Inspection Games

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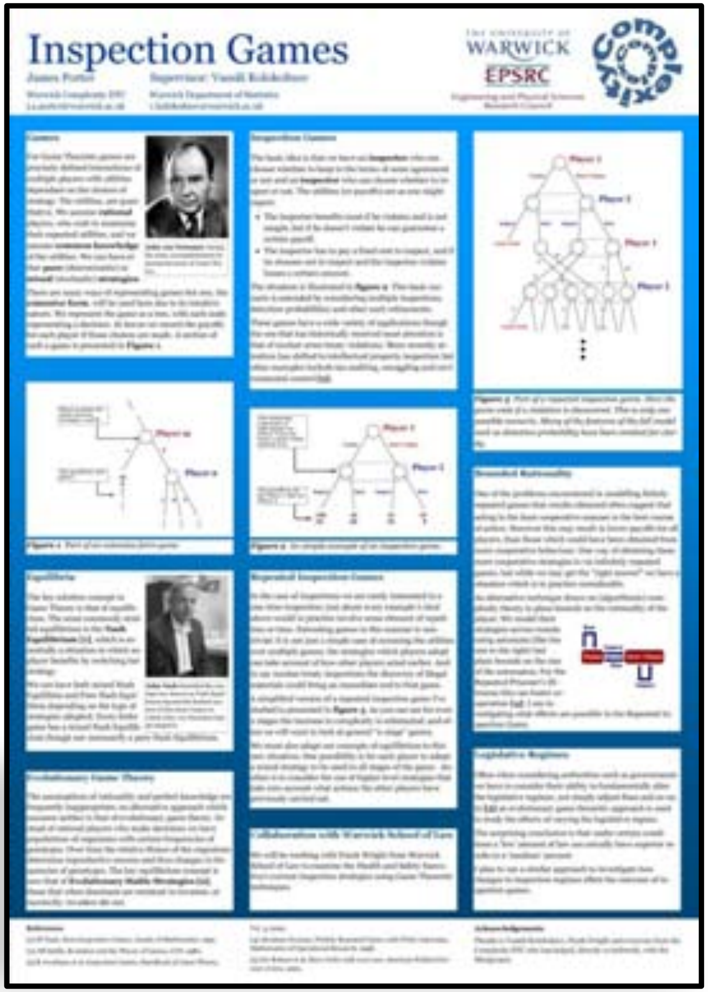
Overview

1. Motivation/Aims
2. Introduction
3. (Two) Models
4. Bounded Rationality via Computational Complexity
5. Evolutionary Approach
6. Conclusions & Prospects



Motivation

- In UK one estimate puts tax evasion at £80 Billion per year [1]
- Tax evasion estimated to cost $160 Billion USD per year in Developing World [2]
- Nuclear arms inspections have obvious political importance

[1] www.**tax**research.org.uk
[2] http://latestnews.virginmedia.com/news/money/2008/05/12/tax_evasion_causing_child_deaths
Obviously these are incredibly difficult to accurately assess

Aim

Assess Empirical Utility of Inspection Game Theoretic Models/Techniques:

- Distinctive Qualitative Features (What are the applications?)
- Inputs and Knowledge Required
- Verification/Testing/Predictive Use

Basic Concepts in Game Theory

- **Games**
- **Rationality**
- **Common Knowledge**
- **Pure Strategies**
- **Mixed Strategies**
- **Nash Equilibrium**
- Extensive Form Games

Basic Concepts in Game Theory

- Games
- Rationality
- Common Knowledge
- Pure Strategies
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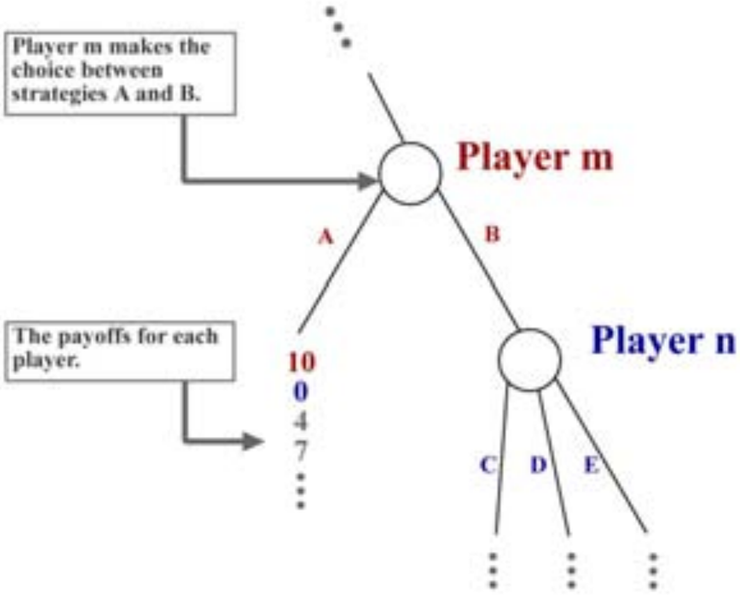
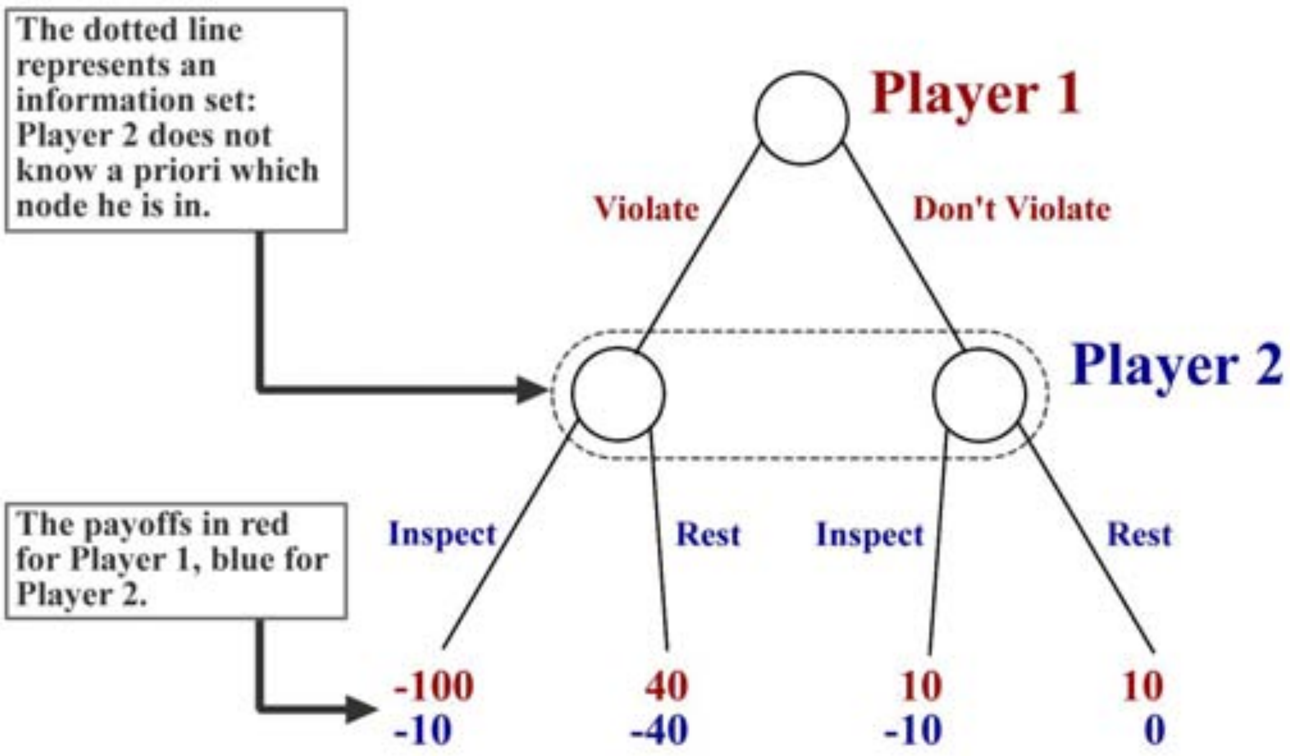
- Nash Equilibrium
- **Extensive Form Games**



