Waving goodbye to traffic jams

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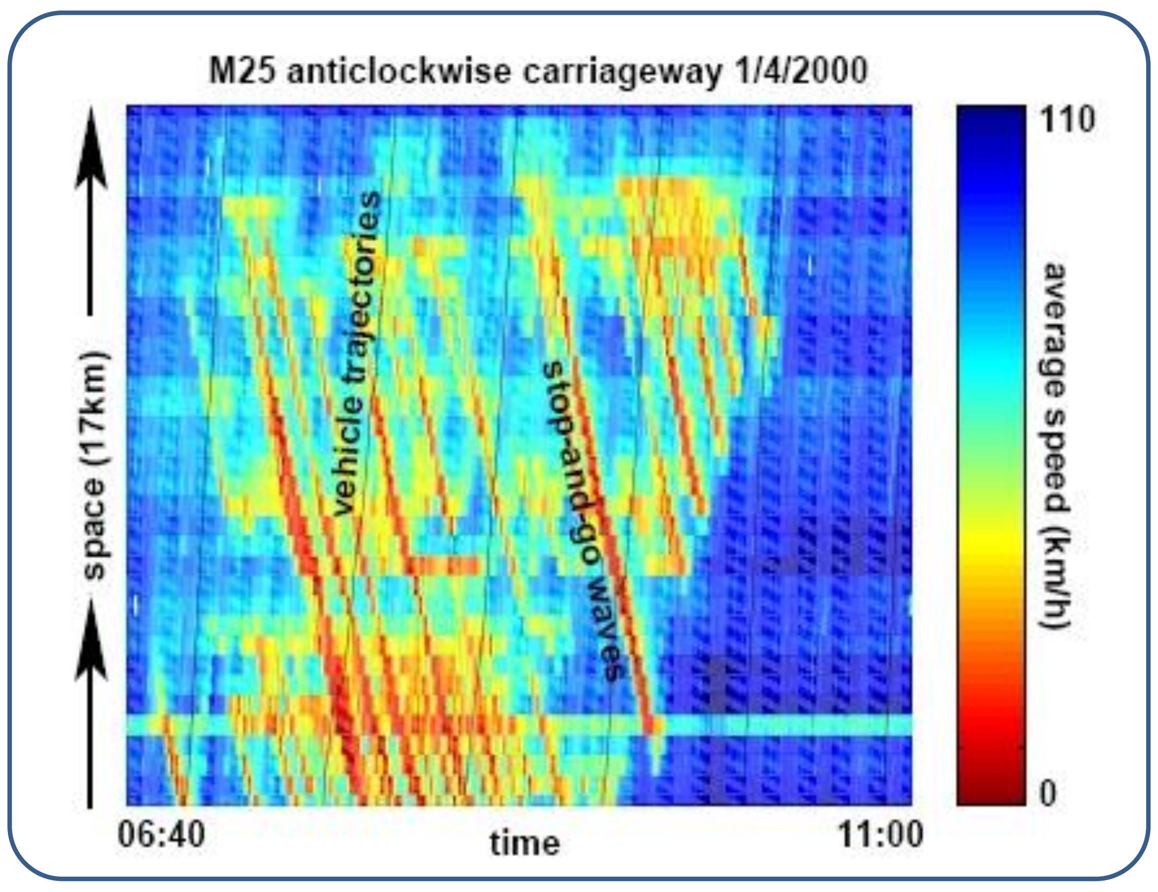
Stop and go waves in traffic

Have you ever been driving along and had the traffic stop ahead of you? If the traffic begins to move freely again after a wait, this is a traffic wave. Despite over 50 years of modelling, the precise mechanisms for the generation and propagation of stop and go waves and associated spatiotemporal patterns are in dispute.
There is now the opportunity to use the empirical datasets collected by new measurement technologies to validate traffic models. The M25 data was collected via inductance loops buried in the road surface, spaced at approximately 500 metre intervals. These are used to record the time and lane number of passing vehicles, and to estimate their speed and lengths.



