Advanced Economic Theory
Models of Elections
Lecture 5

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Agency models of re-election

. Downsian, citizen-candidate and probabilistic voting models are "prospective" theories.

. People vote only on the basis of credible electoral promises or candidate's ideologies.

. "Retrospective" models account for voters dismissing incumbents with poor performance, and retaining effective incumbents.

. Retrospective voting is modelled with repeated games and "simplified contracts."

. The principal (median voter) may only dismiss or retain an agent (politician), performance-based transfers are not allowed.

Moral hazard and adverse selection (Banks and Sundaram 1998)

. In each period $t = 0, 1, ...$, an infinitely-lived principal chooses whether to retain her agent, or hire a new one.

. Each agent is t employed at most 2 periods: t and $t + 1$.

. Each agent's ability $a \in \{a_1, ..., a_K\}$, is private information, and drawn from distribution p. Assume $a_1 < ... < a_K$.

. Each period, employed agent generates a random reward $r \in \mathbb{R}$.

. Reward distribution $F(r|e)$ depends on agent's effort $e \in [\underline{e}, \overline{e}]$.

. $F(r|e)$ has continuous density $f(r|e)$ of compact support R.

. $F(\cdot|e)$ is ranked in first-order stochastic dominance:
for any r, if $e > e'$, then $F(r|e) < F(r|e')$.

- Agent per-period payoff is $u(e, a)$ if employed, and 0 otherwise.
- u is continuous, strictly quasi-concave in e, and increasing in a:
  - opportunity cost of taking higher actions lower for better types;
  - for every $k = 1, ..., K$, there is a unique best effort e_k^* at the second period of employment.
- For each ability type a, there is an effort $e(a)$ with $u(e, a) > 0$.
- The payoff function u is supermodular in (e, a):

  If $(e, a) > (e', a')$, then $u(e, a) + u(e', a') > u(e', a) + u(e, a')$.

  (I.e. $u_{12} > 0$, if u is twice continuously differentiable.)
- The agents' discount factor is $\delta_A \in [0, 1]$.
- The per-period principal's utility for reward r is $v(r)$, strictly increasing in r.
- The principal's discount factor is $\delta_P \in [0, 1)$.

- A strategy s^P for the principal specifies to dismiss (D) time-t agent or not (N), as a function of time-t history, for every time t.
- A strategy $s^{At} = (s^{At}_{k,\tau})_{\tau=0,1}$ for agent t specifies an effort e for both periods $\tau = 0, 1$ as a function of the time-$(t + \tau)$ history.
- Stationary anonymous strategies (s^P, σ^A) are such that
  - time-t retention rule depends only on effort of time-t agent,
  - each agent's effort at $\tau = 0$ depends only on her type a,
  - effort at $\tau = 1$ depends only on a and on reward r at $\tau = 0$.
- s^P is a cut-off strategy if there exists an $\bar{r}$ such that $s^P(r) = D$ if and only if $r < \bar{r}$.
- A mixed strategy σ_A is type-monotonic if
  - there exist $[\underline{e}_k, \overline{e}_k]$ s.t. $\overline{e}_k \leq \underline{e}_{k+1}$ for $k = 1, ..., K - 1$, and $\sigma^A_{0k}([\underline{e}_k, \overline{e}_k]) = 1$ for all k;
  - for all $r \in \mathbb{R}$, $s^A_{1k}(r) \leq s^A_{1,k+1}(r)$ for $k = 1, ..., K - 1$.

- The utility specification covers canonical cases.
- Agent is office motivated politician with two-term limit:
  - $u(e, a) = z - c(e, a)$, z is the office benefit,
  - $c(e, a)$ is opportunity cost of effort e by politician of type a, it is continuous in e, decreasing in a, and submodular in (e, a).
- The agent is an benevolent politician:
  - $u(e, a) = \int v(r) dF(r|e) - c(e, a)$.
- The agent's remuneration is a fixed share of profits $s(r)$:
  - the principal's share is $v(r) = r - s(r)$,
  - the agent's utility is: $u(e, a) = \int s(r) dF(r|e) - c(e, a)$.

