Coupling and Feedback in the Manna Universality Class

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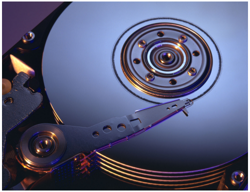
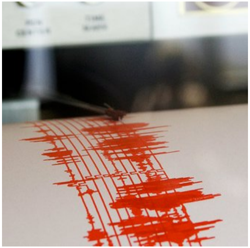
Self-organised Criticality

- Systems exhibiting **self-organised criticality** (SOC) approach criticality without external tuning of parameters.
- **Widespread examples** from earthquake models to interfaces moving through media.
- Understanding has wide ranging **implications across many areas**.



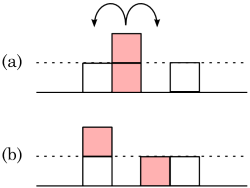


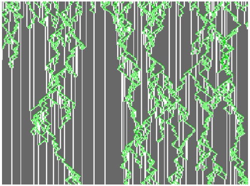
Absorbing Phase Transitions

- We examined the behaviour of the **Manna sandpile model** in the fixed-energy ensemble.
- In the **fixed-energy ensemble** the system no longer approaches criticality by itself.
- An **absorbing phase transition** occurs at the critical value of the control parameter.
- This **establishes a connection** between SOC and phase transitions.

The Manna Model

- Particles sit on a **1d Lattice** with periodic boundary conditions.
- Sites are either **active or inactive**.
  - $z_x > 1 \Rightarrow$ site x is active.
- Updates occur in parallel and in **discrete time**.
  - Particles from active sites are redistributed to neighbouring sites picked with **uniform probability**.
- The Manna model is a **stochastic sandpile model**.





Field Theoretic Description

In the Manna model there are **two fields** to consider

- The activity density $\rho(\mathbf{x}, t)$.
- The particle density $\phi(\mathbf{x}, t)$ (conserved).

Conjectured Langevin equations[1]

$$\frac{\partial \rho}{\partial t} = -r\rho - b\rho^2 + \nabla^2\rho + \sigma\sqrt{\rho}\cdot\eta(\mathbf{x}, t) + \omega\rho\cdot\phi,$$

$$\frac{\partial \phi}{\partial t} = D\nabla^2\rho.$$

Note the similarity to the **Directed Percolation** (DP) Langevin equation highlighted in blue.

[1] Munõz et al (1998) PRL 81(25):5676-5679

DP and Manna Universality Classes

- Directed Percolation includes models such as **the contact process** and Domany-Kinzel Automaton.
- Recently conjectured that the **Linear Interface Model** and Manna classes may be equivalent[1].
- Currently much debate about the nature of the DP and Manna classes and whether they are in fact **distinct**.

[1] Bonachela et al (2007) PRL 98(155702)

DP and Manna Universality Classes

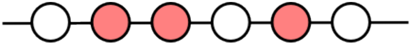
If the DP and Manna classes are distinct, what places a model in one or the other?

- There are **no extra symmetries** in the Manna model over the contact process.
- There is **coupling** of ρ to a conserved background field ϕ.
- Coupling is encapsulated in the **Langevin equations**.

Is the coupling enough to move models to a different universality class?

Contact Process

Well studied model in epidemiology which models **infection spreading** from person to person.



Local infection density is analogous to activity density ρ in the Manna model.

- Infected persons become healed with rate $\omega(\bullet \to \circ) = 1$.
- Healthy persons become infected with rate $\omega(\circ \to \bullet) = (n\lambda)/(2d)$.
- λ is the control parameter, n the number of infected neighbours and d the spacial dimension.

Modified Contact Process

We constructed a **variant of the contact process** where the activity ρ was coupled to a conserved background field ϕ.

- From Manna Langevin equations we construct ϕ:

$$\phi = \int_0^t \nabla^2 \rho(x, \tau)\, d\tau$$

- To emulate the coupling term $\omega\rho \cdot \phi$ we modify the control parameter λ:

$$\lambda \rightarrow \lambda'(\mathbf{x}, t) = \lambda + \omega\phi(\mathbf{x}, t)$$

If this variant is now in the Manna class we would expect to see the **same scaling behaviour** of various quantities as in the Manna model.

Measureables

- Both **natural and constructed** measures are of interest.
- The level of activity

$$\bar{\rho}(t) = \frac{N_A(t)}{L}.$$

- The relative excess of particles

$$S_x(t) = \int_0^x \phi(y,t)\, dy - x\bar{\phi}.$$

- We are interested in the quantities and their variances **at criticality**.

Results

We began by measuring the **scaling exponents** of the Manna model for $S_x(t)$ and $\bar{\rho}$.

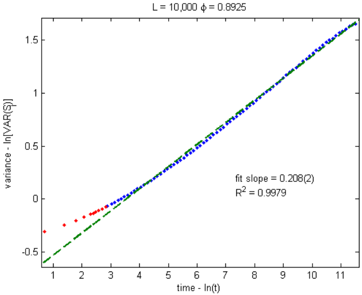
Scaling Relations

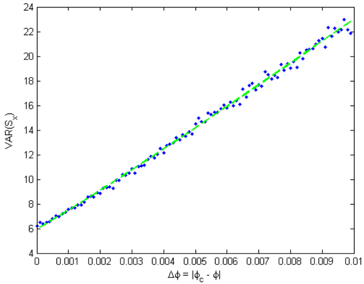
$$\mathrm{VAR}(S_x(t)) \sim t^{\alpha} \qquad \mathrm{VAR}(S_x(t)) \sim \Delta\phi^{\beta}$$
$$\bar{\rho} \sim t^{\delta} \qquad \bar{\rho} \sim \Delta\phi^{\gamma}$$

