Advanced Economic Theory
Models of Elections
Lecture 6

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Valence and aggregate uncertainty (Aragones and Palfrey 2002)

. There are two office motivated candidates.

. One has an electoral advantage (higher valence, incumbency).

. There is aggregate uncertainty on the voters' preferences.

. It cannot be that candidates platforms converge.

. The disadvantaged candidate would lose the election for sure.

. There is no equilibrium in pure strategies.

. In the mixed strategy equilibrium,

  . advantaged candidates locates close to the "expected median,"

  . the disadvantaged one "tries her luck" on extreme platforms.

The model

. The policy space is $X = \{x_j = \frac{j-1}{n-1}, j = 1, ..., n\}$, $n > 1$.

. There are two candidates, A and D, purely office motivated, who simultaneously choose platforms x_A and x_D.

. Each voter j's payoff is $u_j(A) = v - |x_j - x_A|$ if A wins, and $u_j(D) = -|x_j - x_D|$ if D wins; with $0 < v < \frac{1}{n-1}$.

. Each voter votes for the candidate she prefers, and for either candidate with probability $1/2$ if indifferent.

. The unique median policy m is unknown to the candidates, uniformly distributed on X.

Equilibrium

. When $0 < v < \frac{1}{n-1}$, candidate A wins iff $|x_A - m| \leq |x_D - m|$.

Proposition If $0 < v < \frac{1}{n-1}$, then there is no pure strategy equilibrium.

Sketch of proof. If candidate D plays a pure strategy x_D, candidate A can copy x_D and win for sure.

. (At least), the disadvantaged candidate must be mixing.

. Say $n \leq 8$ and even. Consider mixed strategies σ that

  . are symmetric at $1/2$: $\sigma^i_j = \sigma^i_{n-j+1}$ for all j and candidates i,

  . have no gaps in the support: there exist j, ℓ such that $0 \leq j \leq \ell \leq n$ and $\sigma^i_l > 0$ if and only if $j \leq l \leq \ell$.

- Candidate A's winning probability for each platform x_j is:

$\pi_A(x_j, \sigma^D) = \sum_{\ell=1}^{\frac{j-1}{2}} \frac{n-j+\ell+1}{n}\sigma^D_{j-2\ell-1} + \sum_{\ell=1}^{\frac{j-1}{2}} \frac{n-j+\ell+1}{n}\sigma^D_{j-2\ell}$
$+\frac{n-j+1}{n}\sigma^D_{j-1} + \sigma^D_i + \frac{j}{n}\sigma^D_{j+1} + \sum_{\ell=1}^{\frac{n-j}{2}} \frac{j+\ell}{n}\sigma^D_{j+2\ell} + \sum_{\ell=1}^{\frac{n-j}{2}} \frac{j+\ell}{n}\sigma^D_{j+2\ell+1}.$

- Symmetric equilibria with no gaps are solved by
  - equating the payoffs of adjacent platforms,
  - finding one of the endpoints of the support.
- Let ℓ^*_A be the first platform in the support of A's strategy.
- There are $n - 2\ell^*_A + 1$ indifferent conditions, $\ell^*_A \leq \ell \leq n - \ell^*_A$,

  $\ell\sigma^D_\ell + \sum_{j=1}^{[(\ell-1)/2]} \sigma^D_{\ell-2j} = (n-\ell)\sigma^D_{\ell+1} + \sum_{j=1}^{[(n-j-1)/2]} \sigma^D_{\ell+2j+1},$
- With symmetry, these simplify to, for $\ell^*_A \leq \ell < n/2$

  $(\ell-1)\sigma^D_\ell = (n-\ell)\sigma^D_{\ell+1} + \sum_{j=\ell+2}^{n/2} \sigma^D_j.$

