Hot Topics: an Axelrod model with linked features

Anthony Woolcock[*]
Center for Complexity Science, Warwick University, Coventry, CV4 7AL, UK.

Colm Connaughton[†]
Mathematics Institute, Warwick University, Coventry, CV4 7AL, UK.

Yasmin Merali[‡]
Warwick Business School, Warwick University, Coventry, CV4 7AL, UK.

Abstract

The original Axelrod model is a simple model of social interactions between individuals who are each described by a vector of features. Individuals who are more similar are more likely to interact. Upon interaction they become more alike. We study a varient of this model which includes linked features which are more likely to be the ones changed when individuals become more similar.

Our model and the original were compared using results (mean number of cultural regions) from simulations of 100 individuals on square lattice networks. Simulations, with individuals having 3 features and 2 traits per feature, indicate that both models' results are the same, within statistical error. This suggests resilience of the original Axelrod model, with only few features and traits, to the introduction of linked features.

Novel analysis on the dynamics of specific features highlights differences between the results of models with 3 features and with 10 features. Results for models with 10 features and 20 traits suggest different dynamics for the two Axelrod models. Additionally, non-trivial dynamics are seen for both Axelrod models in the cases with 10 features. We conclude that the inclusion of linked traits in the Axelrod model does not significantly affect the dynamics of simulations of the model.

[*]A.J.Woolcock@warwick.ac.uk

[†]C.P.Connaughton@warwick.ac.uk

[‡]Yasmin.Merali@wbs.ac.uk

I. INTRODUCTION

The effect of social interactions on the opinions of individuals over time is not fully understood. Models in cultural dynamics consider the interaction of individuals in a similar way to statistical physicists' consideration of spin-spin interactions. Furthermore "*the statistical physics approach to social dynamics is currently attracting a great deal of interest*" (page 3, [5]). Furthermore, the question of how individuals become radicalised is important in Social Science and to society as a whole. Our initial discussions of the process of radicalisation included the concept of changing the opinions of individuals. Therefore opinion dynamics was the focus of our work.

There has been a wide range of uses for models of opinion dynamics. A well researched area is that of voter dynamics because a key element of model verification is comparison with empirical data and information on voters' choice has been available for many years. Literature on this area ranges from the academic, e.g. Ben-Naim [3], to broader introductions, e.g. Himmelweit [8]. The field of cultural dynamics is one which overlaps with that of opinion dynamics. A key difference is that opinion dynamics models use a scalar for the description of the individuals' choice and cultural dynamics models use a vector of options. We model more than one choice ("feature") for each individual and therefore describe the model and results in the context of cultural dynamics. However the definition between the two fields is essentially arbitrary so terminology is used interchangeably and these descriptions are valid in describing our model in both areas. Simulations are required to elucidate the long-time dynamics of these models. Introductions to a range of models, including agent based modelling, are given in a variety of texts including Halpin [7] and Macey [11].

Our discussions focussed on the idea that upon interacting an individual may find a concept discussed or read/watched/heard resonating with another related opinion, already held by the individual. This related opinion is then more likely to be influenced. We therefore selected the idea of related opinions or ideologies as the specific aspect to be investigated. The term 'related opinion' is interchangeable with 'related feature'.

A salient model in the area of cultural dynamics is that presented by Axelrod in [1]. The original Axelrod model has two key elements. Firstly, when there is interaction between individuals then they become more similar, termed "social influence" by Festinger [6]. Secondly, individuals which are currently similar are more likely to interact, referred to as "homophily". Simulations of this model run towards a final stable where either all individuals' states are identical ("consensus") or there is more than one region present ("hetrogeneity" or "fragmentation"). Surprisingly, despite the presence of social influence the simulation moves towards a hetrogeneous final state. This is because individuals who are completly dis-similar cannot interact (as a consequnce of the implementation of social influence) and so regions which are completely different survive until the final stable state is reached. Some parameters of the Axelrod model give consensus in the final stable state, and others result in hetrogenteity. The phase transition between these two regimes has been studied by Castellano [4], Vazquez [13] and Vilone [14].

Our objective was to understand the inclusion of linked opinions (features) in this established model. It has been well studied from a Statistical Physics perspective, e.g. by Castellano [4], however, here we seek to ask what influence linked features have in the context of Social Science because the motivation for this adaptation is from this background.