(Wilson 2008:2)

Stop and go waves have a structure which propagates upstream against the flow of traffic, and has two sharp interfaces (one at which vehicles brake and one at which they accelerate) bounding a plateau of slow moving traffic. The data shows that they are triggered by lane-changing at congested merges

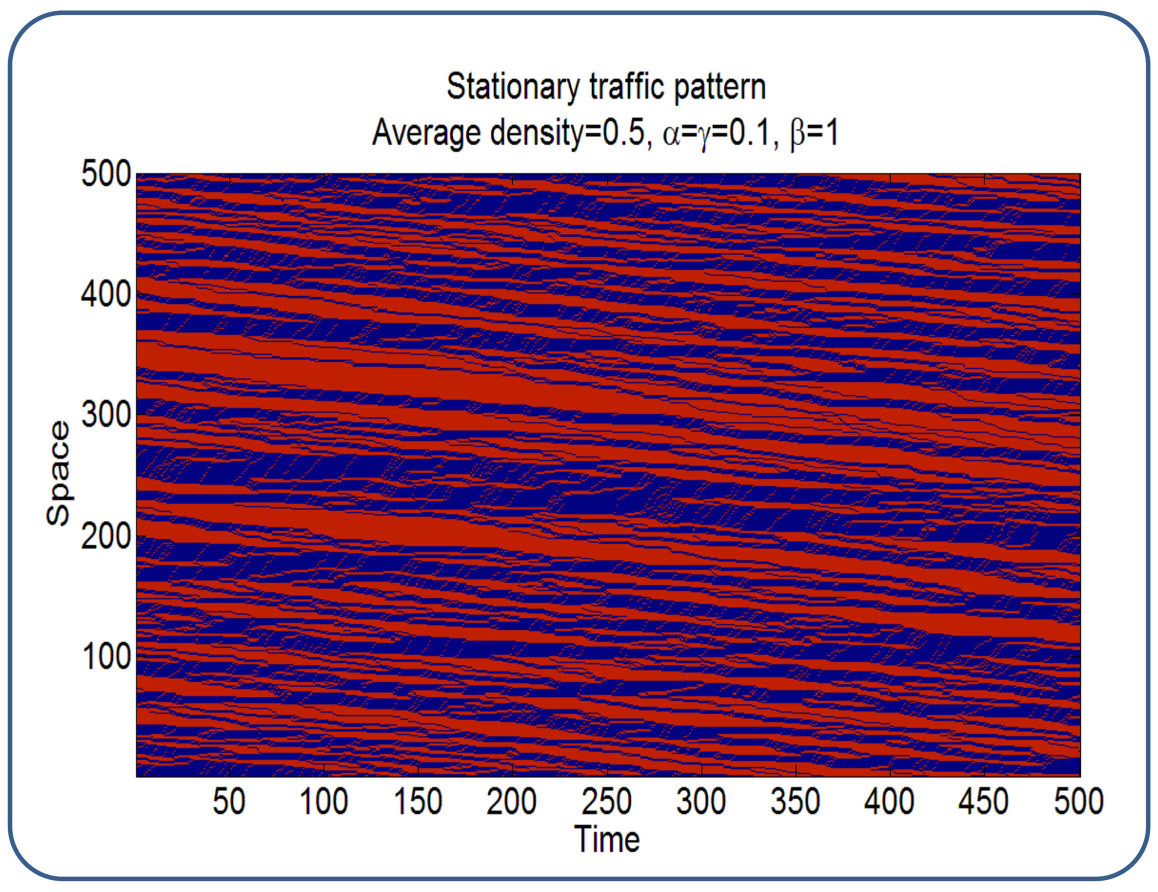
Simulation and solution

We simulate traffic flow by cellular automata on a microscopic level, following the motion of individuals. Continuous time dynamics are implemented via random sequential updating (see below).
Such Markov chain models are capable of producing all the basic space-time patterns of real traffic, with unique stationary distributions guaranteed by the ergodic theorem.
This is compared to the corresponding macroscopic model for vehicle density, $\rho(x,t)$, called the Lighthill, Whitham & Richards (LWR) model (Lighthill & Witham 1955).

$$\frac{\partial}{\partial t}\rho(x,t)+\frac{\partial}{\partial x}j(\rho(x,t))=0$$

This model captures the sharp upstream (decelerating) interfaces, but the downstream interfaces diffuse.