Results

Proposition There exists an anonymous strategy equilibrium (s^P, σ^A) s.t. s^P is a cut-off strategy and σ^A is type-monotonic.

Sketch of Proof. Second-period effort of better agents is higher.

. Supermodularity of u implies also second-period payoff is higher.

. Now, suppose the principal employs a cut-off strategy.

. By FSD, higher effort yields higher expected principal reward.

. Then, better agents' incentive to exert first period effort is higher.

. A cut-off strategy is then a best response:

  . it screens better agents in the first period,

  . these better agents yield better rewards in the second period.

. Environment is "nice," if u and F are continuously differentiable, e_k^* is in the interior of $[\underline{e}, \overline{e}]$ and $u(\overline{e}, a_k) < 0$ for all k, and $\delta_A > 0$. for each a_k, $k = 1, ..., K$.

. Let r^* be the cut-off associated with the strategy s^P.

. Let $v_0(\sigma_0^A)$ be the expected principal reward in period 0, and $v_1(r, s_1^A)$ the reward in period 1.

Proposition When the environment is nice, in any anonimous equilibrium (s^P, σ^A), r^* is interior, $s_{1k}^A(r) < s_{1,k+1}^A(r)$, $\underline{e}_{k+1} > \overline{e}_k$, $\underline{e}_k > s_{1k}^A(r)$ for $k = 1, ..., K-1$, and $v_1(r^*, s_1^A) \geq v_0(\sigma_0^A)$.

. Screening makes each agent type exert more effort in first period.

. Screening leads to higher expected reward in second period.

. Without adverse selection, the equilibrium unravels.

Proposition If all agents have the same type, in equilibrium:

. the agent's effort is e^* in both periods;

. in a nice environment, the cutoff is $r^* \in \{\min R, \max R\}$.

Sketch of Proof. Effort must be weakly lower at $\tau = 1$ than $\tau = 0$.

. I prove it cannot be strictly larger with positive probability.

. If σ_{k0}^P placed positive probability on any effort $e > e^*$,
then the principal's unique best response would be $r^* = \max R$.

. But then agent's unique optimal effort would be e^* at $\tau = 0$.

. Again by contradiction, if $\min R < r^* < \max R$, then the agent's optimal first period effort would be weakly larger than e^*.

. But then principal's unique best response would be $r^* = \max R$.

. Without adverse selection, there is no possibility of selection.

. But then, there are no incentives for high performance either, because the only principal's instrument is retention choice.

. Nevertheless, the principal cannot be better off if "worse" types are added, and cannot be worse off if "better" types are added.

. Instead, the principal can improve with adverse selection, if we "average out" types as follows: $\sum_{k=1}^{K} p_k E(e_k^*) = E(e^*)$.

. Take any equilibrium of the model with adverse selection.

. As all types of agents choose (weakly) higher effort in first period, the first-period principal payoff is $v_0 \geq \sum_{k=1}^{K} p_k E(e_k^*) = E(e^*)$.

. Because $v_1(r^*) \geq v_0$ in equilibrium, also $v_1(r^*) \geq E(e^*)$.

. As v_1 increases in r, the payoff of the principal is strictly higher than without adverse selection.

Candidate preference uncertainty (Duggan 2000)

. There is a continuum of citizen candidates, indexed by ideology b.

. Ideologies are private information and distributed according to the single peaked and symmetric density f on $[-a, +a]$.

. At any time t, the office holder selects a policy $x_t \in [-a, +a]$.

. Candidates for office cannot make credible promises.

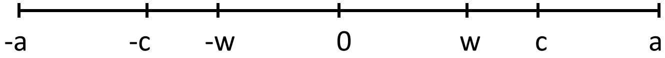
. At any time $t \geq 1$, the incumbent runs against challenger randomly drawn from f.