In these models individuals are defined by cultural features. (Axelrod's model definition uses the word "feature" so for consistency we retain this usage here). Each feature is a number, the "trait", and can take a range of values. When individuals interact their features are compared. For example, if each of the features of two individuals are the same then they are culturally identical. Two features defined to be linked change the probabilities of the outcomes of the interaction. An interpretation of the rule we have introduced is that of related "hot topics". These pairs of topics are discussed first by individuals and are therefore more likely to be the opinions which are changed.Our model is similar to Axelrod's [1] and is therefore named "Axelrod 2".

II. MODEL DESCRIPTION

There are several components to the model:

A. Definition of the state of a single individual

In both models the cultural state of an individual, $a(i)$, is given by $a(i) = (\sigma_1(i), \sigma_2(i), .., \sigma_F(i))$. There are F features for each state. Each feature has q traits. We define the models as 'F by q'. So for example a '2 by 3' model has two features and each feature has three traits. A light example of a 3 by 2 model could be features of: (1) political preference; (2) religious preference; and (3) musical taste. Each feature has two choices (traits) for example: (1) Labour or Conservative; (2) Atheist of Buddhist; (3) jazz or classical. However our model only records numbers for the traits, i.e. the choices are: (1) 1 or 2; (2) 1 or 2; (3) 1 or 2. All the individuals' states (and the network) give the state of the system.

B. Definition of the set of connections between individuals (social network)

The size of the model is 100 individuals arranged on a ten by ten square lattice (with periodic boundary conditions). Each individual has four nearest neighbours exactly. Other networks could be chosen for this and dimensionality of the model has been studied by Klemm [10]. A social network could be used and there are models of how these networks themselves change in time [9]. Results on the effect of different networks on the time to consensus have been previously found [12]. Here we chose a simple lattice to be able to visualise the individuals' states in the structure of the population. There are many time steps in the simulation (of the order $100,000$ steps). Individuals have the chance to interact each time step. However it is only one pair at a time that is considered. One individual, i, is selected at random and then a nearest neighbour, j, is chosen also at random. The social interpretation of an interaction is a conversation between two individuals.

C. Rule for the probability of interaction between two neighbouring individuals

Two individuals have been identified, i and j. Their current states are known, a_i and a_j. There is a probability that these two individuals will interact. This is given by the "overlap". Overlap is the number of identical features. The probability of interaction is overlap divided by the total number of features, F, and is given by ω, where

$$\omega = \tfrac{1}{F}\textstyle\sum_{f=1}^{F} \delta_{\sigma_f(i),\sigma_f(j)}.$$

Where $\delta_{\sigma_f(i),\sigma_f(j)} = 1$ when $\sigma_f(i) = \sigma_f(j)$ and $\delta_{\sigma_f(i),\sigma_f(j)} = 0$ when $\sigma_f(i) \neq \sigma_f(j)$. This captures the idea of homophily because agents which are more similar have a greater overlap so are more likely to interact. The concept of "bounded confidence" is that two individuals must not be too different to interact. In our models the requirement is that at least one of the features must be the same in the two individuals for the possibility of interaction.

D. Definition of the outcome of interactions

This is the only component of the model which is different in Axelrod 2. All other components are identical in both the original Axelrod model and our version.

The final outcome is that the individual i copies the mth feature of individual j, i.e. $\sigma_m(i) = \sigma_m(j)$. The cultural state of i has therefore become more similar to that of j. Here we have the implementation of social influence.

However there may be more than one feature which is different between i and j. For example where $a(i) = (1, 1, 1)$ and $a(j) = (1, 2, 2)$ then there are two features, the second and third, which i could copy the respective feature in j.

We define the probability, $P(m)$, that the mth feature will be copied. In the Axelrod model $P(m)$ is simply one divided by the number of features different between i and j. However in Axelrod 2 this is different. Here linked features are more likely to be changed by the interaction. This is a reflection of the concept that features (opinions or ideologies) may be related and therefore more likely to resonate with the individual during an interaction (e.g. a conversation). The conditions for a linked opinion to affect the outcome of the interaction are as follows:

- more than one feature is different between the individuals,

- there is (at least one) pair of linked features (e.g. features n and p are linked),
- one of the two linked features is the same ($\sigma_n(i) = \sigma_n(j)$),
- the other of the two linked features is different ($\sigma_p(i) \neq \sigma_p(j)$).

If all are met then the different linked feature is more likely to be the one which is changed. Furthermore the parameter α controls how much more likely this linked feature is changed. Where $\alpha = 0$ there is no increased chance of selecting this linked feature. This parameter setting is identical to the original Axelrod model. Where $\alpha = 1$ the probability of selecting the linked feature (if all of the above conditions are in place) is 100%. Between zero and one, the parameter α tunes between these two conditions.

