Inspection Games

James Porter
Supervisor: Vassili Kolokoltsov

Warwick Complexity DTC
j.a.porter@warwick.ac.uk

Warwick Department of Statistics
v.kolokoltsov@warwick.ac.uk

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Games

John von Neumann Among his many accomplishments he pioneered much of Game Theory.

For Game Theorists games are precisely defined interactions of multiple players with utilities dependant on the choices of strategy. The utilities, are quantitative. We assume **rational** players, who wish to maximise their expected utilities, and we assume **common knowledge** of the utilities. We can have either **pure** (deterministic) or **mixed** (stochastic) **strategies**.

There are many ways of representing games but one, the **extensive form**, will be used here due to its intuitive nature. We represent the game as a tree, with each node representing a decision. At leaves we record the payoffs for each player if those choices are made. A section of such a game is presented in **Figure 1**.

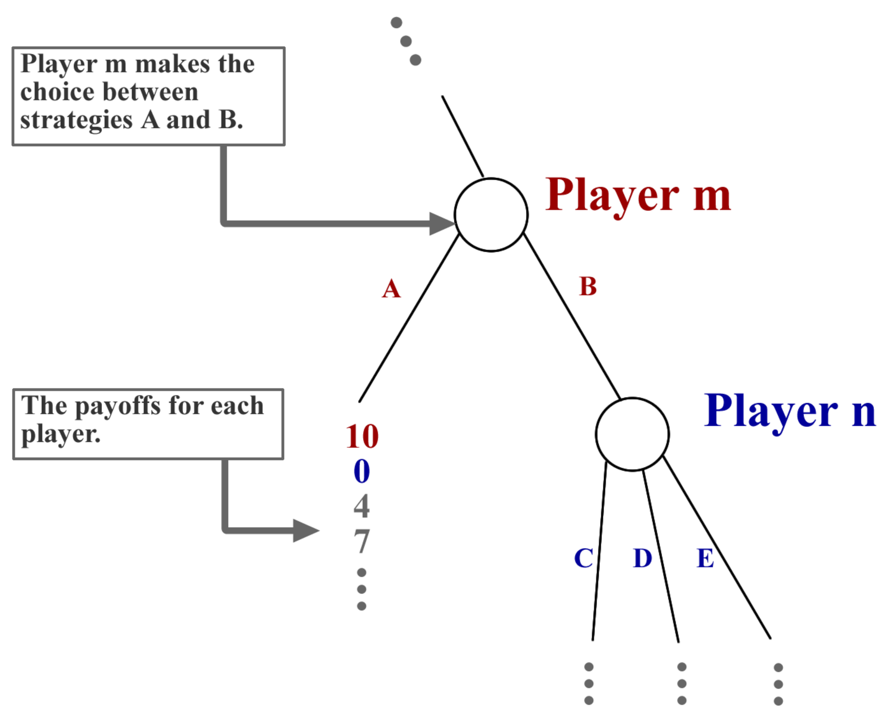


Figure 1 *Part of an extensive form game*

Equilibria

John Nash Extended the concept now known as Nash Equilibrium beyond the limited context of Zero Sum Games in which John von Neumann had developed it.

The key solution concept in Game Theory is that of equilibrium. The most commonly studied equilibrium is the **Nash Equilibrium [1]**, which is essentially a situation in which no player benefits by switching her strategy.

We can have both mixed Nash Equilibria and Pure Nash Equilibria depending on the type of strategies adopted. Every finite game has a mixed Nash Equilibrium though not necessarily a pure Nash Equilibrium.

Evolutionary Game Theory

The assumptions of rationality and perfect knowledge are frequently inappropriate; an alternative approach which assumes neither is that of evolutionary game theory. Instead of rational players who make decisions we have populations of organisms with certain frequencies of genotypes. Over time the relative fitness of the organisms determine reproductive success and thus changes in frequencies of genotypes. The key equilibrium concept is now that of **Evolutionary Stable Strategies [2]**, those that when dominant are resistant to invasion, or succinctly: invaders die out.

Inspection Games

The basic idea is that we have an **inspectee** who can choose whether to keep to the terms of some agreement or not and an **inspector** who can choose whether to inspect or not. The utilities (or payoffs) are as one might expect:

- The inspectee benefits most if he violates and is not caught, but if he doesn't violate he can guarantee a certain payoff.
- The inspector has to pay a fixed cost to inspect, and if he chooses not to inspect and the inspectee violates looses a certain amount.

The situation is illustrated in **figure 2**. This basic scenario is extended by considering multiple inspections, detection probabilities and other such refinements.

