Out-of-Equilibrium Economic Dynamics

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Economics and Equilibria

For those unfamiliar with academic economics you have been wondering about the significance of the qualifier *"out-of-equilibrium"* in the title, for surely that's the natural thing to look at? But for a variety of reasons, principally tractability and inspiration from physics economics has focused on equilibria [5]; although that doesn't generally mean a fixed point of dynamical system: it is usually something more sophisticated. *Roughly speaking an equilibrium is a situation where people know what to expect, where things are stable.*

But this unsatisfactory (or incomplete) as in real economies and markets we frequently see highly volatile unexpected behaviour.


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BILLIONS LOST AS STOCKS CRASH

Pantages In Jail Awaits Sentence To State Prison

GUILTY SAYS JURY AFTER MANY HOURS — Theater Man Declares He 'Got Raw Deal' and Did Not Even Have Chance — MISS PRINGLE GLAD — In Statement Says Not Sure Whether She Will Continue Her Stage Career

SENATE "FARM BLOC" STARTS BATTLE FOR HIGHER TARIFFS

STATE PLANS TO ROUND UP TAX DODGERS — Banks and Corporations to Be Object of Campaign Says Commissioner

Friend Brings Man To Jail On Rum Charge

BINGHAM TAKES EXCEPTION TO SENATE LOBBY — Charges Committee With Unfair Political Tactics During Probe

Alliance Of Two Factions May Win Out — Western Republicans Group With Southern Democrats to Increase Rates

JURY BELIEVES HER STORY

MAINSTAYS OF MARKET IN PLUNGE — Banking Support Unable to Prevent Break Accompanied by Wild Trading — TICKERS FAR BEHIND — General Electric, U. S. Steel, Other Leading Issues Caught In Crash

THREE U. S. C. STUDENTS ARE CRASH VICTIMS — Auto Plunges Into Engine As Rooters Return from Big Football Game

CHRISTIANITY MUST DESTROY WAR OR WAR WILL OVERTHROW CHURCH, SAYS PASTOR SCHROCK

Little work has been done on building tools and concepts within Economics to think about out-of-equilibrium dynamics, yet this being able to model or at least understand the limitations of our ability to model this kind of dynamics would be highly desirable. One mainstream text which moves away from equilibrium approaches is [1], but this is still quite rare.

Bilateral Exchange Economies

One fundamental type of economy is that of an **exchange economy**: one where a set of *agents have bundles of goods and trade directly with other agents*. It's a starting point for thinking about out-of-equilibrium dynamics, but one which is rich enough for us to explore a variety of major issues.

We can specify such an economy along the following lines:

Some relevant work and more extensive accounts can be found in [2,3,4]. This framework is quite general; to narrow things down we make the following assumptions:

(1) Individuals $i \in 1, \ldots, I$ and commodities $j \in 1, \ldots J$.
(2) Endowments $e_i^j \in \mathbb{R}, e_i > 0$ of commodity j for individual i .
(3) A set of periods $t \in 1, 2, \ldots$ in which trade takes place.
(4) Positive real valued utility functions $u_i(x_{it})$.

Agents propose offers to other agents and receive offers which they then decide whether to accept.

"Zero" information

Contrary to most economic models we assume agents do not know much about other agents. In particular they do not know other agent's utility functions or bundles; they make and receive offers.

Cautious Trading

Given agent's lack of information they do not act in anticipation of uncertain future utility, they will only make trades which immediately improve their situation.

Connected Process

The trading process is connected, that is in any period there exists positive probably of any agent making a proposal to any other agent in some future period.

Analytical Results

Economic theory typically focuses on what can be analytically derived from a set of basic assumptions. So what kind of results do we get with **Cautious Trading?**

Convergence of Utility

Utility of individual agents will converge and thus so will the total utility of the economy.

Pairwise Optimality

Not only will utility converge but it will do so to pairwise optimal values, that is for any two agents in the economy with a limiting bundle of goods there is no way of reallocating resources between them that would make on better off without also making one worse off.