. The time-1 incumbent is randomly selected.

. The time-t utility of a citizen b depends on policy x_t, according to symmetric loss function $L(|b - x_t|)$, where $L' < 0$ and $L'' \leq 0$.

. Utilities are discounted with factor δ.

Theorem As long as voters are not too risk averse (i.e., if $|L''|$ is uniformly not too large), there is essentially a unique symmetric stationary PBE. The median voter is decisive.



. Incumbents with centrist *b* in *[0, w]* and extremists with *b* in *[c, a]* adopt their preferred policy *x = b* when in office.

. Centrist are reelected and extremists are voted out.

. Moderates with *b* in *[w, c]* compromise when in power. They adopt policy w and are reelected.

. Symmetrically for *b < 0.*

. Let U_b be the (normalized) equilibrium value for citizen b.

. The equilibrium obeys the following indifference equations:

$$L(w) = U_0, \quad L(c - w) = \delta U_c.$$

. The continuation utility of a voter b for electing challenger is:

$$U_b = \int_{-a}^{-c} [L(x\text{-}b)(1\text{-}\delta) + \delta U_b] dF(x) + \int_{-c}^{-w} L(c+b) dF(x) \\ + \int_{-w}^{w} L(x\text{-}b) dF(x) + \int_{w}^{c} L(c\text{-}b) dF(x) + \int_{c}^{a} [L(x\text{-}b)(1\text{-}\delta) + \delta U_b] dF(x).$$

. Thresholds w and c are determined by 2 conditions:

$$L(w) = 2 \int_c^a [L(x)(1 - \delta) + \delta L(w)] dF(x) \\ + 2 \int_w^c L(w) dF(x) + 2 \int_0^w L(x) dF(x). \quad (1)$$

. Median voter is decisive and indifferent between a random challenger and reelecting incumbent who implements policy w.

$$L(c-w)=\delta\{\int_{-a}^{-c}[L(c-x)(1-\delta)+\delta L(c+w)]dF(x)$$
$$+\int_{-c}^{-w}L(c+w)dF(x)+\int_{-w}^{w}L(c-x)dF(x)$$
$$+\int_{w}^{c}L(c-w)dF(x)+\int_{c}^{a}[L(c-x)(1-\delta)+\delta L(c-w)]dF(x)\}.$$

. Candidate c is indifferent between implementing policy w forever, or policy c once and then be replaced by random challenger.

. To show that $w>0$, suppose by contradiction $w=0$.

. Then any incumbent with $b\in(0,c)$ would deviate from equilibrium and pick policy $x=b$, instead of $x=w=0$.

. To show that $c<a$, note that, if all incumbents with $b>w$ chose w, then it would be the case that $w=0$.

. Else the median voter would not retain an incumbent with policy w, as this would be her worst possible equilibrium policy.

. The proof that $c>w$ is also by contradiction.

Party competition (Bernhardt et al. 2009)

. There are 2 parties: A and B. Party A includes all candidates with $b < 0$, and party B all those with $b > 0$.

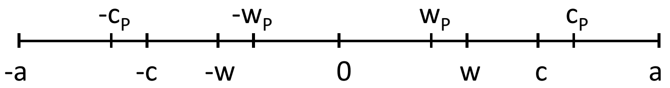
. In every period t, the challenger is selected at random from the opposite party with respect to the incumbent.

Proposition If $|L''|$ is uniformly not too large, there is essentially a unique symmetric stationary PBE. The median voter is decisive.

. Party-B candidates with ideology $b \in [0, w_P]$ and $b \in [c_P, a]$ adopt their preferred policy $b = x$ when in office.

. Centrist are reelected and extremists are voted out.

. Candidates with ideology $b \in [w_P, c_P]$ compromise to policy w_P and are re-elected.



. Party competition makes incumbents' more moderate.

. Incumbents are afraid of being substituted by candidates from the opposite party, with opposite ideology.

. Party competition increases compromise: $c_P > c$.

