KARL POPPER & DAVID MILLER. 'Why Probabilistic Support Is Not Inductive', *Philosophical Transactions of the Royal Society of London*, Series A **321**, 1562, 30/4/1987, pp. 569–591.

Added August 10, 2008: THEOREM 3 on p. 577 can be generalized in the following way:

LEMMA: **If $\vdash x \vee y$ and $y \vdash z$ then $p(x, z) - p(x, y)$ is equal to**

$$[1 - p(x, z)][1 - p(y, z)]/p(y, z) \text{ if } p(y, z) > 0$$

and to $1 - p(x, y)$ if $p(y, z) = 0$.

Proof: In the following chain of identities we use the identity $p(x, y) = p(x, yz)$ (which follows from $y \vdash z$), the multiplication and addition laws, and the identity $p(x \vee y, z) = 1$ (which follows from $\vdash x \vee y$).

$$\begin{aligned}
[p(x,z) - p(x,y)]p(y,z) &= p(x,z)p(y,z) - p(x,yz)p(y,z) \\
&= p(x,z)p(y,z) - p(xy,z) \\
&= p(x,z)p(y,z) - [p(x,z) + p(y,z) - p(x \vee y,z)] \\
&= -p(x,z)[1 - p(y,z)] - [p(y,z) + 1] \\
&= [1 - p(x,z)][1 - p(y,z)]
\end{aligned}$$

If $p(y, z) > 0$, the announced equality is proved. If, on the other hand, $p(y, z) = 0$, then the final product above is also 0, and so its first factor is 0. It follows that $p(x, z) - p(x, y) = 1 - p(x, y)$.

THEOREM: **If $\vdash x \vee y$ and $y \vdash z$ then $p(x, y) \leq p(x, z)$.**

Proof: Since all probabilities lie between 0 and 1 inclusive, the theorem follows from the lemma.

The burden of this theorem, stated informally, is that when a hypothesis h is maximally independent of the evidence — that is, it goes wholly beyond the evidence —, then the probability $p(h, e)$ increases when the evidence e is weakened; and hence, **the weaker is the evidence, the greater is the probabilistic support**.

COROLLARY: **$p(x, y) \leq p(x, x \vee y) \leq p(x \leftarrow y)$.**

Proof: Since $\vdash (x \leftarrow y) \vee y$ and $y \vdash x \vee y \vdash \top$, we have $\vdash (x \leftarrow y) \vee (x \vee y)$. By two applications of the theorem we obtain

$$p(x \leftarrow y, y) \leq p(x \leftarrow y, x \vee y) \leq p(x \leftarrow y, \top) = p(x \leftarrow y).$$

Now $p(w, z) = p(wz, z)$ generally, and hence $p(x \leftarrow y, y) = p((x \leftarrow y)y, y) = p(xy, y) = p(x, y)$ and $p(x \leftarrow y, x \vee y) = p((x \leftarrow y)(x \vee y), x \vee y) = p(x, x \vee y)$. The original THEOREM 3 follows.

Added February 7, 2003: The last eight lines of the proof of Theorem 6 on p. 578 should presumably read:

> Now $xy(x \vee z)$ is equivalent to xy, so collecting terms
>
> $$\begin{aligned} [1 - p(x \vee z)][p(x) - p(xy)] &= p(y)p(x \vee z) - p(y(x \vee z)) \\ &= p(y)[p(x \vee z) - p(x \vee z, y)] \\ &= -p(y)s(x \vee z, y). \end{aligned}$$
>
> The two factors on the left are never negative; and since $p(y)$ is not 0 it follows that $s(x \vee z, y) \leq 0$; and this proves the first part of the theorem.
>
> The addition law and (d) ensure that $p(x) \neq p(xy)$. So if (c) holds, the left side of the above equation is positive. Thus $s(x \vee z, y) < 0$.

In the penultimate line of the proof of Theorem 8 on p. 588, '(G3)' should be '(G2)'.