. Candidate D's winning probability is:

$$\pi_B(x_i, \sigma^A) = \textstyle\sum_{j=1}^{[i/2]} \frac{n-i+j}{n}\sigma^A_{i-2j-1} + \sum_{j=1}^{[i/2]} \frac{n-i+j}{n}\sigma^A_{i-2j}$$
$$+ \textstyle\sum_{j=0}^{[(n-i)/2]} \frac{i+j}{n}\sigma^A_{i+2j+1} + \sum_{j=0}^{[(n-i)/2]} \frac{i+j}{n}\sigma^A_{i+2j+2}.$$

. Proceeding as with candidate A, we obtain, for $\ell^*_D \leq \ell < n/2$,

$$(\ell - 1)\sigma^A_{\ell+1} = (n - \ell)\sigma^A_\ell + \sigma^{n/2}_{i=\ell+2}\sigma^A_i.$$

Proposition In equilibrium with symmetric strategies and no gaps,

. $\sigma^D_\ell \geq \sigma^D_{\ell+1}$ for $1 \leq \ell < \frac{n}{2}$, and $\sigma^D_\ell \leq \sigma^D_{\ell+1}$ for $\frac{n}{2} \leq \ell \leq n-1$;

. $\sigma^A_\ell \leq \sigma^A_{\ell+1}$ for $1 \leq \ell < \frac{n}{2}$ and $\sigma^A_\ell \geq \sigma^A_{\ell+1}$ for $\frac{n}{2} \leq \ell < n-1$.

. A's mixed strategy is hill-shaped, D's strategy is U-shaped.

. Candidate A occupies "more valuable" median platforms, pushing candidate D out to "try her luck" on more extreme platforms.

. The symmetric, no-gap equilibrium is essentially unique.

Valence and policy motivations (Groseclose 2001)

. Suppose candidates have mixed office/policy motivations.

. Unlike in the case of pure office motivations, a pure strategy equilibrium may exist.

. The disadvantaged candidate moves towards extreme platforms.

. As valence advantage grows, the advantaged candidate initially moves closer to the expected median, and then moves away.

. The overall platform divergence increase in valence advantage.

The model

. Policy motivated candidates L and R with bliss points $b_L < 0$, $b_R = -b_L$ simultaneously choose platforms x_L and x_R on $\mathbb{R}$.

. Let $W = A, B$ be the winner of the election.

. Candidates have mixed motivations, i's payoff is:

$$u_i(x_L, x_R) = (1 - w)L(|x_W - b_i|) + w\,\text{Pr}(i \text{ wins election})$$

. Candidate i's valence is v_i. Let $v = v_L - v_R$.

. If i wins election, median voter's utility is $v_i + L(|m - x_i|)$.

. L is smooth, $L' < 0$, $L'' < -\varepsilon$ for small $\varepsilon > 0$ and $L(0) = 0$.

. The median m has a continuous density f symmetric around 0 and c.d.f. F.

Results

- Numerical analysis with $L(z) = -z^2$ demonstrates:
  - in pure strategy equilibrium, candidate divergence increases in the valence advantage v;
  - as v increases from zero, x_L moves toward and x_R away from the expected median;
  - as v grows larger, x_L eventually moves back towards b_L, x_R keeps diverging;
  - for all $v > 0$, x_L is more moderate than x_R.
- Some of these comparative statics results are generalized, but without proving pure-strategy equilibrium existence.
- Let $x_L^*(v)$ and $-x_R^*(v)$ denote eqm. platforms.
- $R(x) = -xL''(x)/L'(x)$ coefficient of relative risk aversion.

Proposition Assume a pure strategy equilibrium exists for $v > 0$. If $R(x_R^*(0)) \geq 2f(0)x_R^*(0)$, then $x_L'(0) > 0$ and $x_R'(0) > 0$. (A sufficient condition is that $L''' < 0$.)

Proposition Assume that a pure strategy equilibrium exists for $v > 0$. Then there exists v_1 such that for all $v > v_1$, $x_R^*(v) > b_R$. Also assume that the support of f is a closed interval. Then there exists v_2 such that for all $v > v_2$, $x_L^*(v) = b_L$.