. Then, median voter tightens re-election standards: $w_P < w$.

. When compromising, one's policy is more moderate.

. The indifference equations characterizing equilibrium are:

$$L(v) = \underline{U}_0 (= \overline{U}_0), \quad L(w_P - c_P) = \delta \underline{U}_{c_P}.$$

. $\underline{U}_b$ is the continuation utility of a voter with $b > 0$ $(b < 0)$ for electing a challenger from the opposite party A (B):

$$\underline{U}_b = 2 \int_{-a}^{-c} [L(x-b)(1-\delta) + \delta \overline{U}_b] dF(x)$$
$$+2 \int_{-c}^{-w} L(c+b) dF(x) + 2 \int_{-w}^{0} L(x-b) dF(x).$$

. $\overline{U}_b$ is the utility from a random challenger from the same party:

$$\overline{U}_b = 2 \int_0^w L(x-b) dF(x) + 2 \int_w^c L(c-b) dF(x)$$
$$+2 \int_c^a [L(x-b)(1-\delta) + \delta \underline{U}_b] dF(x).$$

. Median voter is indifferent between a party-B incumbent that implements v and electing a random challenger from party A.

. Party-B incumbent c_P is indifferent between policy w_P forever, and policy c_P once then replaced by a random party A challenger.

. Thresholds w_P and c_P are determined by:

$$L(w_P) = 2\int_{c_P}^{a}[L(x)(1-\delta)+\delta L(w_P)]dF(x)$$
$$+2\int_{w_P}^{c_P} L(w_P)dF(x) + 2\int_0^{w_P} L(x)dF(x) \qquad (2).$$

$$L(w_P - c_P) = 2\int_{-a}^{-c_P}[L(c_P - x)(1-\delta)+\delta \overline{U}_{c_P}]dF(x)$$
$$+2\int_{-c_P}^{-w_P} L(c_P + w_P)dF(x) + 2\int_0^{-w_P} L(c_P - x)dF(x).$$

. Comparing utility expressions, we obtain: $\underline{U}_{c_P} < U_{c_P} < \overline{U}_{c_P}$.

. Together with $\delta < 1$, this implies that $c_P - w_P > c - w$.

. Because of symmetry, equations (1) and (2) have same form:

$$\phi(w,c) = -L(w) + 2\int_c^a[L(x)(1-\delta)+\delta L(w)]dF(x)$$
$$+2\int_w^c L(w)dF(x) + 2\int_0^w L(x)dF(x).$$

. By implicit function thm., $\frac{dw}{dc} = -\frac{\phi_2(w,c)}{\phi_1(w,c)} < 0$, for $w \leq c$.

. This and $c_P - w_P > c - w$ imply that $w_P < w$ and $c_P > c$.

Proposition All voters prefer party competition over at-larger selection of candidates.

- All voters like insurance because risk averse and discount utilities.
- Parties provide ex-ante insurance against extremist policy:
  - there is less expected office-holder turnover $(c_P > c)$,
  - policies are more moderate over all $(w_P < w$ and $c_P > c)$.

Summary

. I have presented agency models of election.

. Voters do not care about electoral promises.

. They retain effective incumbents, and dismiss incumbents with poor performance to elect the challenger.

. If candidates' valence and ideologies are known, retention rules are ineffective.

. If candidates' valence or ideologies are uncertain, such retention rules encourage high effort/platform moderation.

. Party competition encourage even more moderation and improves voter welfare.

Next Lecture

. I consider candidates' valence: all characteristics that are valuable to all voters, regardless of their ideology.

. In elections with aggregate uncertainty, the advantaged candidate locate close to the expected median and the disadvantaged one takes her chance by diverging.

. Eqm. may be in pure strategy if candidates are policy motivated. With office motivation, equilibrium is in mixed strategies.

. In a retrospective voting model, higher-valence incumbents are retained even with less moderate policies.

. Incentives to compromise make challengers expected policies more moderate, and valence benefits the whole electorate.