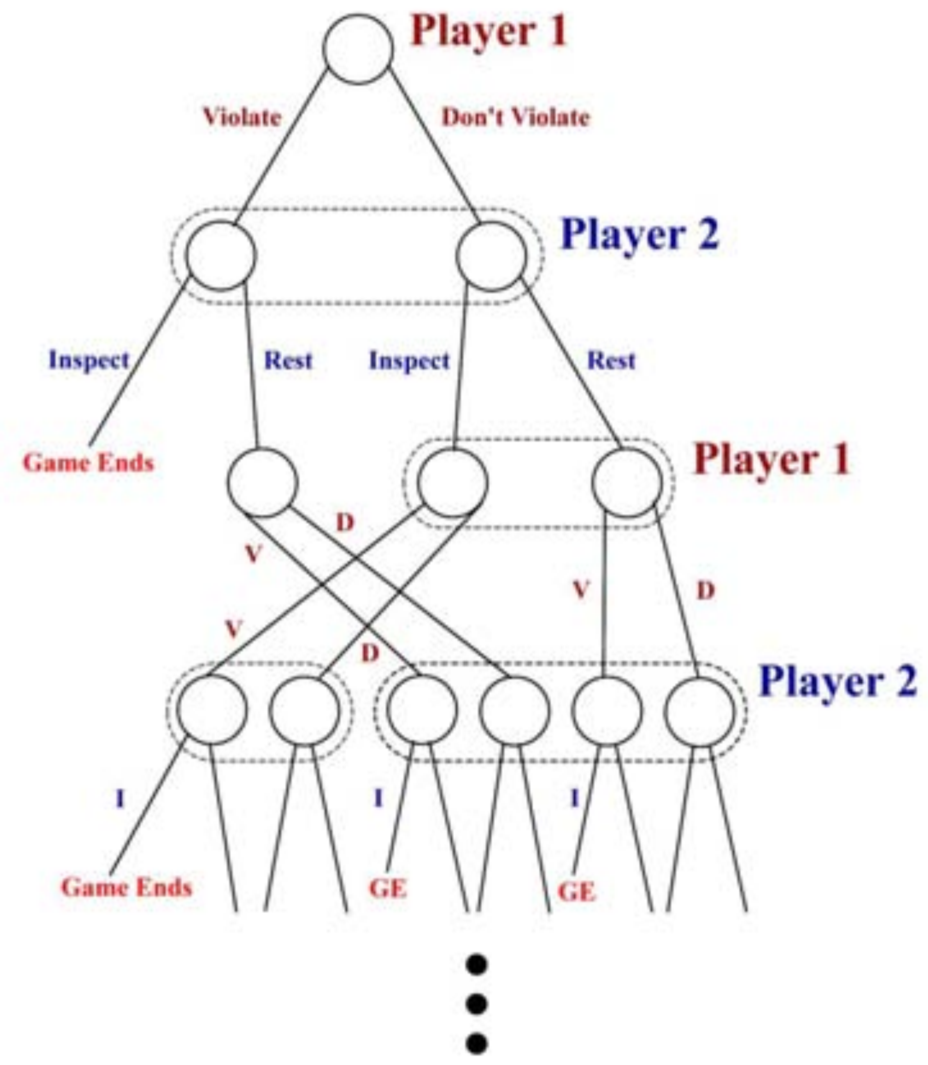
What is an inspection game?

- Simplest form of Inspection Game:



Repeated Inspection Games

- Game now over several stages
- Corresponding increase in complexity.



Basic Concepts in Game Theory (II)

- **Strategic Form Game**
- Payoff Bimatrices

	Cheat	Comply
Inspect	-100 100	5 -5
Don't Inspect	20 -20	5 0

Basic Concepts in Game Theory (II)

- Strategic Form Game
- **Payoff Bimatrices**

$$\begin{bmatrix} 100,-100 & -5,5 \\ -20,20 & 0,5 \end{bmatrix}$$

	Cheat	Comply
Inspect	-100 100	5 -5
Don't Inspect	20 -20	5 0

Model 1: Features

- Model for tax evasion inspections
  - (Tax) Inspector/Auditor, who chooses to either
    - Inspect, fixed cost c, probability of success p
    - Not inspect
  - Tax Payer/Inspectee, who chooses to either
    - Pay tax and earn a legal income r
    - Evade tax: earn legal income r, surplus l
- All of the above can easily be adapted to other scenarios

Model 1: Single Round Version

- Basic form, a single round with bimatrix:

$$\begin{bmatrix} r+\bar{p}l-pf, -c+pf-\bar{p}l & r+l, -l \\ r, -c & r, 0 \end{bmatrix}$$

- Can be easily solved for equilibrium behaviour
- Under reasonable assumptions there is a single mixed NE
- Can obtain **value** (u_1, v_1)

