Trading Effort for Freedom:
Workday Credits in the Stalinist Camp System

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Appendix: Calculating the Economic
Impact of Workday Credits

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In this appendix, we formulate a simple model that captures the problematic nature of the usage of workday credits in the prison camp system with regard to output and profits. This model is intended to serve as an analytical tool for explaining our concrete research problem rather than to provide a possible generalization of this phenomenon. Therefore our assumptions will use the specific workday credit system implemented in the Soviet prison camps. The model is used to test two different camp managers' objective function: output versus profit maximization.

A.I.   Output maximization without time preferences

We take a simplified model of a prison camp (which can equally represent a larger camp system), where total output is determined by labor N and labor productivity P[1]:

(1.) $$Y = N \cdot P$$

where
Y = total output
N = number of prisoners
P = average labor productivity

Workday credits would raise labor productivity but reduce labor through the shortening of effective sentences. To specify the expressions of both N and P, we assume first that prisoners enter the prison camp only by being arrested and subsequently being convicted and sentenced. And we assume, for reasons of simplicity, that all prisoners have to serve the same term, T_0. Second, we assume that the number of arrests is constant in any period, third that prisoners leave the prison camp only through release by the end of their sentence. Thus we exclude the possibility of both mortality and escapes.[2] N therefore depends on three

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[1] It is important to note that expression (1.) does not represent a production function in a stricter sense, as it lacks additional inputs like capital – which, however, also may exist with their effect on output being captured by the value for P. Nevertheless, expression (1.) implies that the "real" underlying production function, whatever its form, exhibits constant returns for labor. This assumption might appear unrealistic in a "normal", civilian economy. Yet in this case we start from the fact that Soviet prison camps in general were subdivided into separate camp departments of standardized size and with, at least officially, equally standardized endowment with infrastructure (although in practice, regulations concerning the latter were often flouted to some extent). Thus, an increase in the overall number of prisoners would not lead to an unlimited raise in inmates in existing places of confinement, but to the establishment of new, similar camps and camp departments, where labor productivity could also be expected to range on a similar level. The fact that at some points in time the Soviet camp system's directors were struck by surprise by high inflows of new prisoners and thus failed to immediately organize new camps and working places, which lead to an temporary fall in overall productivity, does not interfere with this general principle.

[2] This assumption will appear problematic against the background of the Stalinist prison camp system within which, as is generally known, significant mortality did incur. Under such circumstances, one might imagine that the camp management would include a high level of mortality in its calculations and pursue the strategy to extract the maximum output from every prisoner in a short-term perspective, expecting that the average prisoner would not survive the moment of his release so that the receipt of workday credits would have no real implications for him. Yet many former prisoners' testimonies reveal that in the Soviet camp system the highest probability to die was given in the first year after the prisoner's committal to a camp and generally decreased throughout the following periods. Besides that, the far highest mortality in Stalinist camps was observed during the war period when the workday credit system officially was rescinded. During the late 1940s, when it was introduced again, the mortality approached relatively low levels.

factors: the number of arrests, the number of prisoners being released, and the average period of time prisoners are held in captivity. Under these assumptions, we are able to write N at any period t as the difference between *all* arrests and releases that have taken place in the past:

(2.) $$N_t = \sum_{i=0}^{t-1} A_i - \sum_{i=0}^{t-1} R_i \qquad \text{for any period } t \geq 0$$

with A_i, R_i denoting the number of arrests/releases in any period i

From the above assumption of a standardized, constant sentence, no prisoner will be in a prison camp longer than the exact duration of this term. Consequently we can modify equation (2.) as follows:

(3.) $$N_t = \sum_{i=t-T_t}^{t-1} A_i$$

In this equation, we introduce a new expression, where T_t represents at any period t, the duration of the period the one prisoner, who has been kept in the camp longer than any other inmate (and is thus just about to be released in the current period) has spent there. Hence equation (3.) does not contain an expression depicting the number of releases: All prisoners arrested prior to period $t - T_t$ have already been released prior to period t, whereas all prisoners arrested during the interval $(t - T_t ; t)$ will be released in future periods and currently account for the prison camp's labor force. Remember that without any other influences, T_t will equal T_0 in any period. Recalling A_i being constant and normalizing it to unity allows us to simplify this expression and to rewrite it as

(4.) $$N_t = T_t$$

Accordingly, we can rewrite equation (1.):