These games have a wide variety of applications though the one that has historically received most attention is that of nuclear arms treaty violations. More recently attention has shifted to intellectual property inspection but other examples include tax auditing, smuggling and environmental control **[3]**.

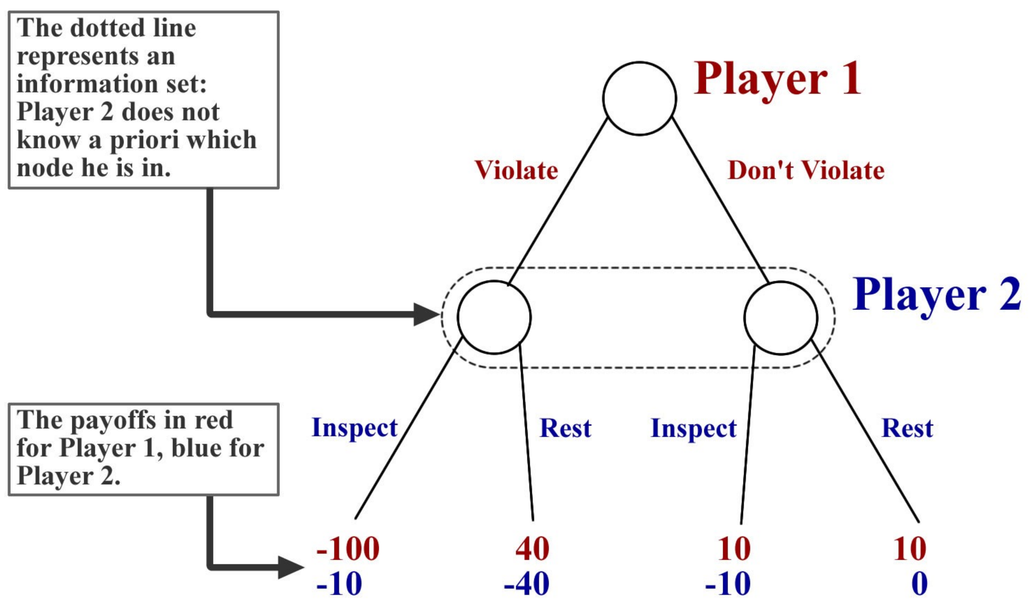


Figure 2 *An simple example of an inspection game.*

Repeated Inspection Games

In the case of inspections we are rarely interested in a one-time inspection: just about every example I cited above would in practice involve some element of repetition or time. Extending games in this manner is non-trivial: it is not just a simple case of summing the utilities over multiple games; the strategies which players adopt can take account of how other players acted earlier. And in say nuclear treaty inspections the discovery of illegal materials could bring an immediate end to that game.

A simplified version of a repeated inspection game I've studied is presented in **figure 3**. As you can see for even 2 stages the increase in complexity is substantial; and often we will want to look at general "n-stage" games.

We must also adapt our concepts of equilibrium to this new situation. One possibility is for each player to adopt a mixed strategy to be used in all stages of the game. Another is to consider the use of higher level strategies that take into account what actions the other players have previously carried out.

Collaboration with Warwick School of Law

We will be working with Frank Wright from Warwick School of Law to examine the Health and Safety Executive's current inspection strategies using Game Theoretic techniques.

