Problem Set 7

Exercise 1. Consider Bertrand's model of duopoly in the case that each firm's unit cost is constant, equal to c. Let $\Pi(p) = (p-c)D(p)$ for any price p,and assume that Π is continuous and is uniquely maximized at the price p^m (the "monopoly price"). Let s be the strategy for the infinitely repeated game that charges p^m in the first period and subsequently as long as both firms continue to charge p^m, and punishes any deviation from p^m by either firm by choosing the price c for k periods, then reverting to p^m.

a. Given any value of δ, for what values of k is the strategy pair (s, s) a Nash equilibrium of the infinitely repeated game?

b. Given any value of δ, for what values of k is the strategy pair (s, s) a subgame perfect equilibrium of the infinitely repeated game?

Exercise 2. Suppose that the game below is infinitely repeated with discount factor δ.

	C	D
A	5,3	2,6
B	6,0	3,1

a. Describe a strategy profile that yields player 1 an average discounted payoff of at most 4, and player 2 an average discounted payoff of at least 4, for $\delta \to 1$.

b. Determine for which values of δ this strategy profile is a subgame-perfect equilibrium.

Exercise 3. Consider the infinitely repeated Prisoner's Dilemma in which the payoffs of the component game are those given in the figure below.

	C	D
C	x,x	0,x+1
D	x+1,0	y,y

Consider the strategy following tit-for-tat strategy:

$$s_i\left(h^t\right) = \begin{cases} C & \text{if } t = 0 \text{ or } h_j^t = C \\ D & \text{if } t > 0 \text{ and } h_j^t = D. \end{cases}$$

a. Under which conditions on the parameters $x > y > 0$, and $0 < \delta < 1$ is the strategy profile (tit-for-tat, tit-for-tat) a Nash equilibrium?

b. Under which conditions on the parameters $x > y > 0$, and $0 < \delta < 1$ is the strategy profile (tit-for-tat, tit-for-tat) a subgame perfect equilibrium?