. As the valence advantage grows large, L locates at her ideal point, and R locates more extremely than her ideal point.

Again, the disadvantaged candidate gets "pushed out."

. Say L satisfies condition α if for $y > 0$ and all $x \in (0, y]$,

$$\frac{[L'(y+x)-L'(y-x)]/x}{[L'(y+x)+L'(y-x)]/y} \geq 1.$$

. Say that f satisfies condition β if $f(b_R) > \frac{2b_R f(0)-1}{2b_R+[w/(1-w)]f(0)}$

Proposition Assume that a pure strategy equilibrium exists for $v > 0$. Suppose f is uniform with support $[-\frac{1}{2f(0)}, \frac{1}{2f(0)}]$; that $b_R \leq \frac{1}{2f(0)}$ and that L satisfies condition α. Then for all $v > 0$, $|x_L^*(v)| < |x_R^*(v)|$: x_L^* is more moderate than x_R^*.

Proposition Assume pure strategy equilibrium exists for $v > 0$. Suppose that $L(x) = -x^2$, and that f satisfies condition β. Then for all $v > 0$, $|x_L^*(v)| < |x_R^*(v)|$: x_L^* more moderate than x_R^*.

Valence and re-election (Bernhardt et al. 2013)

. We take Duggan (2000) agency model of re-election and introduce valence heterogeneity.

. The challenger's valence is unknown to the electorate, the valence of the incumbent is common knowledge at the time of election.

. Long-run correlation between valence and extremism is positive.

. But for newly elected office-holders, the relationship is negative.

. A valence increase in first-order stochastic dominance sense benefits all voters.

. Valence mean-preserving spread benefits median voter.

The model

. In every period t, an office holder with ideology $b \in [-a, a]$ and valence $v \in V$, selects a policy $x \in \mathbb{R}$.

. Voters observe her valence while she is in office.

. At end of term, she runs for re-election with probability $1 - q$.

. Voters have no information on her challenger's valence/ideology.

. If the incumbent retires, a new office holder is chosen randomly.

. Stage utility of a citizen i of ideology b_i when policy x implemented is:

$$u_i(x, v) = L(|b_i - x|) + v, \text{ where } L' < 0 \text{ and } L'' < 0,$$

. Period utilities are discounted at factor δ.

. In addition, the office holder receives ego rents w.

Equilibrium Characterization

Theorem If voters are not too risk averse (if $|L''|$ is uniformly not too large), there is essentially a unique symmetric stationary PBE.

. The median voter is decisive.

. The equilibrium is summarized by increasing thresholds functions $w, c : V \to (0, a)$.

. For each v, $0 < w_v < c_v < a$ for party R.

. Symmetrically $-a < -c_v < -w_v < 0$ for party L.

. The payoff of a voter of ideology b if re-electing an incumbent of valence v who adopts platform x is

$$U_b(x, v) \equiv [L(|x - b|) + v](1 - q\delta) + \delta q \tilde{U}_b$$

. The expected value from electing the challenger is:

$$\tilde{U}_b \equiv \int_V \{ \int_0^{w_v} [[L(|x+b|) + L(|x-b|) + 2v](1-q\delta) + 2\delta q \tilde{U}_b] dF(x)$$
$$+ \int_{w_v}^{c_v} [[L(|w_v+b|) + L(|w_v-b|) + 2v](1-q\delta) + 2\delta q \tilde{U}_b] dF(x)$$
$$+ \int_{c_v}^{a} [[L(|x+b|) + L(|x-b|) + 2v](1-\delta) + 2\delta \tilde{U}_b] dF(x) \} dG(v).$$

. For every v, the threshold w_v and c_v solve:

$$U_0(w_v, v) = [L(w_v) + v](1-q\delta) + \delta q \tilde{U}_0 = \tilde{U}_0,$$
$$U_{c_v}(w_v, v) = [L(c_v\text{-}w_v) + v](1-q\delta) + \delta q \tilde{U}_{c_v} = v(1\text{-}\delta) + \delta \tilde{U}_{c_v}.$$

. Median voter is indifferent between reelecting the incumbent of valence v who adopts policy w_v.

