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Transition to Turbulence in Pipe Flow

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Introduction

- Fluid flow in a pipe undergoes a transition from a laminar state to sustained turbulence as the Reynolds number Re increases.
- Recently [1] there have been experimental results determining the critical point Re_c as the marker for the onset of sustained turbulence
- Currently critical exponents are inaccessible to experiments.
- We model pipe flow in 1+1 dimensions and investigate scaling properties at criticality with effort to measure critical exponents to compare with those of the Directed Percolation (DP) universality class

PDE Model

- We model the turbulence intensity q and mean velocity u in the pipe:

$$q_t + uq_x = q[u + r - 1 - (r + \delta)(q-1)^2] + q_{xx}$$
$$u_t + uu_x = \epsilon_1 (1-u) - \epsilon_2 uq - u_x$$

- The model includes minimum derivatives to allow for diffusion of turbulent regions and left-right symmetry breaking
- r is a control parameter that plays the role of the Reynolds number
- Although this model captures the transition and basic behaviour of pipe flow, it is too simplistic to to show *puff decay* and *puff splitting*

Discrete Model

- A discrete model for pipe flow with transient chaos was introduced in [2]
- We consider a modified model with non-linear downstream advection:

$$q^{t+1}_{x+1} = f^k[q^t_x + d(q^t_{x-1} - 2q^t_x + q^t_{x+1}) - c(1 + u^t_x - \zeta)(q^t_x - q^t_{x-1})]$$
$$u^{t+1}_{x+1} = u^t_x + \epsilon_1(1 - u^t_x) - \epsilon_2 u^t_x q^t_x - c(1 + u^t_x)(u^t_x - u^t_{x-1})$$

- Here f is a tent map given in Figure 1 which introduces chaotic dynamics in the model as observed in real pipe flow.

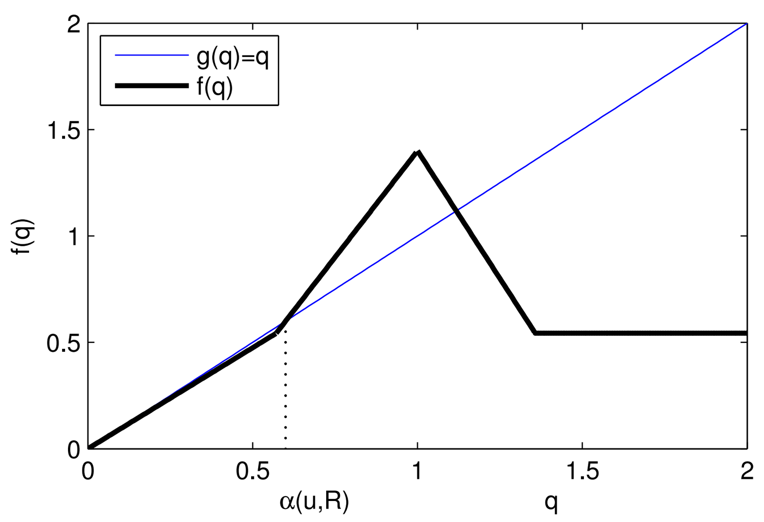

Figure 1 : Chaotic tent map

- $\alpha(u, R)$ separates chaotic and monotonic dynamics of q
- The map is incorporated in the model by setting $\alpha = 2000(1 - 0.8u)R^{-1}$
- R is a control parameter analogous to the Reynolds number
- There is a critical point $R_c \approx 2329.8$ beyond which turbulence in the system is sustained
- This model reproduces the features of real pipe flow shown in Figure 2.