1. *Example: no linked features in 3 by 2 model*

If the two individuals states' are $a(i) = (1, 1, 1)$ and $a(j) = (1, 2, 2)$ and there are no linked traits then the second or third features both have 50% chance of being selected to change.

2. *Example: a pair of linked features in 3 by 5 model*

Here the states are $a(i) = (2, 4, 5)$ and $a(j) = (3, 4, 1)$. We specify in the model that the first two features are linked (i.e. σ_1 and σ_2) and that $\alpha = 1$. Therefore the conditions are all met:

- σ_1 and σ_3 are different, there are more than one different features,
- σ_1 and σ_2 are linked,
- of σ_1 and σ_2 one is the same ($\sigma_2(i) = \sigma_2(j)$), and
- of σ_1 and σ_2 the other is different ($\sigma_1(i) \neq \sigma_1(j)$).

Therefore the probability that the first feature (σ_1) is changed is 100% (because the conditions are met and $\alpha = 1$). Then the individual i changes their state to reflect this and $a(i) = (3, 4, 5)$.

If $\alpha = 0$ then the probability of selecting the first or the third feature would be 50% each, because there are two features to chose from (the first and the third). Then the outcome would either be $a(i) = (3, 4, 5)$ or $a(i) = (2, 4, 1)$. If there are more than one pair of linked features (for example in a 10 by 15 model) and more than one satisfy the above condition then they are chosen with equal probability. See Appendix A for mathematical definitions of the probability of choice of feature.

E. Expectations based on the model

The model alters the probability of which features are changed in the interactions. Therefore we expected the dynamics of the simulations to be altered by the inclusion of this rule. Furthermore the types of individuals surviving to the stable final state were expected to be different under the Axelrod 2 rules.

III. RESULTS OF NUMERICAL SIMULATIONS

A. Descriptions of states of the model: Cultural Zones

A cultural zone, as defined in [1], is a group of connected (in the nearest neighbour sense) individuals where, additionally, each pair of nearest neighbour individuals must have least one feature with the same trait. E.g. In the 3 by 2 model, $a(i) = (1, 2, 2)$ and $a(j) = (1, 1, 1)$ are in the same cultural zone if they are nearest neighbours. This is because the first feature is trait of 1 in both individual i and individual j.

1. Descriptions of states of the model: Cultural Regions

A cultural region is also a group of connected individuals. However membership requires them to have identical cultural states, i.e. all features having the same trait. In the 3 by 2 model, for example, $a(i) = (1, 2, 2)$ and $a(j) = (1, 2, 2)$ are in the same cultural region (if they are nearest neighbours).

We plot the number of cultural regions in our results because there are richer dynamics in this data than the number of cultural zones, as demonstrated in FIG.1. The starting state of each individual is randomly selected so there are initially many regions because there are very few connected individuals which are culturally identical. However many individuals have at least one feature in common with a neighbour so there are fewer cultural zones. In a simulation over a long number of time steps, both the number of regions and zones decrease to the same value, in FIG.1 this is one (i.e. consensus). (Time to consensus has been studied, e.g. by Sood [12], however here we focus on the features of the curve because the model network is consistent across all models considered). The curve representing the dynamics of the number of cultural regions is of more interest than cultural zones so this is the choice of measurement for our models' dynamics. We note here that there are large fluctuations about the mean in the dynamics of a single simulation. Therefore the curve of the mean cannot be taken to be the typical behaviour of a specific simulation.

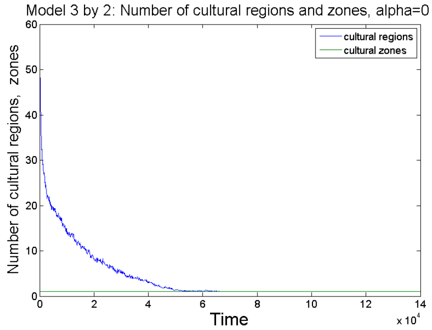


FIG. 1: The mean number of cultural regions and cultural zones for 50 runs. The 3 features and 2 traits per feature model with $\alpha = 0$ (Axelrod model) was simulated.

B. 3 by 2 and 10 by 2 models

The first model studied was a simple 3 by 2 model to enable the analysis of the final state (from the total possible number of 8). A 10 by 2 model was also studied to compare the individual feature dynamics for two models, each with only one pair of linked traits. A comparisson was made between a 3 features model (with one pair linked features) and 10 features model (with one pair of linked traits).