(5.) $$Y_t = T_t P_t$$

Having specified N, P_t remains to be discussed. Since we are interested in comparative-static and dynamic analyses of Y, we define the initial, constant level of labor productivity as P_0. Now let us consider the introduction of workday credits, which offer all prisoners achieving increased productivity P_{wc} (with $P_{wc} > P_0$) a reduction in the time effectively spent in detention by a factor q (with $1 > q > 0$) equaling the ratio of working days to the sum of working days plus the number of workday credits granted. For the sake of simplicity, we first assume that the system offers just one alternative – either a prisoner meets the target of P_{wc} and gets his term reduced by the factor q, or he does not meet this requirement and receives no credit. Furthermore, since the workday credit system rewards prisoners meeting a precisely defined minimum standard, we assume that all prisoners willing and able to earn workday credits will show exactly the required productivity P_{wc}, but will refrain from making any effort to perform even better.[3] Finally, we assume that all prisoners respond to the incentive, achieving a productivity of P_{wc} and accordingly receiving workday

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[3] This implies that prisoners are able to exactly estimate the yields of their efforts. Hence we exclude the possibility of prisoners being motivated by the perspective of getting workday credits but narrowly or temporarily failing to show the required productivity. Under this assumption prisoners in such cases would know in advance that they would not be able to meet the target and therefore abstain from any attempts to reach it. This is important since otherwise workday credits could serve as a stimulus, although the prisoners would not (always) be rewarded and thus the "price" the camp management would have to pay for an increase in productivity would be lower. Of course, this assumption can be softened in order to allow for a certain range of error in the prisoners' attempt to predict their performance to the point, as we have done in the simulation above by choosing a value of $P_{wc}= 1,25$ instead of 1,21 (see above, p. 11, note 15).

credits at all times.[4]

Having set up our model, we shall try to elicit the preferences that stood behind the decision for or against such a system. With all prisoners taking advantage of a workday credit system from the beginning to the end of their detention, the value of T_t will be reduced by the factor q, so that in the long run $T_t = qT_0$. Consequently, the output in the long-term perspective is

(8.) $$Y_t^{wc} = qT_0P_{wc}$$

and, with regard to output maximization, the question of whether to opt for workday credits would depend on the values for q and P_{wc}. From the perspective of a long-term planner, who equally values the output in all future periods, workday credits would be introduced if condition (9.) were fulfilled.

(9.) $$Y_t^{wc} = qT_0P_{wc} > T_0P_0 = Y_0$$

which yields

(10.) $$\frac{1}{q} < \frac{P_{wc}}{P_0}$$

The application of this formula to the data provided in Table 1 formally corroborates the finding formulated in section III: the inequality was not fulfilled by any of the empirical values of the scale applied in the Soviet camp system in late 1940s.[5]

## A.II.   Output maximization with time preferences

After the introduction of workday credits, the number of prisoners decreases not immediately, but gradually over a finite number of periods. This effect must be considered if we turn to a planner with time preferences, who values output produced in the nearer future at a higher rate. In our model, it implies a gradual decrease in the value of T_t, inflicted by a temporary increase in releases that will last until the last prisoner arrested prior to the introduction of workday credits is released. Due to the assumption of a constant inflow of prisoners with identical sentences, this decrease will be linear. At the end of this period, the permanent number of prisoners will have been reduced by the same factor by which the average term of confinement is actually reduced by the workday credit system, which means that no prisoner receiving workday credits will remain in the camp longer than qT_0. Interestingly, the duration of this period also equals qT_0, which implies that, the lower q (i.e. the higher the workday credits), the shorter this period will last and the sooner the full reducing effect will occur. So higher workday credits mean not only a more significant, but also a quicker reduction of labor resources (see Fig. 2).

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[4] This assumption is not as strict as it might sound; it could be equally interpreted in such a way that only the group of prisoners receiving workday credits will be considered, since for all other inmates nothing would change. See discussion above on p. 9.

[5] The same applies to scales with the requirements for workday credits in the 1930s. The main difference to those established in late 1940s was that the former were less sophisticated.

