University of Warwick, Department of Economics
EC941: Game Theory Prof. Francesco Squintani

Problem Set 6

Exercise 1. There are 3 players. Player 1 first chooses between C and D. If choosing C, it is then player 2's turn to move. She may choose either c or d. If player 2 chooses c, then the game is over, and each player's payoff equals to 1. If either player 1 chooses D, or player 1 chooses C and then player 2 chooses d, then player 3 is called to play. Player 3 must choose between L and R, without knowing the past history of play. Her choice results in the following payoffs: $u_1(D,L) = 3$, $u_2(D,L) = 3$, $u_3(D,L) = 2$; $u_1(D,R) = 0$, $u_2(D,R) = 0$, $u_3(D,R) = 0$; $u_1(C,d,L) = 4$, $u_2(C,d,L) = 4$, $u_3(C,d,L) = 0$; and $u_1(C,d,R) = 0$, $u_2(C,d,R) = 0$, $u_3(C,d,R) = 1$.

a. Model this situation as an extensive form game, and draw an appropriate tree diagram.

b. Find its pure-strategy perfect Bayesian equilibria.

Exercise 2. A hungry bird has a piece of food that it may give to its offspring, or keep for itself. It does not detect whether or not its offspring is hungry. In either case, the offspring may signal that it is hungry to its parent (by squawking, for example). An animal is stronger and thus produces more offspring (i.e. has a higher "biological fitness") if it gets the food than if it does not. Normalize the parent's strength if it keeps the food to be 1, and denote its strength if it gives the food to its offspring by $S < 1$. If the offspring does not squawk, its strength is 1 if it gets the food, $V < 1$ if it is not hungry and does not get the food, and 0 if it is hungry and does not get the food. If the offspring squawks, its strength is multiplied by the factor $1 - t$ where $0 \leq t \leq 1$ (i.e. squawking is costly). Denote the degree to which the parent and offspring are related by r, and take each player's payoff to be its strength plus r times the other player's strength. Evolutionary pressure will lead to behavior for each player that maximizes that player's payoff, given the other player's behavior.

a. Model this situation as an extensive form game, and draw an appropriate tree diagram.

b. Find the conditions on r, in terms of S, V, and t, under which the game has a separating equilibrium in which the offspring squawks if and only if it is hungry and the parent gives it the food if and only if it squawks.

c. Show that if the offspring's payoff from obtaining the food exceeds her payoff from not obtaining it whether or not she is hungry (which means that $r < (1 - V)/(1 - S)$, then

the game has such an equilibrium only if$t > 0$. That is, in this case an equilibrium exists in which the signal is accurate only if the signal is costly.

c. Show that if $r < (1 - S)/(1 - V)$ then the game has a pooling equilibrium in which the offspring is quiet whether or not it is hungry and the parent keeps the food whether or not the offspring squawks.