It has been shown by Axelrod [1] and Axtell [2] that the Axelrod model with a low number of traits, e.g. 5 for each feature, will eventually develop to consensus. Therefore the expectation was that our model with $\alpha = 0$ would also return a single stable group of culturally identical individuals in the long time simulation for the 3 by 2 model. We observe this in FIG.2. Furthermore the dynamics of the model move smoothly to this equilibrium state.

We ran the simulation for a range of α as seen in FIG.2. We see from FIG.3(a) and FIG.3(b) that there is no clear difference in the distribution of the types of final state for Axelrod model and Axelrod 2. This suggests that the introduction of the linked features to the Axelrod model does not affect the dynamics of the whole state of all the individuals together as they become more similar and eventually reach consensus.

The results of the final state were analysed for the 3 by 2 model because this was feasible. The number of different permutations of final states is $q^F = 2^3 = 8$. With the other models considered, e.g. 10 by 15, and 10 by 20 this was not feasible. The results in FIG.3(a) and FIG.3(b) suggest that there is no first order effect of linking features (i.e. $\alpha = 1$) on the type of final state.

The results in FIGS.4(a), 4(b), 5(a) and 5(b) show the dynamics of the models in more detail than purely the changing number of regions. In these results we can see which opinions are changing as the simulation runs. For the, 3 by 2, Axelrod model we see in FIG.4(a) that each of the three features are roughly equally likely to be selected to change throughout the

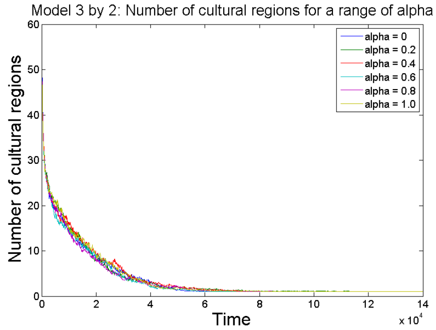


FIG. 2: Results for the model with 3 features and 2 traits for each feature model, and a range of α values: time evolution of the mean (of 50 runs) number of cultural regions. Models with each different α value follow the same dynamics and both have the same number of stable final states, i.e. one. The system moves to consensus (homogenous final state) for each model plotted here.

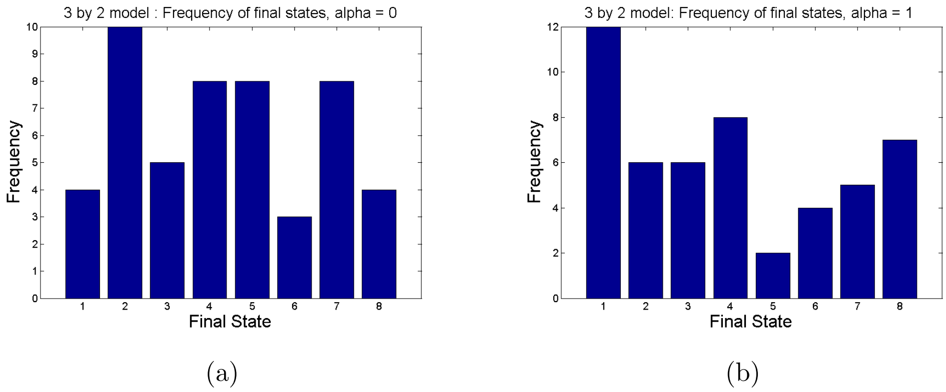
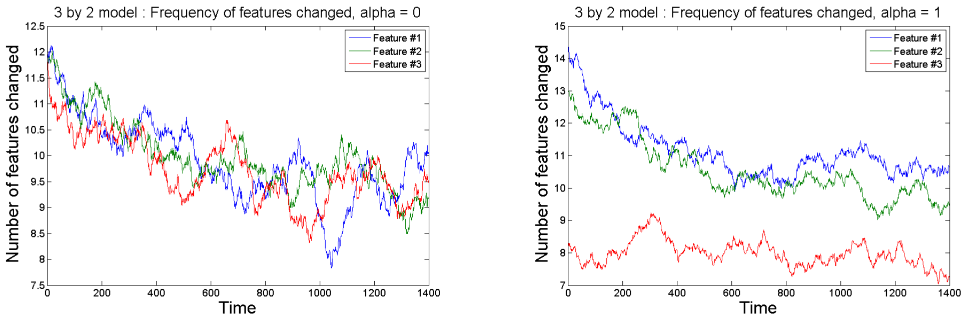