Fig. 2.
Dynamics of the number of prisoners during the transitory period, following the introduction of workday credits (for two alternative values for q)
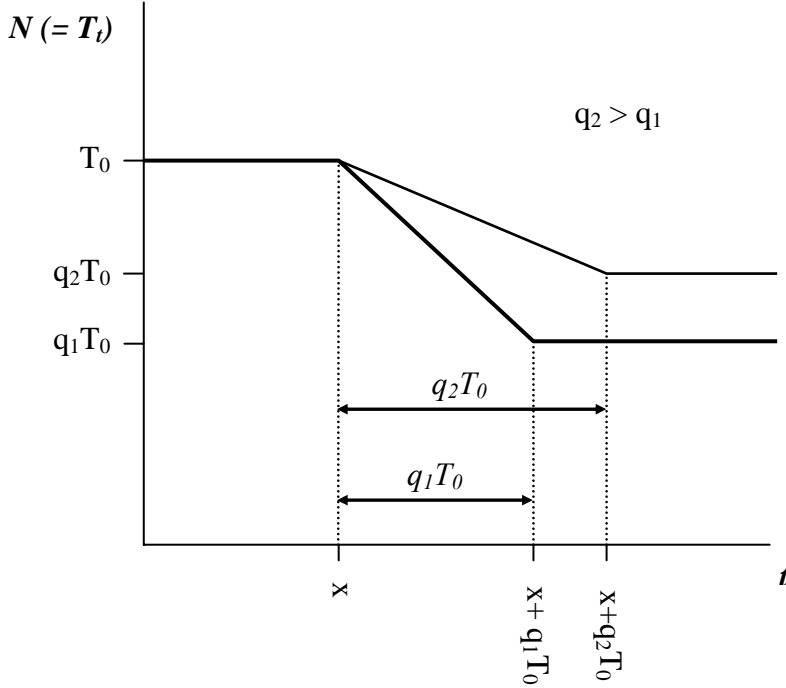
According to this effect and to our further assumptions we have to modify T_t for all periods in the wake of the implementation of workday credits in the following manner:

(11.) $\qquad N_t = T_t^{wc} \qquad\qquad$ for all $t \geq x$; with:

$$
\begin{cases}
T_t^{wc} = T_0 \left(q + (1-q)\dfrac{T_0 - \dfrac{1}{q}(t-x)}{T_0} \right) & \text{if} \qquad t - x < qT_0 \quad \textit{for all } t \geq x \\[4em]
T_t^{wc} = qT_0 & \text{if} \qquad t - x \geq qT_0
\end{cases}
$$

with x denoting the period in which workday credits are implemented

In this specification, the first term reflects the linear decrease of T_t over the transition period. Note that the term in brackets equals 1 in period $t = x$, i.e. at the very moment of the introduction of workday credits, whereas in any of the following periods its value will be less than 1. Thus T_t^{wc} will be less than T_0 in any period after the introduction of workday credits (see also Fig. 1).

We consider an agent (e.g. the manager of the camp system) with usual time preferences, represented by a discount factor δ with $\delta = \dfrac{1}{1+r}$ and r depicting the discount rate with $0 \leq r$. Such an agent would opt for or against the introduction of workday credits depending on which option would yield a higher value for the discounted future output stream. This can be denoted as follows (in the following the period of introduction of workday credits is given as $x = 0$):

Without workday credits:

(12.)
$$\sum_{t=0}^{\infty} \delta^t Y_t = \sum_{t=0}^{\infty} \delta^t T_0 P_0 = \frac{1}{1-\delta} T_0 P_0$$

With workday credits:

(13.)
$$\sum_{t=0}^{\infty} \delta^t Y_t = \sum_{t=0}^{qT_0} \delta^t T_0 \left(q + (1-q)\frac{T_0 - \frac{1}{q}t}{T_0} \right) P_{wc} + \sum_{t=qT_0+1}^{\infty} \delta^t q T_0 P_{wc}$$

In order to determine the values of the parameters that yield a higher discounted future output flow with a workday credit system in place than without, we juxtapose the expressions for those two values (after slightly converting the expression in (13.) and dividing both sides through T_0):

(14.)
$$\frac{1}{1-\delta} P_0 < P_{wc} \left[\sum_{t=0}^{qT_0} \delta^t \left(q + (1-q)\frac{T_0 - \frac{1}{q}t}{T_0} \right) + q\frac{\delta^{qT_0+1}}{1-\delta} \right]$$

In inequality (15.) several parameters could be varied to make the workday credit system more attractive. An increase in q (i.e. the allotment of less generous workday credits) would increase the term on the right hand side. So would an increase in P_{wc}. This jibes with the intuition that less generous credits alongside with more demanding qualification requirements would diminish the negative impact on output and thus increase the value of the discounted output stream. However, under the assumption of perfect information, a camp manager considering the introduction of a workday credit system would choose the optimal combination of P_{wc} and q from the very beginning, presumably with high values for both variables, in order to achieve the highest possible profitability for the lowest possible price. So there are plausible reasons to take both these values as determined by exogenous parameters. In that case, the only remaining variable on which the decision could depend is the discount factor δ. Specifically, as we argued in section III, there will be a critical discount factor below which it would be optimal to opt for the introduction of workday credits.