. Politician of ideology c_v is indifferent between compromising to w_v to be re-elected or choosing policy c_v and being dismissed.

Lemma High valence office-holders can take more extreme policy positions and be reelected, that is, for any $v_H, v_L \in V$,

$$v_H > v_L \Rightarrow w_H > w_L.$$

Lemma There is a positive correlation between valence and probability of being reelected: for any $v_H, v_L \in V$,

$$v_H > v_L \Rightarrow c_H > c_L.$$

Lemma The compromise set is strictly increasing in valence: for any $v_H, v_L \in V$,

$$v_H > v_L \Rightarrow c_H - w_H > c_L - w_L.$$

- High valence incumbents compromise more, since they
  - internalize cost of being replaced by a low valence candidate
  - find it less costly to compromise, as they can compromise to more extreme positions.

Proposition The correlation between valence and extremism is positive both for reelected incumbents and those not reelected. But there is a negative correlation between valence and extremism for newly elected office holders.

. This is because the compromise set of newly elected politicians is larger the higher is valence.

Proposition Unless the probability of retirement q is large, there is a positive correlation between valence and extremism in the expected stationary distribution of office holders.

. This result holds because if an office-holder is reelected once, then she is reelected until she retires.

Valence and Welfare

. Suppose we increase every valence $v \in V$ by the same amount d.

. Then eqm. thresholds w', c' are s.t. $w'_{v+d} = w_v$, $c'_{v+d} = c_v$, and the expected utility of each citizen increases by d.

Proposition A first order stochastic dominance shift of the distribution of valences G makes all voters strictly better off.

. We now specialize the model:

. Ideologies are uniformly distributed between $[-a, a]$,

. loss function is a power function $L(|x|) = -|x|^z$ where $z \geq 1$,

. incumbents always run for re-election, $q = 0$,

. ego rents are zero, $W = 0$.

. We consider the case in which a challenger has valence v_H with probability $p \in (0,1)$ or valence $v_L < v_H$ with probability $(1-p)$. The associated equilibrium thresholds are w_H, c_H, w_L, c_L.

. And compare to the case in which candidates' valence is $\tilde{v} = pv_H + (1-p)v_L$ with probability 1 and thresholds are $\{\tilde{w}, \tilde{c}\}$.

Proposition The equilibrium threshold are such that:

$w_H > \tilde{w} > w_L$, $c_H > \tilde{c} > c_L$, and $c_H - w_H > \tilde{c} - \tilde{w} > c_L - w_L$.

The change in the median voter expected utility is:

$U_0(v_L, v_H) - U_0(\tilde{v}) = \tilde{w} - w_L - (w_H - \tilde{w}) + v_H - v_L > 0.$

. The median voter prefers the "riskier" environment with good and bad candidates to the "average" case.

Summary

- I have considered candidates' valence: all characteristics that are valuable to all voters, regardless of their ideology.
- In elections with aggregate uncertainty, the advantaged candidate locate close to the expected median and the disadvantaged one takes her chance by diverging.
- Eqm. may be in pure strategy if candidates are policy motivated. With office motivation, equilibrium is in mixed strategies.
- In a retrospective voting model, higher-valence incumbents are retained even with less moderate policies.
- Incentives to compromise make challengers expected policies more moderate, and valence benefits the whole electorate.

Next Lecture

. I present a model in which lobbies make campaign contributions conditional on support for their priorities.

. Impressionable voters more likely vote for the candidate with greater campaign contributions.

. In equilibrium, platforms maximize a weighted sum of the lobbies and informed voters payoff.

. Instead, activists are second-movers: they mobilize and choose effort in response to candidates' platforms.

. In elections with aggregate uncertainty, I show that ideologically opposed activists lead to more moderate platforms.

. As activists polarize, platforms may diverge less or more.