Extension of Model 1: Multi Round Version

- Define game recursively:

$$\Gamma_n = \begin{bmatrix} r + \bar{p}(l + \mathrm{u}_{n-1}) - pf, -c + pf + \bar{p}(-l + \mathrm{v}_{n-1}) & r + l + \mathrm{u}_{n-1}, -l + \mathrm{v}_{n-1} \\ r + \mathrm{u}_{n-1}, -c + \mathrm{v}_{n-1} & r + \mathrm{u}_{n-1}, \mathrm{v}_{n-1} \end{bmatrix}$$

- Then NE values will be:

$$(\mathrm{u}_n, \mathrm{v}_n) = \mathrm{val} \begin{bmatrix} r + \bar{p}(l + \mathrm{u}_{n-1}) - pf, -c + pf + \bar{p}(-l + \mathrm{v}_{n-1}) & r + l + \mathrm{u}_{n-1}, -l + \mathrm{v}_{n-1} \\ r + \mathrm{u}_{n-1}, -c + \mathrm{v}_{n-1} & r + \mathrm{u}_{n-1}, \mathrm{v}_{n-1} \end{bmatrix}$$

There isn't time or space here to do this in full – see written report for a fuller account.

Extension of Model 1: Multi Round Version

- Rewriting: $(\mathrm{u_n}, \mathrm{v_n}) = (\mathrm{u_{n-1}}, \mathrm{v_{n-1}}) + \mathrm{val}(\mathrm{M_n})$
- Where:

$$\mathrm{M_n} = \begin{bmatrix} r + \bar{p}(l) - p(f + \mathrm{u_{n-1}}), -c + p(f - \mathrm{v_{n-1}}) - \bar{p}l & r + l, -l \\ r, -c & r, 0 \end{bmatrix}$$

- For 2 round case this we have:

$$(u_2, v_2) = (u_1, v_1) + val(M_2)$$

- If certain conditions are satisfied we can obtain the Mixed NE value in a straightforward (though algebraically awkward) way.

More than 2 Rounds

- We can continue in this fashion to obtain the NE for such a game with an arbitrary number of rounds.

In Avenhaus's "Compliance Quantified" (section 5.4) a general analytical solution is obtained for a similar though simplified zero sum, single-violation-possible model.

Model 1: Numerical Approach

- As you can see analytically messy for even a small number of stages
- When we do obtain results, they are invalidated by slight changes to model

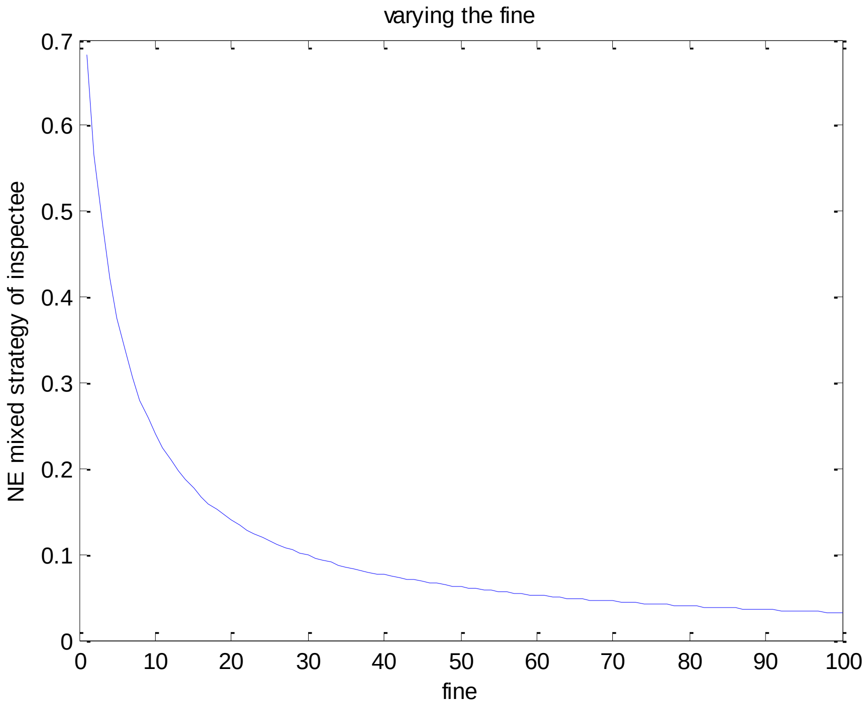
- But recursive definition gives algorithm for obtaining N.E. of games
- For “realistic” examples computationally unproblematic (within certain parameter regimes)

Model 1: Numerical Results (I)

DEFAULT PARAMETER VALUES

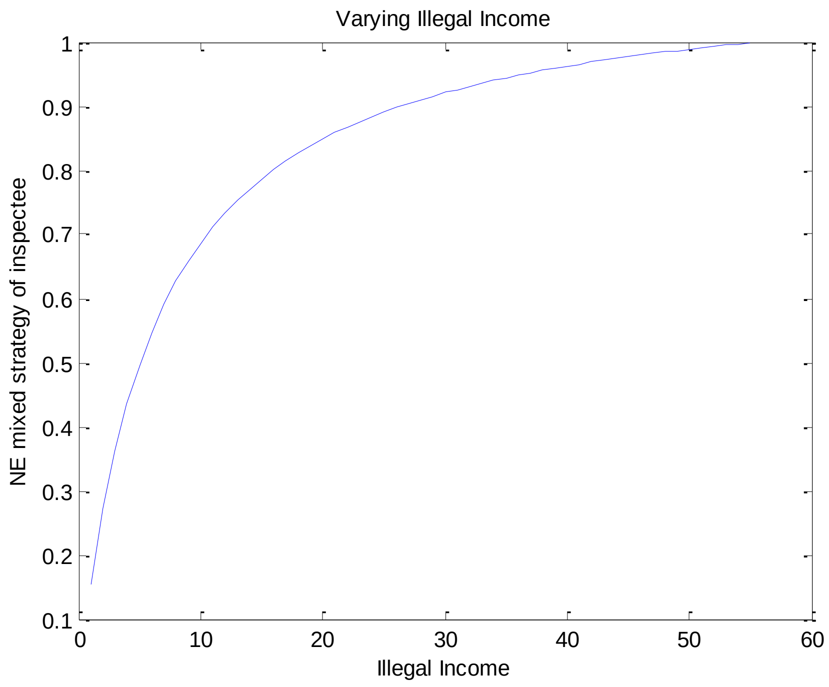
- n = 3
- r = 10
- f = 100
- c = 10
- l = 30
- p = 0.9