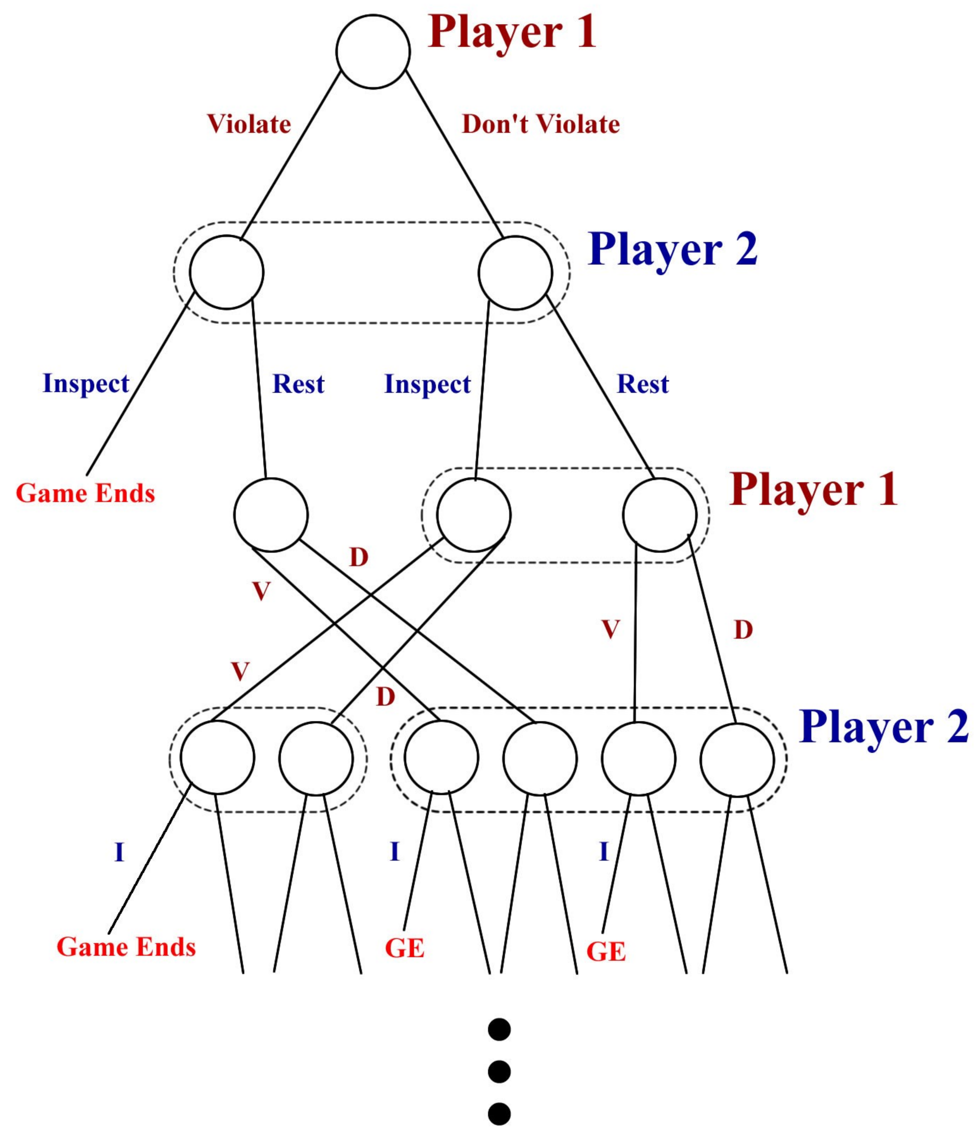
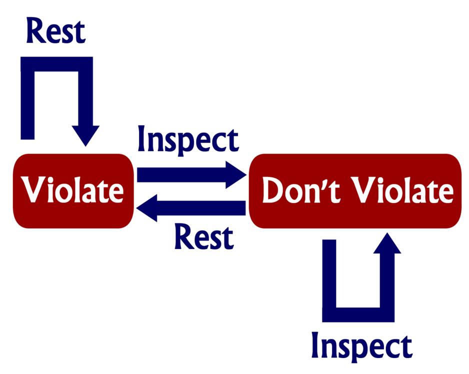


Figure 3 *Part of a repeated inspection game. Here the game ends if a violation is discovered. This is only one possible scenario. Many of the features of the full model such as detection probability have been omitted for clarity.*

Bounded Rationality

One of the problems encountered in modelling finitely repeated games that results obtained often suggest that acting in the least cooperative manner is the best course of action. However this may result in lower payoffs for all players, than those which could have been obtained from more cooperative behaviour. One way of obtaining these more cooperative strategies is via infinitely repeated games, but while we may get the "right answer" we have a situation which is in practice unrealisable.

An alternative technique draws on (algorithmic) complexity theory to place bounds on the rationality of the player. We model their strategies across rounds using automata (like the one to the right) but place bounds on the size of the automaton. For the Repeated Prisoner's Dilemma this can foster cooperation **[4]**. I am investigating what effects are possible in the Repeated Inspection Game.



Legislative Regimes

Often when considering authorities such as governments we have to consider their ability to fundamentally alter the legislative regimes, not simply adjust fines and so on. In **[5]** an evolutionary game theoretic approach is used to study the effects of varying the legislative regime.

The surprising conclusion is that under certain conditions a 'low' amount of law can actually have superior results to a 'medium' amount.

I plan to use a similar approach to investigate how changes in inspection regimes effect the outcome of inspection games.

References

[1] JF Nash, Non-Cooperative Games, Annals of Mathematics, 1951.
[2] JM Smith, Evolution and the Theory of Games, CUP, 1982.
[3] R Avenhaus et al, Inspection Games, Handbook of Game Theory, Vol. 3, 2002.
[4] Abraham Neyman, Finitely Repeated Games with Finite Automata, Mathematics of Operational Research, 1998.
[5] Iris Bohnet et al, More Order with Less Law, American Political Science review, 2001.

Acknowledgements

Thanks to Vassili Kolokoltsov, Frank Wright and everyone from the Complexity DTC who has helped, directly or indirectly, with the Miniproject.