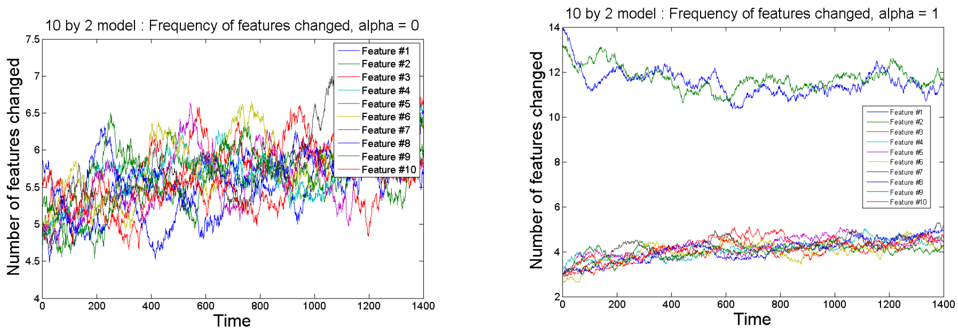
FIG. 3: (a) Axelrod model ($\alpha = 0$) and (b) Axelrod 2 ($\alpha = 0$). Both tally the frequency (from 50 simulations) of each possible final state from the 3 features (each with 2 possible trait values). Where the following stable final states are shown: 1 is (111); 2 is (222); 3 is (112); 4 is (221); 5 is (121); 6 is (212); 7 (122) and 8 is (211).

simulation, as expected from the model definition. Whereas in the Axelrod 2 model results, FIG.4(b) we see that the two linked opinions (feature 1 and feature 2) are more likely to be selected throughout the whole simulation. Again this is expected from the definition of the Axelrod model with $\alpha = 1$. A similar result is seen in the difference between FIG.5(a) (10 by 2 Axelrod model) and FIG.5(b) (10 by 2 Axelrod 2 model). The relative increase, from $\alpha = 0$ to $\alpha = 1$ in the probability of the linked features changing is greater in the 10 by 2 model because there were originally less more features (i.e. ten as opposed to three) that could be changed.



(a) (b)

FIG. 4: The number of times each feature is changed in the simulation (in 100 time step groupings). Both plots are from 50 simulations of a 3 features with 2 traits per feature model. (a) Axelrod model ($\alpha = 0$). (b) Axelrod 2 model ($\alpha = 1$) with feature number 1 and feature number 2 being linked.



(a) (b)

FIG. 5: The number of times each feature is changed in the simulation (in 100 time step groupings). Both plots are from 50 simulations of a 10 features with 2 traits per feature model. (a) Axelrod model ($\alpha = 0$). (b) Axelrod 2 model ($\alpha = 1$) with feature number 1 and feature number 2 being linked.

C. 10 by 15 model

We studied a model with 10 features to enable the inclusion of multiple pairs of linked features. Results by Axelrod, [1], indicate that the 10 by 15 model moves towards a low mean number of stable regions (consensus or 2 regions). However the 10 by 10 model moves to consensus and the 5 by 15 model to many mean number of stable regions. Therefore we considered 10 by 15 a judicious choice of model to study. If the number of regions increased or decreased after implementing linked traits then we would have similar models and their respective results to compare our model to.

The 10 features by 15 traits model was constructed from 5 pairs of linked features. We see in the plot of cultural regions FIG.6 nontrivial time evolution of the model. There is a

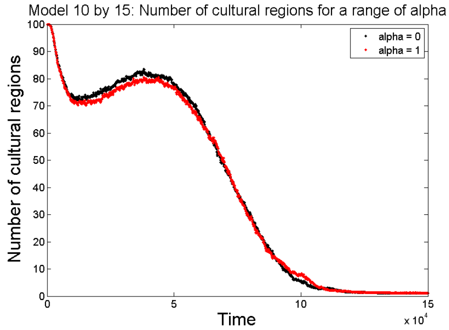
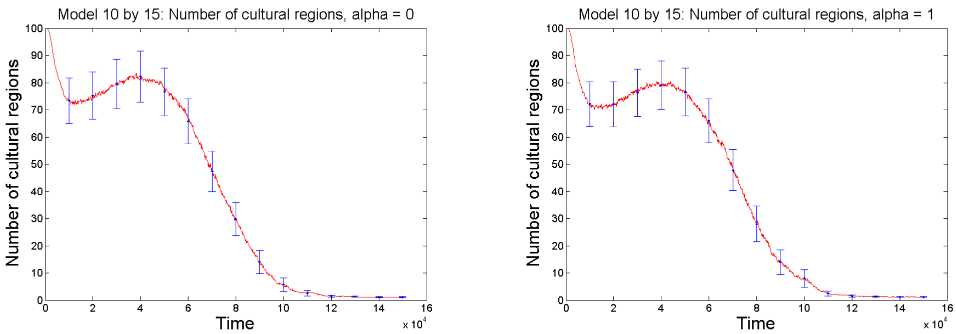