VARY THE FINE

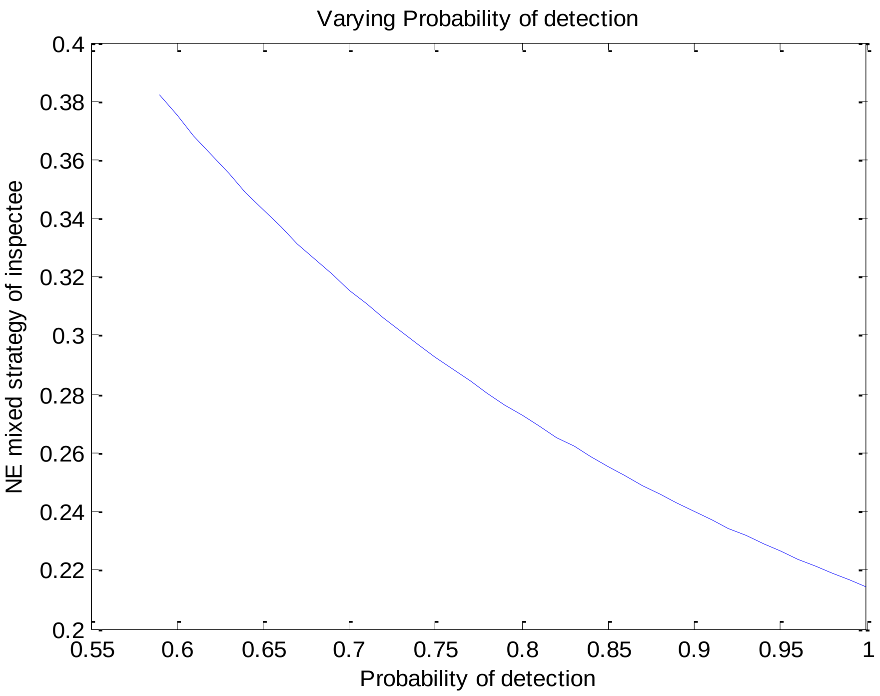


Model 1: Numerical Results (II)

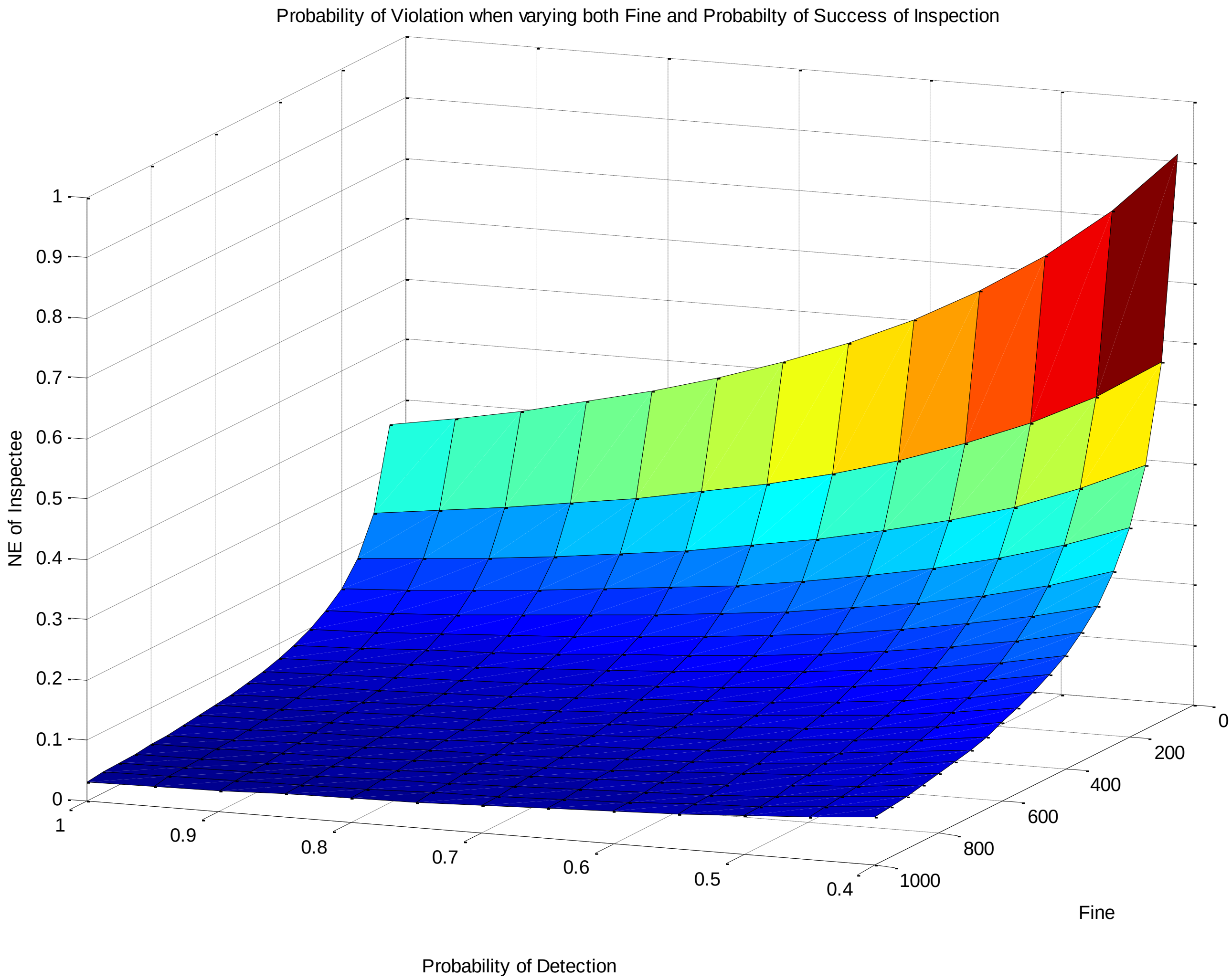
VARY ILLEGAL INCOME



VARY PROBABILITY OF DETECTION



Model 1: Numerical Results (III)



