumption activity has on a utility function, a consumption set or a production set.

An indirect effect is an effect that is:

- created by an economic agent other than the one who is affected;
- not transmitted through prices.

Example: two firms that pollute each other environment, each one imposes a negative external effect on the other.

Ownership Rights and Markets

Notice that:

- If the two firms merge the pollution effect on each other is not an external effect any more but part of the firm's technology.
- If a market in pollution rights is created then firm i must buy from firm j a pollution right such as it would buy any other intermediate input: the externalities are incorporated into the market transactions.

The general equilibrium model does not treat as endogenous the size of agents and the number of markets.

It takes them for given.

Examples

A firm polluting a river and thus decreasing the possibilities for swimming is an externality: the external effect of a production activity on a consumption set.

In this case the consumption feasible set is a correspondence that depends on the production levels of the firms and the consumption levels of the other consumers:

$$X^i(x^1, \dots, x^{i-1}, x^{i+1}, \dots, x^I, y^1, \dots, y^J)$$

Examples (2)

The noise emanating from the stereo system of one's neighbor is a typical consumption externality.

The utility function of the concerned consumer i depends on consumer's j's music consumption x^j_m:

$$u^i(x^i, x^j_m)$$

In general, a consumption externality is characterized by:

$$u^i(x^1, \ldots, x^I)$$

Examples (3)

Meade (1952)'s famous example of the beekeeper and the orchard is a typical example of a mutual production externality.

The production function of each firm depends on the input of the other firm:

$$f^1(x^1, x^2), \qquad f^2(x^1, x^2)$$

In general, the PPS of both firms is a correspondence that depends on the production plan of all firms:

$$Y^j(y^1, \ldots, y^J)$$

Robinson Crusoe's Economy with Externalities

There exists a single consumer, two goods and two firms.

Clearly in this economy there is no issue of who owns the firms.

Assume that there are two externalities imposed on firm 2:

- an externality generated by the consumer's consumption of good 1: x_1;
- an externality generated by firm 1's production of good 1: y_1^1.

Preferences and Technology

The consumer's preferences are: $u(x_1, x_2)$.

Firm 1's technology: $y_1^1 = f^1(y_2^1)$ (differentiable and concave). Recall that by sign convention y_2^1 is negative.

Firm 2's technology: $y_2^2 = f^2(y_1^2, y_1^1, x_1)$ (differentiable and concave). Recall that by sign convention y_1^2 is negative.

Let $\omega = (\omega_1, \omega_2)$ be the consumer's endowment vector.

Pareto Efficiency

We consider first the *Pareto efficient allocation*.

This is the solution to the following central planner's problem:

$$
\begin{aligned}
\max_{\{x_1, x_2, y_1^1, y_2^1, y_1^2, y_2^2\}} \quad & U(x_1, x_2) \\
\text{s.t.} \quad & x_1 \leq \omega_1 + y_1^1 + y_1^2 \\
& x_2 \leq \omega_2 + y_2^1 + y_2^2 \\
& y_1^1 = f^1(y_2^1) \\
& y_2^2 = f^2(y_1^2, y_1^1, x_1)
\end{aligned}
$$

Pareto Efficiency (2)

Consider now the necessary and sufficient first order conditions:

$$\begin{aligned}
\frac{\partial U}{\partial x_1} - \lambda_1 + \mu_2 \frac{\partial f^2}{\partial x_1} &= 0 \\
\frac{\partial U}{\partial x_2} - \lambda_2 &= 0 \\
\lambda_1 - \mu_1 + \mu_2 \frac{\partial f^2}{\partial y_1^1} &= 0 \\
\lambda_2 + \mu_1 \frac{\partial f^1}{\partial y_2^1} &= 0 \\
\lambda_2 - \mu_2 &= 0 \\
\lambda_1 + \mu_2 \frac{\partial f^2}{\partial y_1^2} &= 0
\end{aligned}$$

Pareto Efficiency (3)

They can be re-written, eliminating the multipliers:

$$\frac{\dfrac{\partial U}{\partial x_1} + \dfrac{\partial U}{\partial x_2}\,\dfrac{\partial f^2}{\partial x_1}}{\dfrac{\partial U}{\partial x_2}} = -\frac{\partial f^2}{\partial y_1^2} = -\frac{1 + \dfrac{\partial f^2}{\partial y_1^1}\,\dfrac{d\,f^1}{d\,y_2^1}}{\dfrac{d\,f^1}{d\,y_2^1}}$$

This corresponds to the equality of the:

- the *social marginal rate of substitution*, that takes the consumption externality into account,
- the *social marginal rate of transformation of firm* 2, that coincides with the private one,
- the *social marginal rate of transformation of firm* 1, that takes the production externality into account.