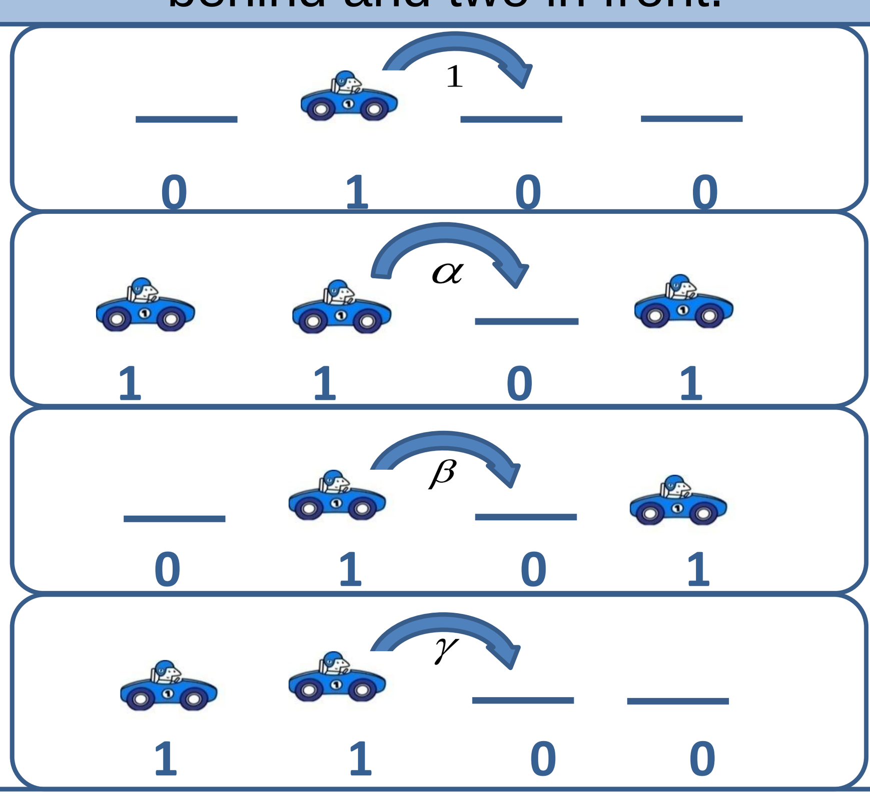
This simulation is run with 250 cars on a system size of 500 with periodic boundary conditions, to mimic a ring road.
After equilibration, it is run for 500 time steps and shows striped patterns of high traffic density (jams) and free road similar to the real data.



Can we understand the connection between descriptions on different scales?
Is the fundamental diagram key to understanding the emergent patterns?
What are the relevant admissibility criteria for the LWR model solutions?