Model 2: Features

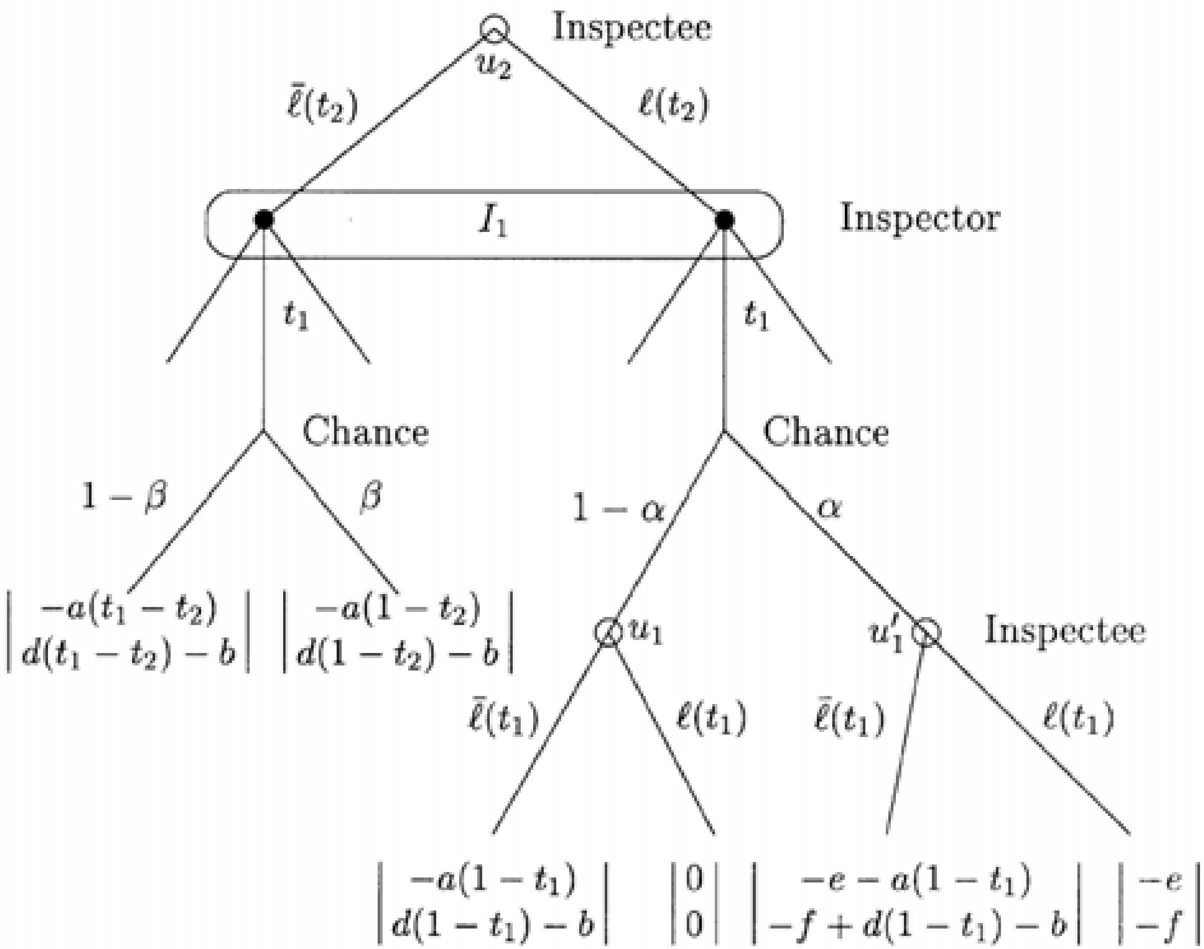
- Single Inspected Object
- Thorough initial inspection, then interim inspections
- Detection probability 1-β
- False alarm probability α
- *k* inspections, labelled backwards

Avenhuas & Canty, Playing for Time a Sequential Inspection Game, European Journal of Operational Research 167 (2005), 475-492.
There have been several extensions proposed to this model, see bibliography in written report.

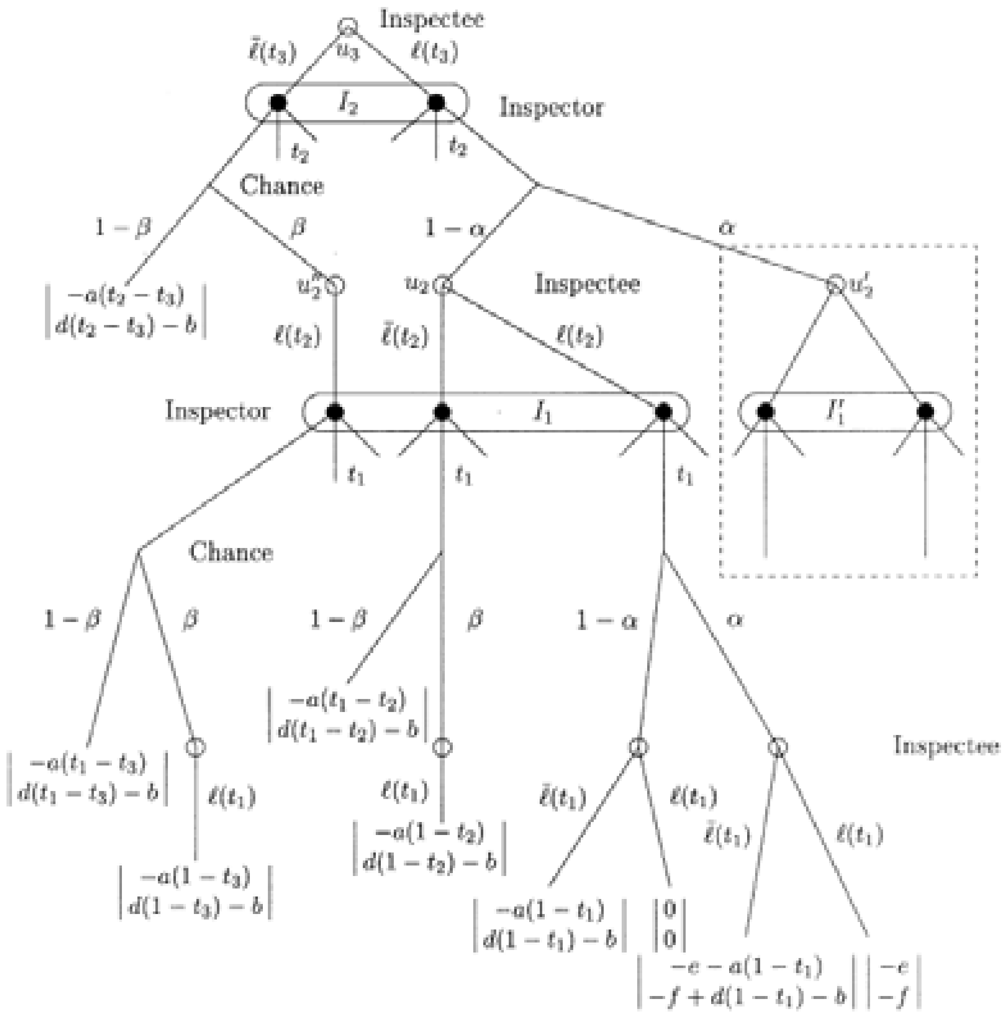
Model 2

- Utilities: (Inspector, Inspectee)
  - *(0,0)* legal action, no false alarm
  - *(-le,-lf)* legal action, l false alarms
  - *($a\Delta t$, $d\Delta t - b$) detection of illegal activities after time Δt.*

