

All results in quantum mechanics can be derived from a small number of principles, which summarize the most important features of the whole theory. The classical quantities  $x$  and  $p$  are substituted by operators, defined in an Hilbert space, that satisfy these commutation rules:

$$\begin{aligned}[x_i, x_j] &= 0 \\ [p_i, p_j] &= 0 \\ [x_i, p_j] &= i\hbar\delta_{ij}\end{aligned}$$

Time evolution is given by the hamiltonian operator.

The starting point in my analysis is the following relation:

$$\psi(x', t) = \int d^3x \langle x' | \exp \left[ -\frac{i}{\hbar} H(t - t_0) \right] | x \rangle \psi(x, t_0)$$

This relation is quite obvious, knowing that:

$$\begin{aligned}\psi(x) &= \langle x | \psi \rangle \\ \int d^3x | x \rangle \langle x | &= 1\end{aligned}$$

The previous relation can be viewed also as some kind of integral operator acting on the wave function  $\psi(x)$ .

I next introduce the concept of propagator, defined as:

$$K(x', t; x, t_0) = \langle x' | \exp \left[ -\frac{i}{\hbar} H(t - t_0) \right] | x \rangle$$

I next analyze some of the properties of the propagator and I show that the propagator is just the transition probability amplitude between  $x_i$  and  $x_f$ . Using the closure relation and the properties of the propagator, I then show that the transition between  $x_i$  and  $x_f$  can be seen as a transition from  $x_i$  to all the intermediate points  $x$  and then as another transition from  $x$  to  $x_f$ . This leads us rightly to the path integral formulation of quantum mechanics, which was first studied and derived by Feynman. I derive Feynman's relation for the propagator:

$$\langle x' | \exp \left[ -\frac{i}{\hbar} H(t - t_0) \right] | x \rangle = \int \mathcal{D}x \exp \left[ \frac{i}{\hbar} S(x) \right]$$

I then consider two examples that can be quite easily analyzed with path integral: free particle and harmonic oscillator. The free particle propagator

can be calculated with the previous relation by completing the square in the exponent and solving the integrals. I also obtain the same result in another method, inserting a completeness in  $p$ . It is quite satisfying that the results I get are exactly the same.

The harmonic oscillator is solved with another method: the classical action is expanded around the classical solution so that I can drop one term because it vanishes due to the minimum action principle. I also study the limit  $\omega \rightarrow 0$ , which corresponds to the free particle, and I show that in this limit the propagator reduces to the one obtained before.

I then consider the WKB approximation and I show that I can get to the same results considering the path integral formulation of the propagator, in the steepest descent approximation. The wave functions obtained are the same as in the WKB approximation.

Studying a particle in a magnetic field, I prove that I can obtain Landau energy levels by expanding the propagator in powers of  $t$ . This is quite interesting, because it shows that with path integral I can also get the energy spectrum.

In all this examples, I have tried to show that the path integral formulation of quantum mechanics is a powerful tool (which will be very useful in quantum field theory) and it gives a different interpretation of quantum mechanics itself. I have also proved that the results obtained are the same as those obtained in modern quantum mechanics.