Pareto Optimality

In fact if the utility functions are continuous we have the stronger results of Pareto Optimality, that is at the limit there is no way of reallocating resources between *all agents* that improves at least one agent's bundle, without making at least one agent worse off.

These results are all limit results, there is little we can say about short term events.

Numerical Approaches

While we seem to get "good" results from analytical approaches; this does not tell us the whole story. We can ask many questions by using the numerical approach outlined above that would be difficult (if not impossible) to answer analytically. This becomes even more important if we are to augment the model with more realistic features which will force us to drop some of the strong assumptions we originally made. *It also forces us to specify an explicit mechanism* or behaviour rather than making assumptions of the form "agents maximise utility" without even providing a possible way this could be achieved.

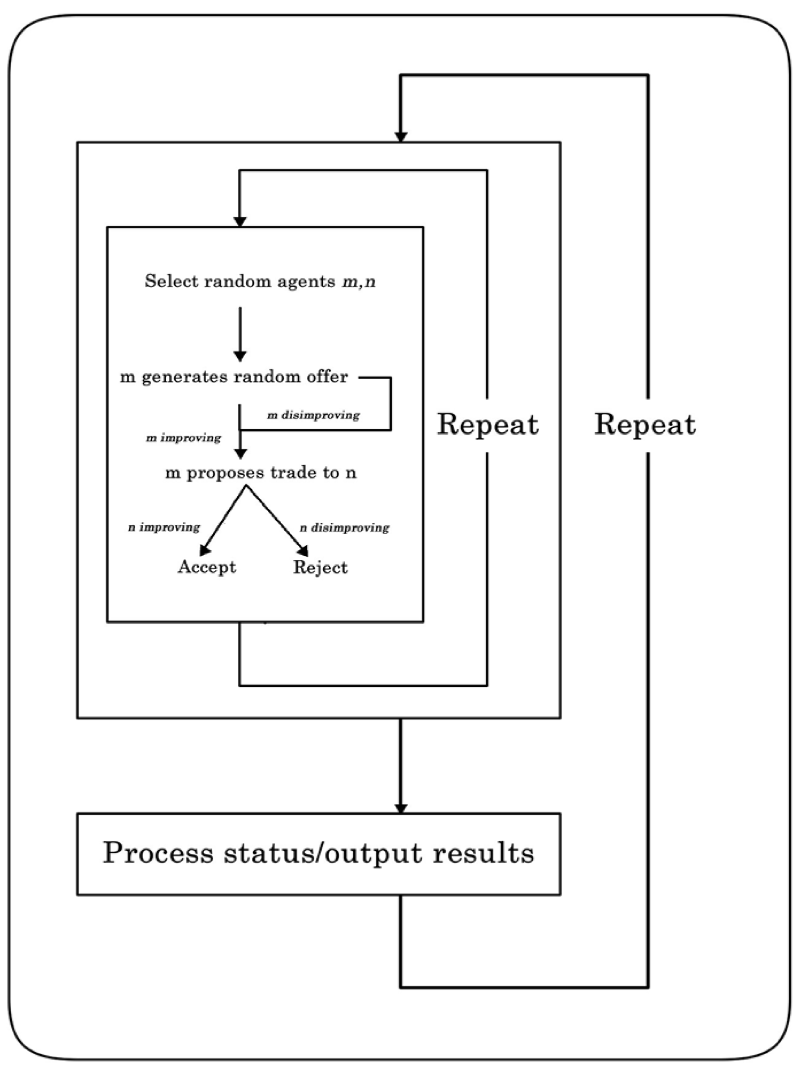

Figure 1 *An outline of an algorithm to simulate an exchange economy. This not only formalises the above model but requires us to explicitly implement the random pairing of agents and the proposal of offers.*

Numerical Results

Figures 2,3 and 4 show some of the results we can obtain numerically for an economy with Cobb-Douglas utility functions and random parameters and endowments.