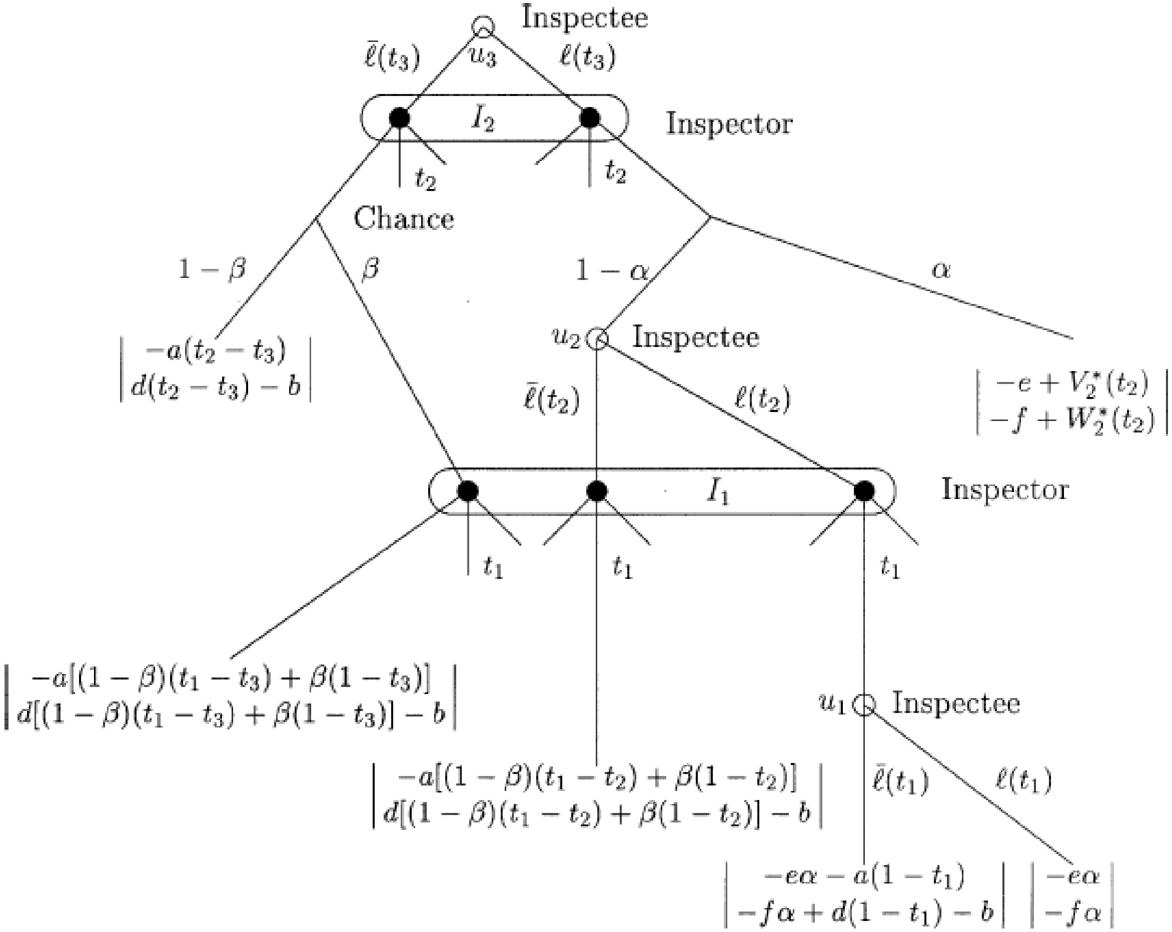
Model 2: Single Inspection Version



Model 2: 2 Inspection Version (Full)



Model 2: 2 Inspection Version (Reduced)



Model 2: Analysis

- We can continue with the analysis to derive optimal number of inspections

Other Models

1. Ferguson and Melolidakis 1997
2. Pradiptyo 2006

- Actually aren't that many recently published models.
- Many classic Nuclear Arms Inspection models.

See written report for further details.

Computational Complexity and Bounded Rationality

- Model strategies of players by finite state automata
- Of certain Complexity
- If Bounded obtain more cooperative behaviour

A. Heyman, Finitely Repeated Games and Finite Automata, Mathematics of Operational Research, 1998.
On Bounded Rationality and Computational Complexity, C.H. Papadimitriou and Mihalis Yannakakis

Applied to Finitely Repeated Inspection Game

- Folk Theorem(s)
- Complexity of Automata
- Basic idea “prove your automata is genuine”

- Probably of little practical use

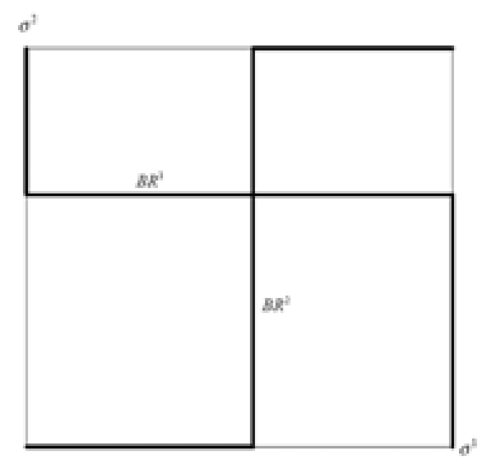
Evolutionary Approach: Fictitious Play

- Re-examine earlier model
- Fictitious Play
- (With Enhancements)