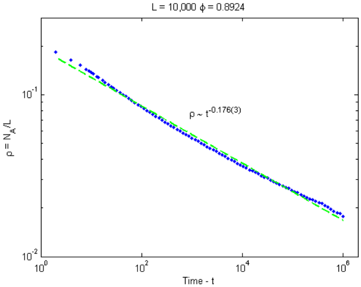
Simulations run using natural initial conditions, lattice size of 16,384, and averaged over 1000 realisations give the following values

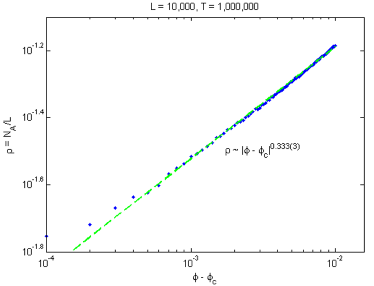
- $\alpha = 0.208(2)$, $\beta = 1.055(5) \simeq 1$
- $\delta = -0.176(3)$, $\gamma = 0.333(7) \simeq 1/3$

Scaling Exponents









Finite Size Scaling

Scaling with system size was measured.

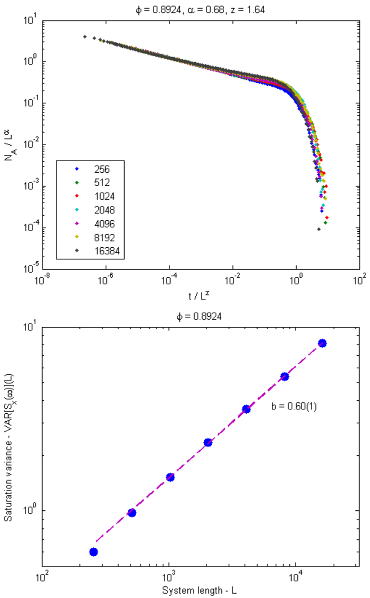
$$\rho = L^{\nu} \tilde{\rho}(L^{-z} t)$$

Data collapse is obtained for the correct critical exponents

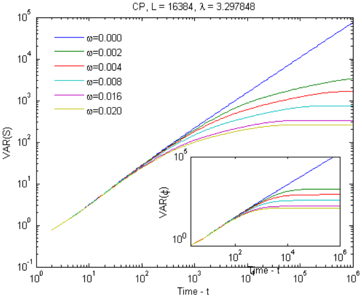
- $\nu = -0.32$
- $z = 1.64.$

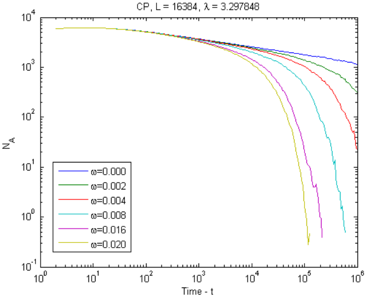
The **saturation variance** of S_x scales with system size as

- $\mathrm{VAR}(S_x(\infty)) \sim L^{0.60(1)}$



Quantities were measured for the modified contact process with **varying coupling strength** ω.





Coupling **shifts the critical point** so the system is now sub-critical.

Recovering Criticality

Coupling shifts the critical point therefore we

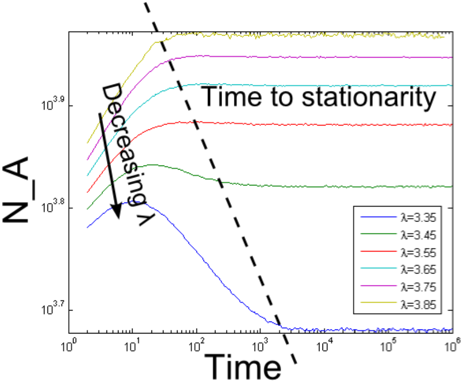
- Fix coupling strength $\omega = 0.01$
- Vary λ to find new critical point

Best marker is the **saturation activity level** $\bar{\rho}(\infty)$.

Despite simple scheme this requires **massive compuational effort**.

- Near the new critical point **time to reach stationarity increases**.
- This makes an accurate location of $\lambda_c(\omega = 0.01)$ tough.

Recovering Criticality

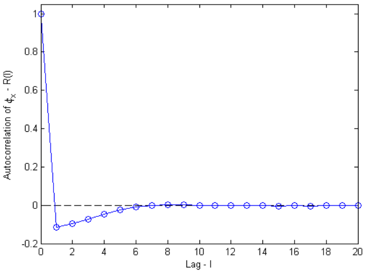


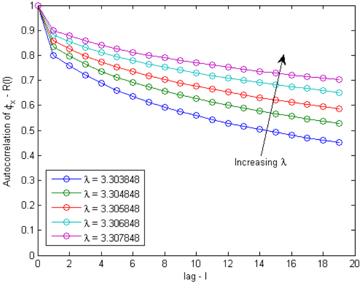
Numerics show the critical point is in the interval $\lambda_c \in (3.07848, 3.12848)$.

This is not good enough to identify the Universality class by scaling exponents.

Correlations

Spacial correlations might shed some insight.





Autocorrelation for ϕ in the Manna model can be understood from **microscopic rules** of the model.

Autocorrelation of ϕ looks **vastly different** for the two models.

Summary

- Measured Scaling Exponents and Finite Size Scaling for the **Manna model**.
- Constructed a variant of the contact process with activity coupled to a **conserved background field**.
- Found **interesting behaviour** but could not identify a new universality class.
- Further investigation needed into **varying coupling strength** and more precise location of critical point.

Thanks

- Prof. Haye Hinrichsen
- Dr. Stefan Großkinsky
- DTC Staff

... and finally, thank you for listening.