Walrasian Equilibrium

Consider now the *Walrasian equilibrium* of this economy.

Notice that the key assumption is that each individual agent considers as parameters not only the prices but also the other variables that characterize his decision set.

In particular, these variables — y_1^2 and x_1 for firm 2 — must be equal to the choices of the other agents.

Let $p = (p_1, p_2)$ the vector of prices in this perfectly competitive economy.

Walrasian Equilibrium (2)

Firm 1's maximization problem is clearly not affected by externalities:

$$\max_{\{y_1^1, y_2^1\}} \quad p_1\, y_1^1 + p_2 y_2^1$$

$$\text{s.t.} \quad y_1^1 = f^1(y_2^1)$$

The private marginal rate of transformation equals the price ratio:

$$-\frac{1}{\frac{d\, f^1}{d\, y_2^1}} = \frac{p_1}{p_2}$$

Let $y^{*,1} = (y_1^{*,1}, y_2^{*,1})$ be the solution to this problem.

Walrasian Equilibrium (3)

The consumer's utility maximization problem is also not affected by externalities:

$$\max_{\{x_1, x_2\}} \quad U(x_1, x_2)$$

$$\text{s.t.} \quad p_1\, x_1 + p_2 x_2 \leq p_1\, \omega_1 + p_2\, \omega_2 + \Pi(p_1, p_2)$$

where

$$\Pi(p_1, p_2) = (p_1\, y_1^{*,1} + p_2\, y_2^{*,1}) + (p_1\, y_1^{*,2} + p_2\, y_2^{*,2})$$

The private marginal rate of substitution equals the price ratio:

$$\frac{\partial U / \partial x_1}{\partial U / \partial x_2} = \frac{p_1}{p_2}$$

Let $x^* = (x_1^*, x_2^*)$ be the solution to this problem.

Walrasian Equilibrium (4)

Firm 2 is affected by the externalities, from firm 2's view point $y_1^{*,1}$ and x_1^* are given, it cannot control them:

$$\max_{\{y_1^2, y_2^2\}} \quad p_1\, y_1^2 + p_2 y_2^2$$

$$\text{s.t.} \quad y_2^2 = f^2(y_1^2, y_1^{*,1}, x_1^*)$$

The private marginal rate of transformation equals the price ratio:

$$-\frac{\partial f^2(y_1^2, y_1^{*,1}, x_1^*)}{\partial y_1^2} = \frac{p_1}{p_2}$$

Let $y^{*,2} = (y_1^{*,2}, y_2^{*,2})$ be the solution to this problem.

Walrasian Equilibrium (5)

Therefore the Walrasian equilibrium of this economy is a vector of prices

$$p^* = (p_1^*, p_2^*)$$

and an allocation

$$\{x^*, y^{*,1}, y^{*,2}\}$$

such that:

- The allocation $\{x^*, y^{*,1}, y^{*,2}\}$ solves the three problems above given the vector of prices p^*;
- Markets clear:

$$\begin{aligned} x_1^* &= \omega_1 + y_1^{*.1} + y_1^{*,2} \\ x_2^* &= \omega_2 + y_2^{*.1} + y_2^{*,2} \end{aligned}$$

The key comparison is the one between *the two marginal conditions that define the Pareto efficient allocation*:

$$\frac{\dfrac{\partial U}{\partial x_1} + \dfrac{\partial U}{\partial x_2}\,\dfrac{\partial f^2}{\partial x_1}}{\dfrac{\partial U}{\partial x_2}} = -\frac{\partial f^2}{\partial y_1^2} = -\frac{1 + \dfrac{\partial f^2}{\partial y_1^1}\,\dfrac{d\,f^1}{d\,y_2^1}}{\dfrac{d\,f^1}{d\,y_2^1}}$$

and the two marginal conditions that define the Walrasian equilibrium allocation:

$$\frac{\dfrac{\partial U}{\partial x_1}}{\dfrac{\partial U}{\partial x_2}} = -\frac{\partial f^2}{\partial y_1^2} = -\frac{1}{\dfrac{d\,f^1}{d\,y_2^1}}$$

Failure of First Welfare Theorem

Result

This allow us to concludes that in general the Walrasian equilibrium of an economy with externalities is not Pareto efficient.