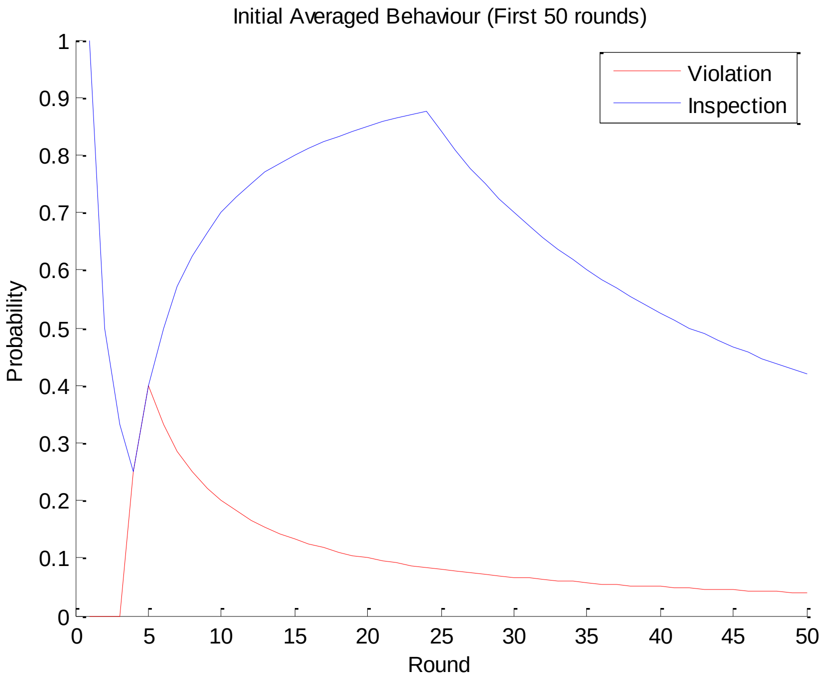
- How equilibria form (if they form)
- How quickly they form

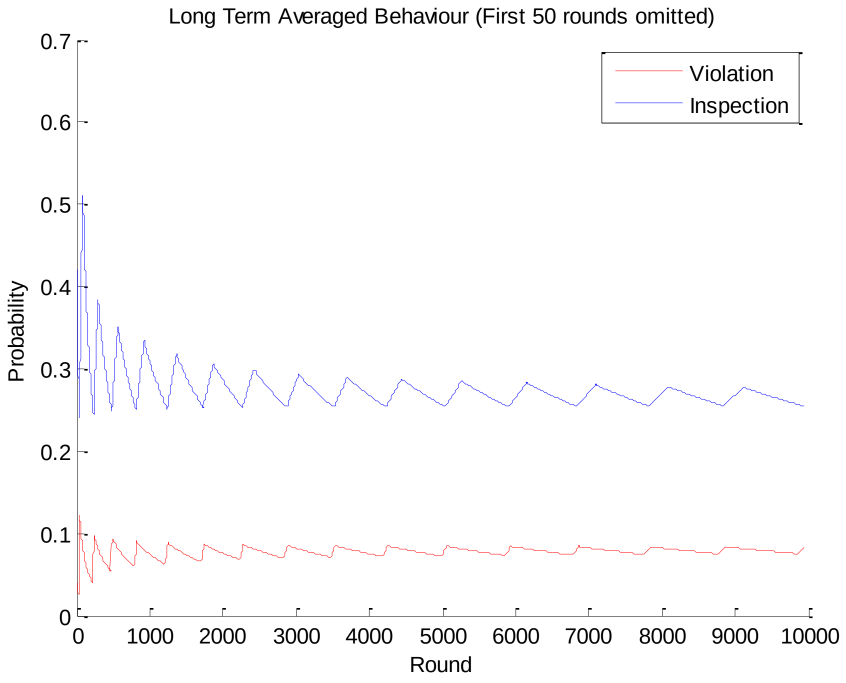
Fictitious Play

- Each player has initial weight function (prior belief about other player's strategies) κ_0^i
- This is updated to κ_t^i by count of plays of strategies
- This allows us to obtain a probability distribution γ_t^i on those strategies
- And via a rule ρ_t^i we obtain the best response