FIG. 6: The mean number of cultural regions (for 100 runs) for the models with 10 features and 15 traits per feature. The Axlerod model (with $\alpha = 0$) and Axelrod 2 ($\alpha = 1$) are plotted.

local minimum in the early time dynamics. A consequence is that there are two times in the simulation that the model has the same number of regions. At the local minimum and then again at just after the local maximum, in FIG.6, the simulated model has the same number of regions. One hypothesis to explain this is different shape types for the regions at the early and later time. Thinner shapes, at the early time, may be broken into more to cause the local maximum in number of regions.

Rank sum hypothesis testing (at 5%) on the curves in FIG.6 indicates that with 100 simulations we are not able to identify separate curves for Axelrod and Axelrod 2 models. Therefore this suggests that the 10 features by 15 traits Axelrod model number of regions dynamics are resilient to the change of introducing linked features. Whether the apparent difference in curves is statistically significant was also tested in FIG.7(a) and FIG.7(b). The error bars here are greater than the difference between the curves of the mean number of regions. Therefore we again cannot claim that this 10 by 15 model is different between Axelrod and Axelrod 2.



(a) (b)

FIG. 7: The mean number of cultural regions (for 100 simulations) for the 10 features with 15 traits per feature model. (a) Axelrod model ($\alpha = 0$). (b) Axelrod 2 model ($\alpha = 1$). Error bars are 3 sigma confidence intervals.

Model parameter	Number final stable regions
$\alpha = 0$	1.180 $\pm$ 0.173
$\alpha = 1$	1.180 $\pm$ 0.168

TABLE I: Number of final stable state regions for 10 by 15 model

With this model not all the simulations run towards a stable final state of consensus, as seen in TABLE I. We have fragmentation where there are two or more stable regions of cultural type surviving. This was measured for the 100 simulations and the mean calculated. The error on the number of final states was estimated using the 3 sigma confidence, error calculated through standard deviation divided by root of the number of simulations. The Axelrod ($\alpha = 0$) and Axelrod 2 ($\alpha = 1$) models give the same mean number of final stable regions for the 100 simulations. This again suggests robust dynamics for this model type.

D. 10 by 20 model

We studied the 10 by 20 model as an extention of the work to investigate the effect of linked features. Upon finding that 10 by 15 results for Axelrod and Axelrod 2 were comparable we obtained results to investigate a model with more traits. We expected the number of stable final regions to be greater in the 10 by 20 model the 10 by 15 model. This was see in the comparisson of TABLE I and TABLE II.

The 10 features by 20 traits model was constructed from 5 pairs of linked features. As in the 10 by 15 model we do not always have consensus in the final stable state, but find hetrogeneity. There appear to be more final states in the Axelrod 2 model, as shown in the TABLE II of the mean (over 50 simulations) number of final stable regions. However, the error measure is large compared to the difference, so further simulations would be required to comfirm this. Error margins are given by the three sigma confidence intervals. The standard deviation of the simulations run was used to calculate these. More regions in the final stable state with linked features is suggestive of the state of the system being more easily locked into a stable configuration with more regions when the Axelrod 2 model is implemented.

Model parameter	Number final stable regions
$\alpha = 0$	1.590 ± 0.478
$\alpha = 1$	1.680 ± 0.353

TABLE II: Number of final stable state regions for 10 by 20 model

Rank sum hypothesis testing for the mean number of regions for this model, as seen in FIG. 8, suggest there may be statistically different dynamics between Axelrod and Axelrod

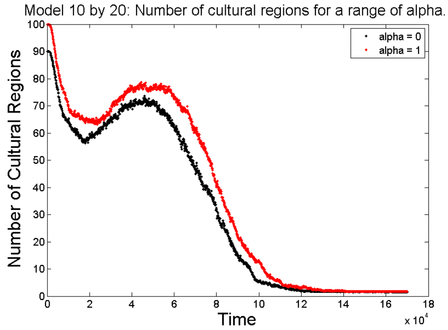
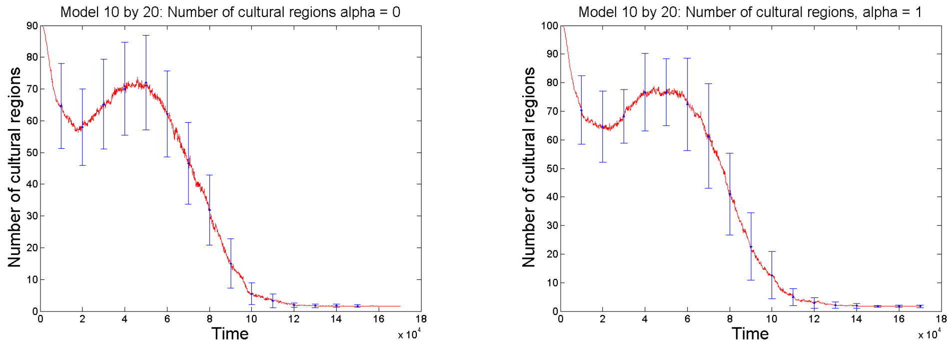