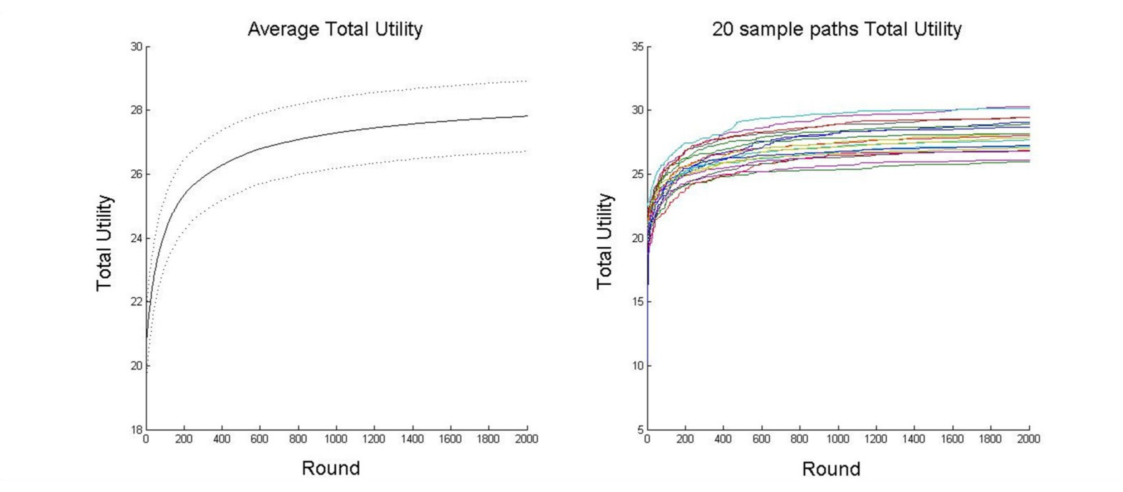

Figure 2 *We get apparent, rapid convergence of both average total utility averaged across multiple realisations (above left) and total utility of individual realisations (above right).*

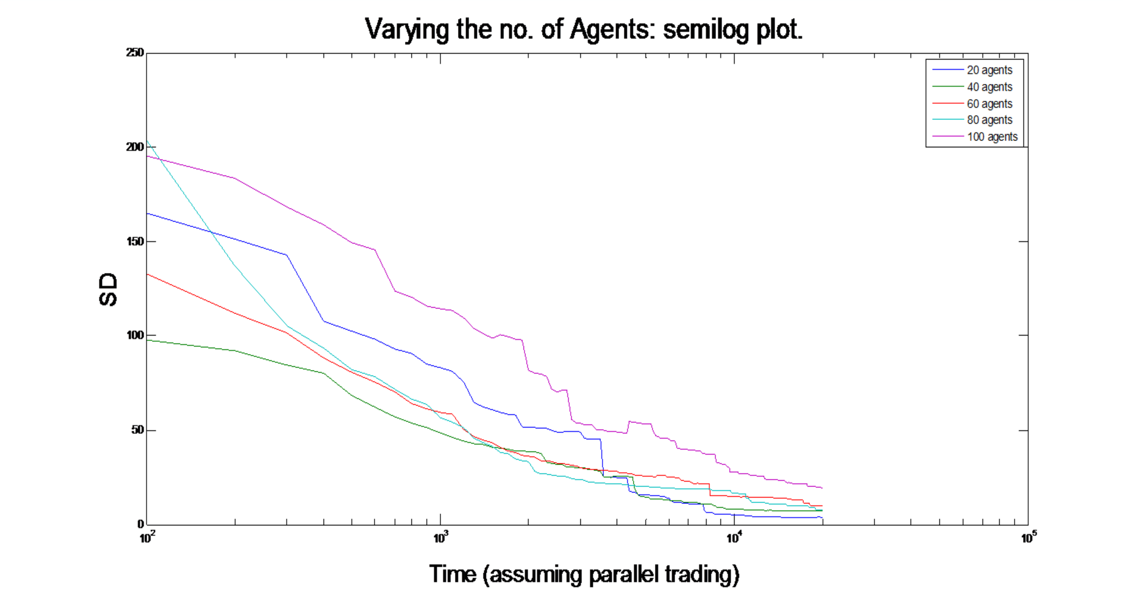

Figure 3 *We see only slow convergence in the standard deviation of marginal rates of substitution (a way of thinking about price).*

