Word Games for Formal Logic

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Abstract

Some students in the humanities take fright when introduced to the formal manipulations characteristic of elementary sentential & predicate logic. One way to lessen the pain of initiation is to start with word games, of which Lewis Carroll's DOUBLETS (section 1) is a familiar example. The paper presents some other games that successively introduce more of the features of systems of natural deduction (such as the system of Lemmon's *Beginning Logic*). All the examples are in English, but in principle the games can be implemented in any written language based on an alphabet.

0 Preface

To my knowledge Charlesworth [1981] was the first to draw attention to the similarity between various word games, such as DOUBLETS (Carroll [1879]), and the symbolic manoeuvres involved in proofs in formal logic. Such games may be of some help in commending logic to those who protest (implausibly) that they 'cannot cope with symbols'. Starting with DOUBLETS itself, this paper presents five games that mimic one by one some standard features of natural deduction systems. In this prefatory section I presuppose acquaintance with such systems, and summarize the principal comparative features of the five games.

Plays in each game consist in the transformation of one or more initial words, which are called *premises*, by means of repeated application of one or more rules, into a final word, which is called the *conclusion*. The rules are *formal* in the sense that their correct use depends only the letters present in the premises and the conclusion, and not on what the words mean. On the other hand, to avoid triviality, nothing but a genuine word is admitted on any line of a derivation. (What counts as a genuine word must be agreed at the outset. In this paper a *word* is anything of three or more letters that is either an entry, or an unforced inflexion of an entry, in Kirkpatrick [1983].) The games could be played by an appropriately programmed mechanical device equipped with a complete lexicon.

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Game 1 DOUBLETS employs a single one-premise rule of derivation. Each line may be used at most once, for the derivation of the next line (if any). Game 2 STUDDLETS, which is due to Charlesworth, *op.cit.*, has a single two-premise rule. In this game each line may be used as a premise on an unlimited number of occasions. Game 3 SPLICES introduces the requirement that the content of the conclusion be included in the content of the premises taken together. Here the content of a word is understood as the class of its constituent letter types. Game 4 SPLITS allows in addition the use of assumptions, provided that they are discharged before the end of the derivation; that is, the use of words that are neither among the premises nor derived from them alone. To conclude, Game 5 SPLINTERS divides the rule of derivation of SPLICES into two weaker and partly complementary rules that correspond, in a rough way, to the usual introduction and elimination rules for logical constants. In English at least, SPLINTERS (and its more restrictive variant SPLINTS) are not easy games to play; certainly they are more demanding than the system of natural deduction of Lemmon [1965], to which they bear a palpable likeness.

A comprehensive treatment of word games and their analogies (and disanalogies) to systems of natural deduction is to be found in the author's [1985].

1 Doublets

The task of this very familiar game (also known as LADDERGRAMS, WORD CHAINS, WORD PING-PONG, and STEPWORDS, & by several other names) is to transform an initial word into another of the same length, changing exactly one letter at each step, the intermediate steps all being words themselves.

Example 1 *Reduce* **MIND** *to* **BODY** *in 7 lines.*
Solution **MIND** ⇒ **MINE** ⇒ **TINE** ⇒ **TONE** ⇒ **BONE** ⇒ **BODE** ⇒ **BODY**.

Problem 1 *Develop* **RATIO** *into* **LOGIC** *in 24 lines.*

2 Studdlets

A move in DOUBLETS involves just two words, and its correctness is determined by the relation between them. Thus a play of DOUBLETS can be written as a single column of words, and may be checked by checking that each word after the first is correctly obtained from its predecessor. In STUDDLETS each new word is derived from two parent words (possibly identical), which need not be spatially contiguous to each other or to their progeny. The rule of transformation is that with at most one exception each letter of the offspring word must occur in the same position (marking from the left) in one of the parents. Words can be mated as often as desired, with themselves or with others. Plays in STUDDLETS may be set out in family trees, which display graphically the genealogical connections, or (more compactly) in annotated columns, where a note at each line indicates its provenance. In the Example that follows lower case is used in each generation for the letter (if there is one) that is not inherited directly from the parents.