A.III. Profit maximization without time preferences

To change the focus from output to profit maximization we modify our framework by assuming that there are costs incurred in using forced labor, namely constant per-unit costs C, i.e. constant marginal costs.[6] This would imply the following modifications to our model, in which we now examine profits Π instead of output Y:

(15.)
$$\Pi = N(P - C)$$

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[6] Labor costs in Soviet camps were largely constant. Each prisoner had to be supplied with the same provisions determined by orders and other administrative documents applicable to the entire camp system (see, for instance, order of NKVD No. 00943 of August 14, 1939; order of MVD No. 0313 of February 11, 1952.). Furthermore, every camp was organized in departments of standardized size that had to be endowed with the same infrastructure (GARF. F. 9414. Op. 1. D. 374. LL. 60-116. See also order of MVD No. 0190 of March 29, 1947). Finally, the ratio of guards to inmates was established irrespective to camp size. For these reasons, differences that did exist in this field depended mostly on geographic factors and on the specific economic profile of a given camp, rather than on its size.

or, according to the steps from equation (1.) to equation (5.):

(16.) $$\Pi_t = T_t(P_t - C)$$

Asking for the optimal choice in a long-term perspective, i.e. ignoring the short-term effects during the transition period, we can denote instead of equation (9.):

(17.) $$\Pi_t^{wc} = qT_0(P_{wc} - C) > T_0(P_0 - C) = \Pi_0$$

which leads us to:

(18.) $$\frac{1}{q} < \frac{P_{wc} - C}{P_0 - C}$$

(for all $P_0 > C$, which implies that $P_{wc} > C$ is equally fulfilled)

Assuming both P_{wc} and $P_0 > C$,[7] the ratio on the right hand side in inequation (18.) is greater than that in inequation (10.) for any value for $C > 0$. Therefore the value of q could be lower than in inequality (10.) while the entire expression would still remain true: If there are positive labor costs, calculating with regard to profits instead of output makes workday credit systems more attractive from the camp manager's perspective.

A.IV. Profit maximization with time preferences

Finally we consider the cost issue in the case in which time preferences do affect the camp manager's decision. Inequality (14.) now reads as follows:

(19.) $$\frac{1}{1-\delta}(P_0 - C) < (P_{wc} - C)\left[\sum_{t=0}^{qT_0}\delta^t\left(q + (1-q)\frac{T_0 - \frac{1}{q}t}{T_0}\right) + q\frac{\delta^{qT_0+1}}{1-\delta}\right]$$

The results are similar: As the only modification consists in replacing P_0 by $(P_0 - C)$ and, respectively, P_{wc} by $(P_{wc} - C)$, we can argue that, since the ratio $(P_{wc} - C) / (P_0 - C)$ for positive values of C is greater than the ratio P_{wc} / P_0,[8] the maximization of profits instead of output makes it more likely to opt for workday credits in this case, too. We could also assume a case in which $(P_0 - C) < 0$ while $(P_{WC} - C) > 0$, i.e. in which the labor productivity of the prisoners without motivation through the workday credit system is lower than the factor price, whereas the introduction of workday credits raises the productivity above the threshold of profitability. In this case the term on the inequality's left side would be negative and the term on the right hand side would be positive for all possible values of $\delta > 0$ and $q > 0$, i.e. the camp manager would strictly prefer to introduce *any* workday credit system, irrespective of his time preferences.

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[7] If either the nominator, or the denominator of the fraction on the right hand side in inequation (18.) were negative, the inequation sign would be inverse. Specifically, if $P_{wc} > C > P_0$, the term on the right hand side would be negative and the inequation would be fulfilled for any value of q: The camp manager would be willing to pay every price for a workday credit system that would enable him to turn losses into profits. If $P_0, P_{wc} < C$, the term would be positive. In this case the bigger the (according to our assumptions: positive) difference $(P_{wc} - P_0)$, the higher the possible values for q for which the term would be true. This can intuitively be interpreted in such a way that if a camp manager interested only in profitability would face losses both with and without using workday credits, he would by all means endeavor to release prisoners quicker in order to avoid incurring losses.

[8] Given $P_{wc}, P_0 > C$. For all other cases: see discussion in the previous footnote.