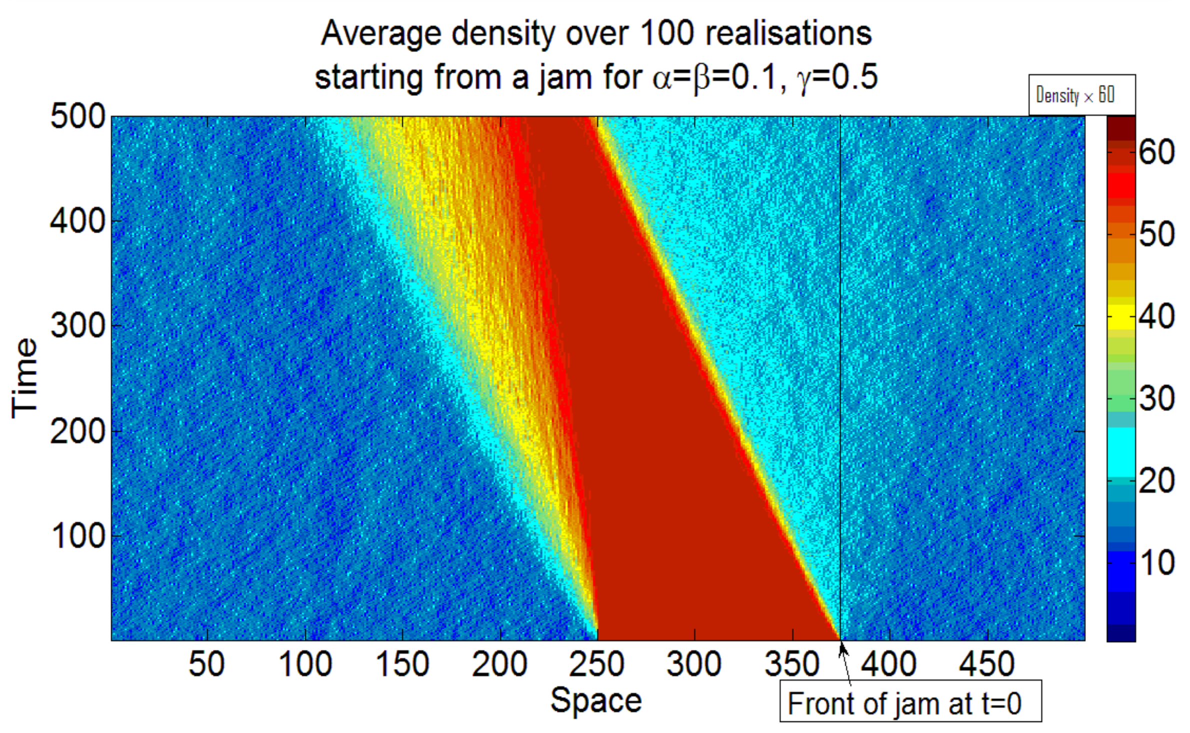
Markov chain traffic model

Each car moves at a rate which depends on the presence or absence of a car one space behind and two in front.



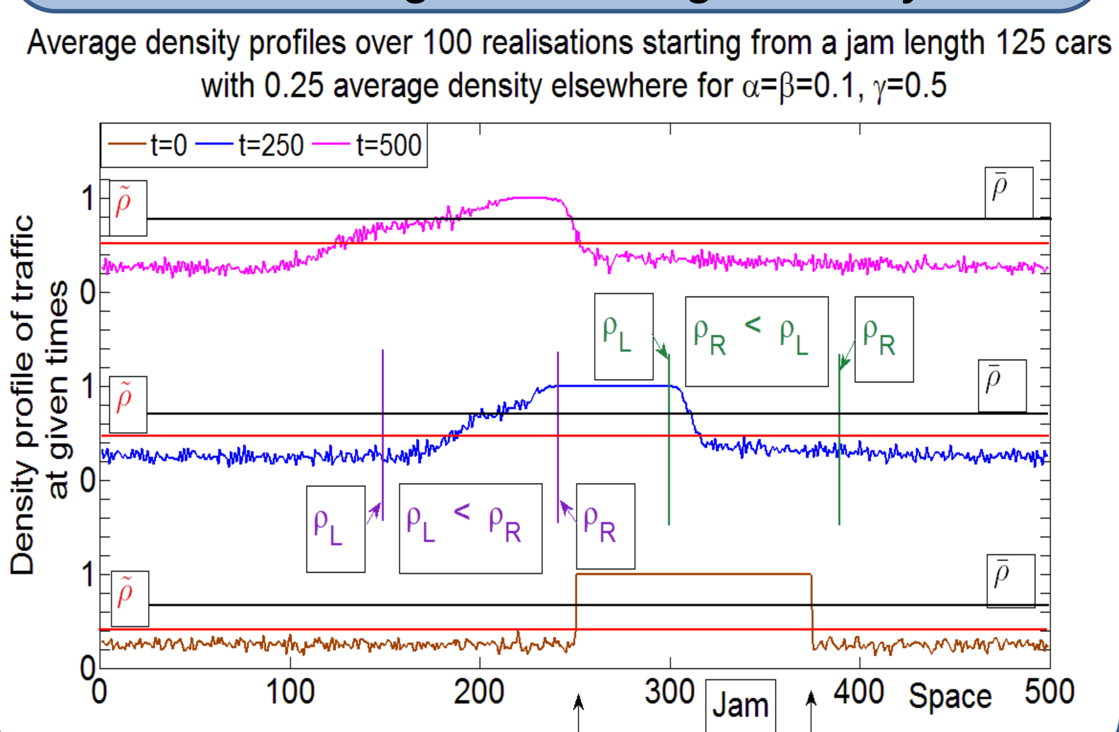
Time evolution of a jam

Simulation of a jam illustrates the behaviour of upstream and downstream interfaces predicted by the LWR model.