Initial Results

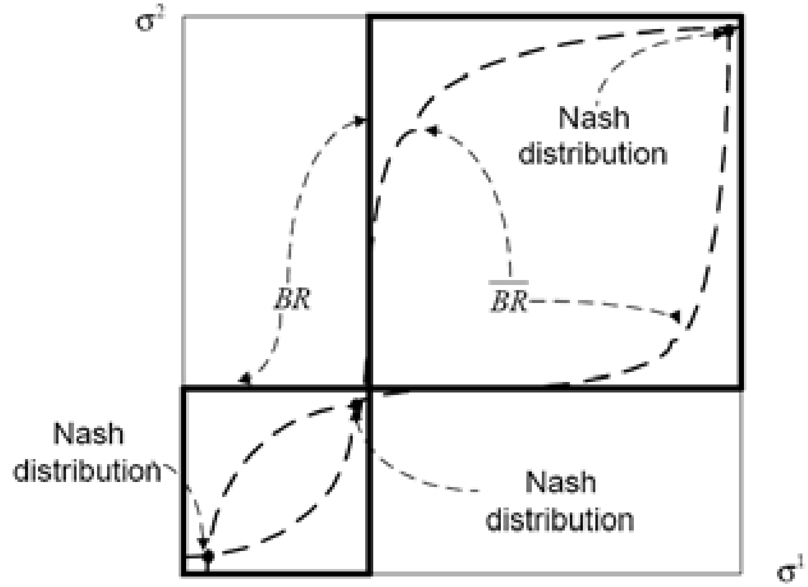




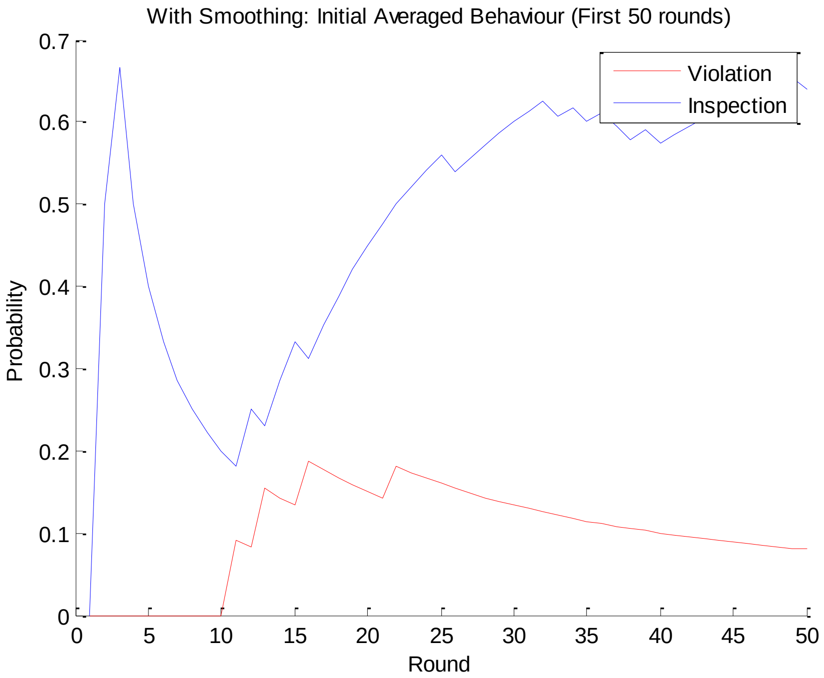
Enhancements

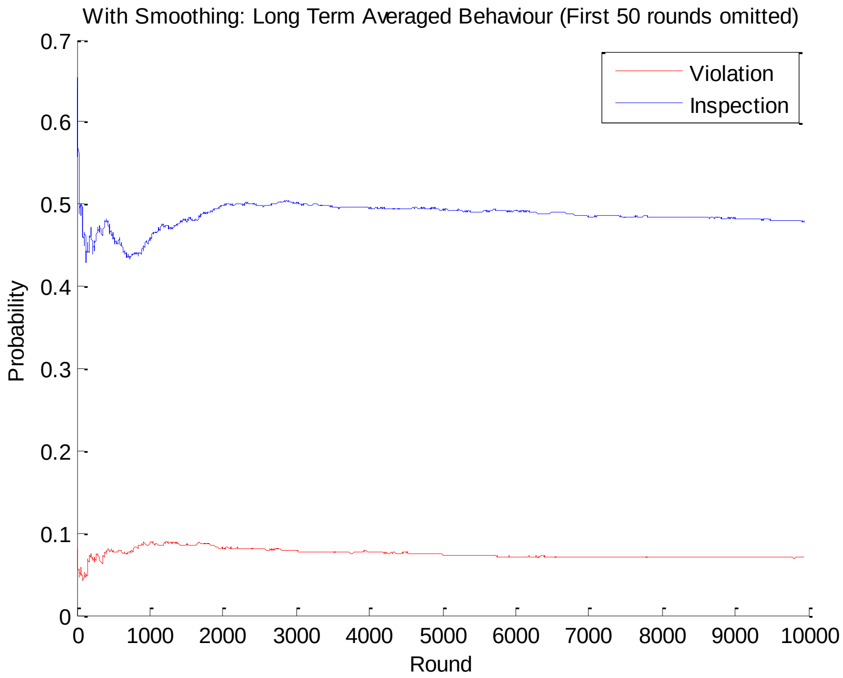
- Smoothed Best Response

$$P(s_j) = \frac{e^{E(s_j)/\gamma}}{\sum_i e^{E(s_i)/\gamma}}$$

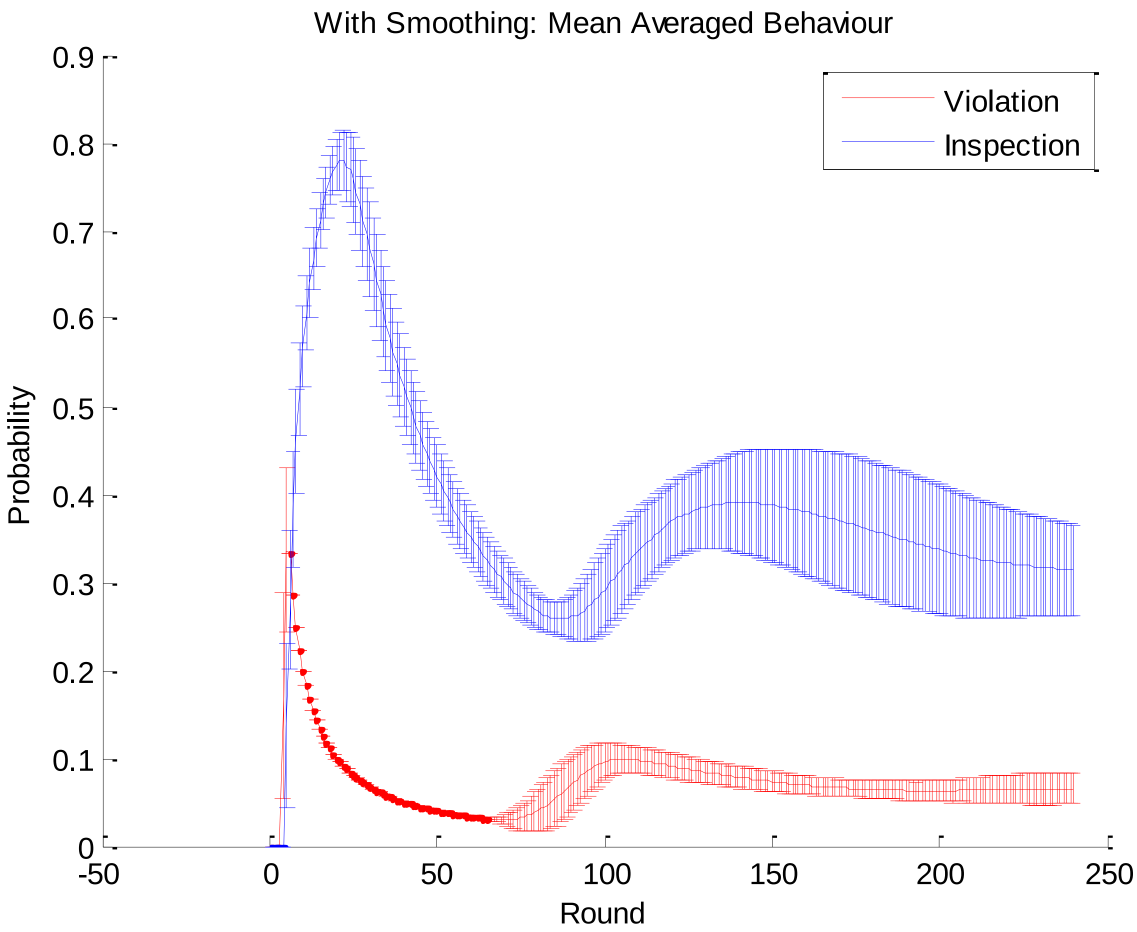


Results


With Smoothing: Initial Averaged Behaviour (First 50 rounds)
Violation
Inspection
Probability
Round
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
5
10
15
20
25
30
35
40
45
50


With Smoothing: Long Term Averaged Behaviour (First 50 rounds omitted)
Violation
Inspection
Probability
Round
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000

Results



With Smoothing: Mean Averaged Behaviour
Violation
Inspection
Probability
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-50
0
50
100
150
200
250
Round

Conclusions

- Variety of flexible models available
- Very direct applications
- However the necessary data can be difficult to obtain
- Computational Complexity not useful as model of bounded rationality
- Evolutionary model(s) very useful as (potentially) allow us to look at short term behaviour and formation of equilibria

Prospects

- "Real world" applications
  - Collaboration with Warwick School of Law and HSE
  - Collaboration with Aston Business School and Home Office
- Modelling Work
  - Many possible extensions to models
  - More general setting
  - 2nd Mini Project


Questions