FIG. 8: Mean number of cultural regions (for 50 simulations) for the model with 10 features and 20 traits per feature. Both Axelrod model ($\alpha = 0$) and Axelrod 2 ($\alpha = 1$) results are shown.



(a) (b)

FIG. 9: The mean number of cultural regions (for 100 simulations) for the 10 features with 20 traits per feature model. (a) Axelrod model ($\alpha = 0$). (b) Axelrod 2 model ($\alpha = 1$). Error bars are 3 sigma confidence intervals.

2. Further simulations are required to establish this as there are large errors associated with the plotted mean from 50 simulations, as seen in FIGs. 9(a) and 9(b).

IV. CONCLUSION AND SUMMARY

A variety of individual types have been modelled for Axelrod and Axelrod 2. Each were modelled for 100 individuals on a square lattice, with periodic boundary conditions. Interactions between individuals were simulated over a large number of time steps (over 100,000).

The results for the mean number of regions for the 3 by 2 and 10 by 15 cases give the same overall dynamics for both Axelrod and Axelrod 2 models. Moreover, the number of final stable regions is the same for both models. This suggests that the original Axelrod model's overall dynamics are resilient to the introduction of linked traits for these cases.

However, it has been shown through the results for individual features in the 3 by 2 and 10 by 2 models that linked features affects which ones change, as expected from the definition of the model. This effect is in place throughout the whole simulation to stable state. Therefore the details of the dynamics are different within Axelrod 2. An interpretation of this effect may be relevant to Social Science.

The 10 by 15 and 10 by 20 simulations for both versions of the Axelrod model show non-trivial dynamics not seen in the 3 by 2 model. There is a local minimum and a local maximum in the plot of mean number of regions for the simulations. This suggests that the number of features and traits is crucial to the dynamics of both Axelrod and Axelrod 2 models. The types of shapes of regions at different times or the dynamics of individual features may be causes of these curve shapes.

There is inconclusive data for comparing Axelrod and Axelrod 2 with the 10 by 20 model. It appears that there are more regions in the final stable state with linked features included. More simulations are required to establish this. If this is the case it is plausible that the linked features are helping to freeze the system into a state with more regions.

V. FURTHER WORK

There are two types of further work. One is to extend the work here. The other is to introduce different features to the Axelrod 2 model which are pertinent to this area of social dynamics.

Of the first type, investigating the variance in results between the 10 by 20 Axelrod 2 and original Axelrod models would be the priority. Furthermore we propose studying the nontrivial time evolution of the 10 by 15 model and analysing the state of the system at the local minimum compared to later times (with the same number of regions). This analysis would focus on the size and length of the cultural regions in the state of the model. In addtition, we suggest to investigating the distribution of the data of single simulations to see for a given time what distributions these have.

Features to combine with Axelrod 2 include simulating the influence of the media. This could easily be implemented by the inclusion of an individual who interacts with all the other ones and acts as the source of the media broadcast. In this case we are moving away from simply having a lattice and starting to introduce a social network. Examples of a social networks which could be implemented include the small world network [7], [15] or a growing network [9]. Furthermore, we suggest adding an effect on the probability of interaction due to the presence of the linked features.

APPENDIX A: AXELROD 2 MODEL DEFINITIONS

1. Definitions

- Individuals are given by i and j.
- The current state of i is a vector of features, $a_i = (\sigma_1(i), ..., \sigma_F(i))$.
- Each σ_f is a feature there are F in total.

Now the probability that two individuals will interact is $\omega = \frac{1}{F}\sum_{f=1}^{F} \delta_{\sigma_f(i)\sigma_f(j)}$ where the i and j subscript are implicit for compactness.