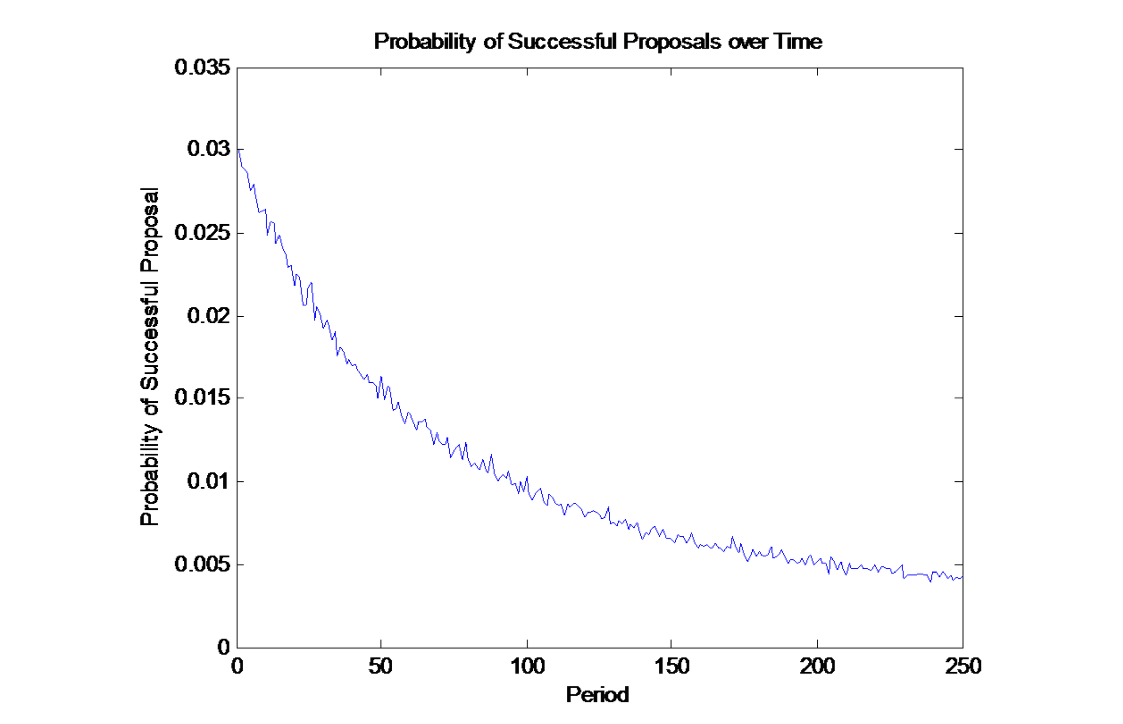

Figure 4 *This shows how the probability of making a successful proposal rapidly declines.*

Experimentation

In order to both make our model more realistic and to escape problems such as the slow convergence we uncovered numerically we augment it with ideas such as experimentation: the acceptance of offers that aren't utility improving but which aren't "bad".

When we numerically simulate this we find that we get quicker convergence in utility, though the total utility values may be slightly lower.

Prediction

We would also like agents to endogenously make predictions and determine their behaviour accordingly. There are many possible schemes the could be used, but they should both respect the limited amount of information available to agents and not require unfeasible computational capacity.

References

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[3] Duncan K. Foley. Statistical equilibrium in economics: Method, interpretation, and an example. 1999.

[4] Dhananjay K. Gode and Shyam Sunder. Allocative efficiency of markets with zero intelligence traders: Market as a partial substitute for individual rationality. The Journal of Political Economy, 1993.

[5] Empirical Content of General Equilibrium Models, CRETA minicourse, http://www2.warwick.ac.uk/fac/soc/economics/research/centres/creta/forthcoming/minicourses/