In other words in the presence of externalities the First Welfare Theorem does not hold.

In general, economic decisions appear to be too decentralized at a Walrasian equilibrium allocation: *they do not take into account the external effect that individual decision have on other agents*.

In general:

- a firm exercising a negative externality will produce too much,
- while a firm exercising a positive externality will produce too little.

Incomplete Markets

One way to interpret the inefficiency we identified is in terms of *incomplete markets*

In other words, in the economy we considered two markets do not exist:

- the market through which firm 1 acquires the right to exert an externality on firm 2;
- the market through which the consumer acquires the right to exert an externality on firm 2.

One way to amend the inefficiency is *to establish firm 2's ownership rights on its production activity* and hence create the missing markets.

Completing Markets

Assume that both these markets are perfectly competitive (strong assumption).

Let

- $q_1^{1,2}$ be the price at which firm 1 must buy from firm 2 the right to exercise its externality,
- $p_1^{1,2}$ be the price at which the consumer must buy from firm 2 the right to exercise her externality.

Let now $(p_1, p_2, q_1^{1,2}, p_1^{1,2})$ be the vector of prices of this redefined perfectly competitive economy.

Completing Markets (2)

Firm 1's maximization problem is now:

$$\max_{\{y_1^1, y_2^1, y_1^{1,2}\}} \quad p_1\, y_1^1 + p_2 y_2^1 - q_1^{1,2}\, y_1^{1,2}$$

$$\text{s.t.} \quad y_1^1 = f^1(y_2^1)$$

$$y_1^{1,2} = y_1^1$$

Enforcement of ownership rights implies that $y_1^{1,2} = y_1^1$. The marginal condition is now:

$$-\frac{1}{\frac{d\, f^1}{d\, y_2^1}} = \frac{(p_1 - q_1^{1,2})}{p_2}$$

Let $(\hat{y}_1^1, \hat{y}_2^1, \hat{y}_1^{1,2})$ be the solution to this problem.

Completing Markets (3)

The consumer's utility maximization problem is now:

$$\begin{aligned} \max_{\{x_1, x_2, x_1^{1,2}\}} \quad & U(x_1, x_2) \\ \text{s.t.} \quad & p_1\, x_1 + p_2 x_2 + p_1^{1,2}\, x_1^{1,2} \leq p_1\, \omega_1 + p_2\, \omega_2 + \hat{\Pi} \\ & x_1^{1,2} = x_1 \end{aligned}$$

where

$$\hat{\Pi} = (p_1\, \hat{y}_1^1 + p_2\, \hat{y}_2^1 - q_1^{1,2}\, \hat{y}_1^{1,2}) + (p_1\, \hat{y}_1^2 + p_2\, \hat{y}_2^2 + q_1^{1,2}\, \hat{y}_1^{1,2} + p_1^{1,2}\, \hat{x}_1^{1,2})$$

The marginal condition is then:

$$\frac{\partial U / \partial x_1}{\partial U / \partial x_2} = \frac{p_1 + p_1^{1,2}}{p_2}$$

Let $(\hat{x}_1, \hat{x}_2, \hat{x}_1^{1,2})$ be the solution to this problem.

Completing Markets (4)

Firm 2 controls the supply of the externality rights $(\bar{y}_1^{1,2}, \bar{x}_1^{1,2})$ therefore its maximization problem is now:

$$\max_{\{y_1^2, y_2^2, \bar{y}_1^{1,2}, \bar{x}_1^{1,2}\}} \quad p_1\, y_1^2 + p_2 y_2^2 + q_1^{1,2}\, \bar{y}_1^{1,2} + p_1^{1,2}\, \bar{x}_1^{1,2}$$

$$\text{s.t.} \quad y_2^2 = f^2(y_1^2, \bar{y}_1^1, \bar{x}_1)$$

The marginal conditions are then:

$$p_2 \frac{\partial f^2}{\partial y_1^2} + p_1 = p_2 \frac{\partial f^2}{\partial \bar{y}_1^{1,2}} + q_1^{1,2} = p_2 \frac{\partial f^2}{\partial \bar{x}_1^{1,2}} + p_1^{1,2} = 0$$

Let $(\hat{y}_1^2, \hat{y}_2^2, \hat{\bar{y}}_1^{1,2}, \hat{\bar{x}}_1^{1,2})$ be the solution to this problem.