Example 2 *Add* **ONE** *to* **TWO** *to make* **THREE** *in 7 lines*

Solution			
	(1)	**O N E**	Given
	(2)	**T W O**	Given
	(3)	**T h E**	1,2
	(4)	**T H E e**	3,3
	(5)	**T i E**	3,3
	(6)	**T I r E**	4,5
	(7)	**T H R E e**	4,6

Problem 2 *Add* **SEVEN** *to* **EIGHT** *to make* **FIFTEEN** *in 12 lines.*

3 Splices

In this game, in contrast to DOUBLETS and STUDDLETS, the letters in the conclusion must be drawn wholly from among those in the premises. The transformation rule of the game is simple. Take any two (possibly identical) words that have appeared in a derivation, place them end to end (in either order), and extract any word to be found in the resultant string. For instance, a **SWORD** may be drawn from **WORDS** and **THINGS**, and **TWO** from **WORD** and **OBJECT**. As in STUDDLETS, plays are best set out in family trees or in annotated columns.

Example 3 *Scour* **NEWSPAPER** *for* **ANSWER** *in 10 lines*

Solution			
	(1)	**N E W S P A P E R**	Given
	(2)	**S P A**	1,1
	(3)	**P A N**	2,1
	(4)	**P A N S**	3,2
	(5)	**A P E**	1,1
	(6)	**E R A**	1,5
	(7)	**E R E**	1,6
	(8)	**N E W**	1,1
	(9)	**W E R E**	8,7
	(10)	**A N S W E R**	4,9

Problem 3 *Mix* **MARTINI** *from* **GIN AND VERMOUTH** *in 17 lines*

4 Splits

SPLICES can be a difficult game, for letters in the interiors of words are often inaccessible. For example, there is no way to persuade a **PENGUIN** to lay an **EGG**, as neither the **E** nor the **G** can be extricated. To ease such transformations SPLITS adds to SPLICES a rule allowing the temporary borrowing of words. All letters from borrowed words must eventually be repaid. We annotate with W those lines that are obtained by *wordsplicing*, and with A those that are obtained by assuming a debt, as well as those that are given. Premises (in capitals) and debts (in lower case) are referred to collectively as *assumptions*. To the left of each line are recorded the assumptions on which it depends.

Example 4 *Pilfer* **EGG** *from* **PENGUIN** *in 7 lines*

Solution				
	1	(1)	**PENGUIN**	A
	2	(2)	**ear**	A
	1,2	(3)	**GUINea**	1,2 W
	4	(4)	**negro**	A
	1,4	(5)	**groPE**	4,1 W
	1	(6)	**PEG**	5,3 W
	1	(7)	**EGG**	6,3 W

Problem 4 *Prepare* **SUPPER** *from* **PRUNES** *in 18 lines*

5 Splinters

Having made Splices easier by passing to Splits, we now make it harder by replacing the wordsplicing rule W by two weaker rules, those of *word excision* WE and *word integration* WI. The rule WI is just like the old rule W except that one of the two words concatenated must be preserved intact. It is no longer possible for a **SHY BRIDE** to give birth unaided to a **HYBRID**. The rule WE allows the excision of any subword of a given word.

Example 5 *Develop* **EPISTEMOLOGY** *into* **SYSTEM** *in 8 lines*

Solution				
	1	(1)	**EPISTEMOLOGY**	A
	1	(2)	**STEM**	1 WE
	3	(3)	**pan**	3 A
	1,3	(4)	**panS**	2,3 WI
	5	(5)	**early**	5 A
	1,5	(6)	**Yearly**	1,5 WI
	1	(7)	**panSY**	4,6 WI
	1	(8)	**SYSTEM**	7,2 WI

Problem 5 *Become* **POPPERIAN** *after* **DEEP DELIBERATION** *in 36 lines*

In Example 5 the loans too are preserved intact throughout the derivation, and repaid intact, rather than by instalments. To impose this as an additional rule (as in the game Splints) does, however, make the game much harder.

References

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