We also define F', the number of features which are different between i and j, i.e.

$$F' = \sum_{f=1}^{F} \left(1 - \delta_{\sigma_f(i),\sigma_f(j)}\right).$$

2. Probability of changing feature m

If two individuals interact we have a probabilty that a certain feature will be changed, $P(m)$. We define the probability for a linked feature, P_l and a standard feature P_s seperately for clarity. To define these probabilities we require two intermediate values for this pairing of individuals.

We define L, the number of features which are:

- different between i and j, and
- in a pair of related features, and
- paired with a feature which is different between i and j,

. because these are the conditions for a linked feature to have an effect on the probability (see later for a way to calculate this).

Now we have:

$$\begin{aligned} P_l(m) = &\ \tfrac{1}{F'} + \tfrac{\alpha(F'-L)}{F'\,L}, \\ P_s(m) = &\ \tfrac{1}{F'} + \tfrac{\alpha}{F'}. \end{aligned}$$

Where α is the parameter tuning between the original Axelrod model ($\alpha = 0$) and Axelrod 2 ($\alpha = 1$).

3. Calculating L

For completeness we give the method for calculating L. We firstly define R_{mn} as the matrix of which agents are linked. For example if the model is $F = 3$ by $q = 2$ with features one and two linked then $R = [010; 100; 000]$. We also define the following (with i and j being implicit for compactness):

$$\begin{aligned} d_m &= \delta_{\sigma_m(i),\sigma_m(j)}, \\ D_{mn} &= \quad \delta_{d_m d_n}. \end{aligned}$$

We note that:

- $D_{mn} = 1$ when the pair of features are either both the same between i and j ($d_m = d_n$)
- $D_{mn} = 0$ when the pair of features are respectively the same between i and j, and the other different ($d_m \neq d_n$)

We can then write mathematically the conditions that a feature, m, is:

- different between i and j: $(1 - d_m)$,
- in a pair of related features, with feature n: R_{mn},
- paired with feature, n, which is different between i and j: $(1 - D_{mn})$

so we can write the definition

$$Ł = \sum_{n=1}^{F} \sum_{m=1}^{F} (1 - d_m) R_{mn} (1 - D_{mn}) .$$

ACKNOWLEDGMENTS

This work was supported by EPSRC and A. Woolcock would also like to thank C. Connaughton and Y. Merali for their support.

[1] R. Axelrod. The dissemination of culture. *Journal of Conflict Resolution*, 41(2):203–226, 1997.

[2] R. Axtell, R. Axelrod, J. Epstein, and M. Cohen. Aligning simulation models: A case study and results. *Computational and Mathematical Organization Theory*, 1(2):123–141, 1996.

[3] E. Ben-Naim. Opinion dynamics: Rise and fall of political parties. *Europhysics Letters*, 69:671–677, 2005.

[4] C. Castellano, M. Marsili, and A. Vespignani. Nonequilibrium phase transition in a model for social influence. *Physical Review Letters*, 85(16), 2000.

[5] C. Castellano, F. Santo, and L. Vittorio. Statistical physics of social dynamics. *Reviews of Modern Physics*, 81:591–646, 2009.

[6] L. Festinger, S. Schachter, and K. Back. *Social Pressures in Informal Groups: A Study of Human Factors in Housing*. Harper, USA, 1950.

[7] B Halpin. Simulation in sociology. *American Behavioural Scientist*, 42(10):1488–1508, 1999.

[8] H. T. Himmelweit, Humphreys P., and M. Jaeger. *How Voters Decide*. Open University Press, third edition, 1988.

[9] E. M. Jin, M. Girvan, and Newman E. J. Structure of growing social networks. *Physical Review E*, 64:1–8, 2001.

[10] K. Klemm, V. Eguiluz, R. Toral, and M. San Miguel. Role of dimensionality in axelrod's model for the dissemination of culture. *Physica A*, 327(1), 2003.

[11] M. W. Macy and R. Willer. From factors to actors: Computational sociology and agent-based modelling. *Annual Review of Sociology*, 28:143–166, 2001.

[12] V. Sood and S. Redner. Voter model on hetrogenous graphs. *Physical Review Letters*, 94:178701, 2005.

[13] F. Vazquez and S. Redner. Non-monotonicity and divergent time scale in axelrod model dynamics. *Europhysics Letters*, 78(1):18001–18007, 2007.

[14] D. Vilone, A. Vespignani, and C. Castellano. Ordering phase transition in the one-dimensional axelrod model. *Euro Physics*, 30:399, 2002.

[15] D.J. Watts. Networks, dynamics, and the small-world phenomena. *American Journal of Sociology*, 105(2):493–527, 1999.