Completing Markets (5)

Therefore the Walrasian equilibrium of this economy is a vector of prices

$$(p_1, p_2, q_1^{1,2}, p_1^{1,2})$$

and an allocation

$$\{(\hat{x}_1, \hat{x}_2, \hat{x}_1^{1,2}), (\hat{y}_1^1, \hat{y}_2^1, \hat{y}_1^{1,2}), (\hat{y}_1^2, \hat{y}_2^2, \hat{\hat{y}}_1^{1,2}, \hat{\hat{x}}_1^{1,2})\}$$

such that:

- The allocation solves the three problems above given the vector of prices;
- Markets clear:

$$x_1^* = \omega_1 + y_1^{*.1} + y_1^{*,2} \qquad \hat{\hat{y}}_1^{1,2} = \hat{y}_1^{1,2}$$

$$x_2^* = \omega_2 + y_2^{*.1} + y_2^{*,2} \qquad \hat{\hat{x}}_1^{1,2} = \hat{x}_1^{1,2}$$

Completing Markets (6)

Putting together the marginal conditions that define the Walrasian equilibrium we now conclude that:

$$\frac{\dfrac{\partial U}{\partial x_1} + \dfrac{\partial U}{\partial x_2}\,\dfrac{\partial f^2}{\partial x_1}}{\dfrac{\partial U}{\partial x_2}} = -\frac{\partial f^2}{\partial y_1^2} = -\frac{1 + \dfrac{\partial f^2}{\partial y_1^1}\,\dfrac{d\,f^1}{d\,y_2^1}}{\dfrac{d\,f^1}{d\,y_2^1}}$$

In other words, when markets are complete the Walrasian equilibrium allocation is Pareto efficient: the First Welfare Theorem holds.

Coase's Observation

Result (Coase 1960)

Provided markets are complete it does not matter for Pareto efficiency how property rights are allocated.

Assume that firm 1 is allocated ownership rights on a quantity $\bar{Q}$ of externality and any reduction in this quantity has to be purchased from firm 1.

Similarly assume that the consumer is allocated ownership rights on an amount $\bar{x}$ of externality and any reduction has to be purchased from the consumer.

Let once again $(p_1, p_2, q_1^{1,2}, p_1^{1,2})$ be the vector of prices of this redefined perfectly competitive economy.

Coase's Observation (2)

The consumer's utility maximization problem is then:

$$\max_{\{x_1, x_2, x_1^{1,2}\}} \quad U(x_1, x_2)$$

$$\text{s.t.} \quad p_1\, x_1 + p_2 x_2 \leq p_1\, \omega_1 + p_2\, \omega_2 + \bar{\Pi} + p_1^{1,2}\, (\bar{x} - x_1^{1,2})$$

$$x_1^{1,2} = x_1$$

where $\bar{\Pi}$ is the total profit of firm 1 and 2, and $(\bar{x} - x_1^{1,2})$ is the amount of externality that the consumer supplies.

The budget constraint can then be re-written as:

$$p_1\, x_1 + p_2 x_2 + p_1^{1,2}\, x_1^{1,2} \leq p_1\, \omega_1 + p_2\, \omega_2 + \bar{\Pi} + p_1^{1,2}\, \bar{x}$$

Coase's Observation (3)

Therefore the marginal condition is the same as the one above:

$$\frac{\partial U/\partial x_1}{\partial U/\partial x_2} = \frac{p_1 + p_1^{1,2}}{p_2}$$

Putting together the marginal conditions that define the Walrasian equilibrium we obtain once again:

$$\frac{\dfrac{\partial U}{\partial x_1} + \dfrac{\partial U}{\partial x_2}\dfrac{\partial f^2}{\partial x_1}}{\dfrac{\partial U}{\partial x_2}} = -\frac{\partial f^2}{\partial y_1^2} = -\frac{1 + \dfrac{\partial f^2}{\partial y_1^1}\dfrac{d\,f^1}{d\,y_2^1}}{\dfrac{d\,f^1}{d\,y_2^1}}$$

In other words, the allocation of property right affects the distribution of surplus but *does not affect the Pareto efficiency* of the allocation.