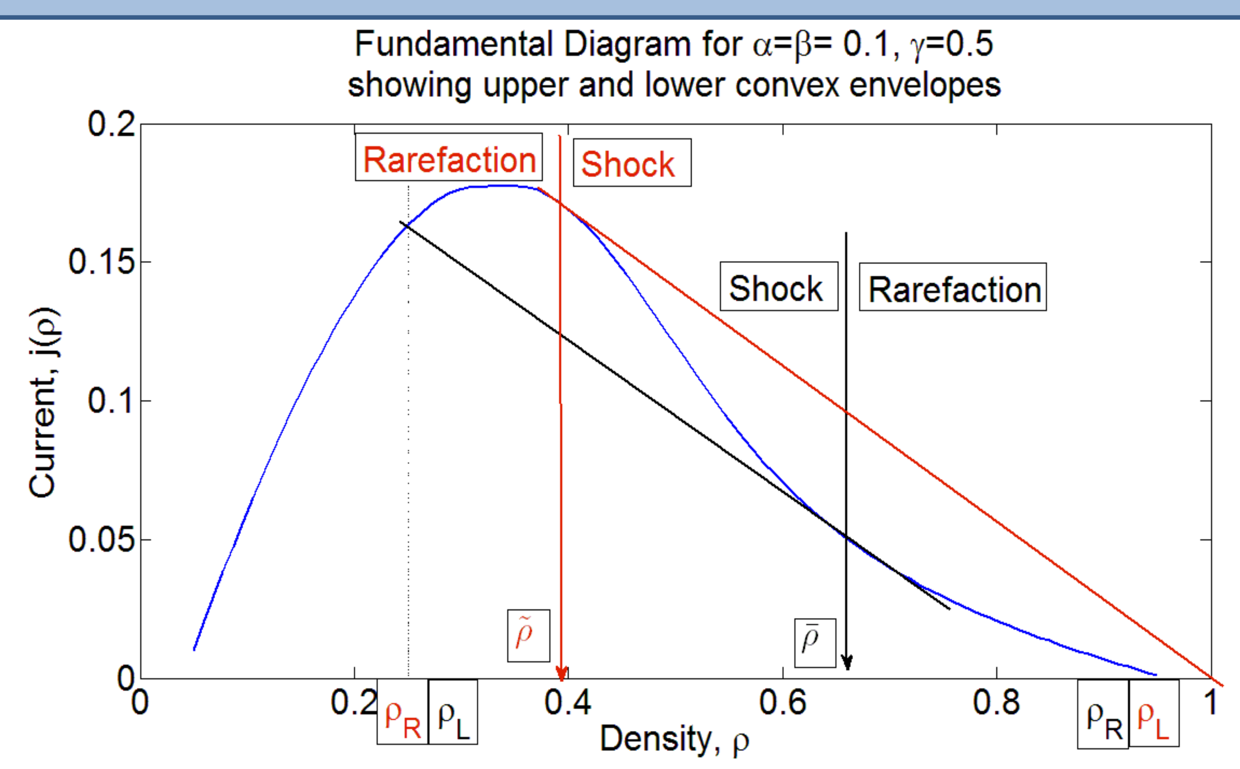
Density profiles

Snapshots are taken of the density profile at certain times, and show the interfaces and the moving area of high density.



Fundamental Diagram

$\rho_L < \rho_R$ is a rarefaction (diffusion) followed by a shock (sharp interface) $\rho_R < \rho_L$ is a shock followed by a rarefaction.



Existing proposals

There are several established admissibility criteria to single out physically relevant solutions to the LWR model, given by characteristics of the equation.

Theory of characteristics analysis depends qualitatively on the shape of the fundamental diagram. (Fowkes & Mahoney 1996).
Entropy conditions rule out physically inappropriate solutions to models like LWR, which generally have non-unique, weak (piecewise continuous) solutions. These work well for traffic flow when the fundamental diagram is strictly concave (Knowles 2008).
Since those of models capable of predicting stop & go waves are neither concave nor convex Gasser (2003) proposes, for Riemann problems, a single shock for $\rho_R < \rho_L$ and entropy solution for $\rho_L < \rho_R$.

Fowkes, N.D and Mahony, J.J. (1996) *An introduction to mathematical modelling.* Sussex: John Wiley & Sons. **Gasser**, I. (2003) 'Short Communication on non-entropy solutions of scalar conservation laws for traffic flow' Math. Mech. 83, No.2, 137-143 **Knowles**, J.K. (2008)'On entropy conditions and traffic flow models' Math. Mech. 88, No.1, 64-73. **Lighthill, M.J and Witham, G.B** (1955) 'A theory of traffic flow on long, crowded roads' Proc. R, Soc. A 229, 317-345 **Wilson**, R.E. (2008)'Mechanisms for spatiotemporal pattern formation in highway traffic models'