Turbulent Patches

- Figure 2 shows different profiles of turbulence intensity and velocity
  - $R > R_c$, turbulent puffs excite nearby regions and split into more puffs
  - $R >> R_c$, puffs expand into *slugs* - featureless turbulence

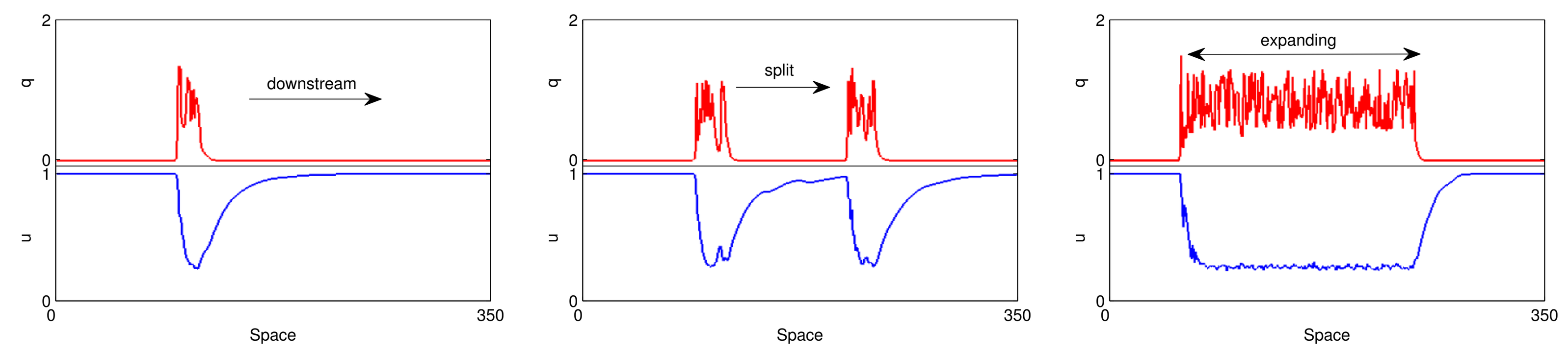

Figure 2 : Snapshots of turbulence: equilibrium puff, puff spliting, slug formation

- Figure 3 shows turbulence intensity for $R > R_c$ in a comoving frame
- The system starts with a single puff that splits into several others.
- More puffs survive than die out, asymptotically the system tends to a constant fraction of turbulent points F_t

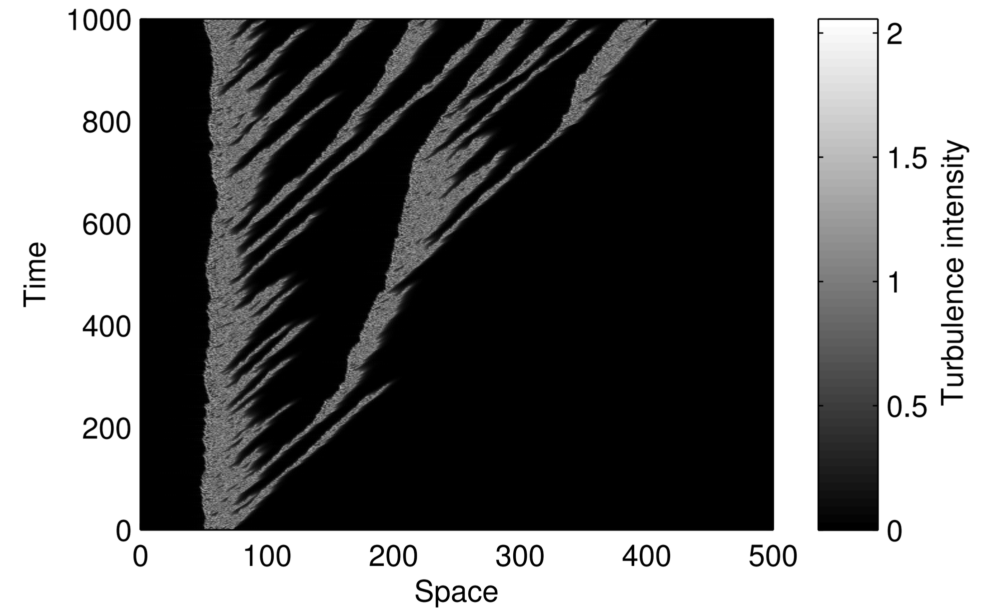

Figure 3 : Space-time plot of a splitting puff

Time Decay of the Order Parameter

- The order parameter of the system is the turbulence fraction F_t

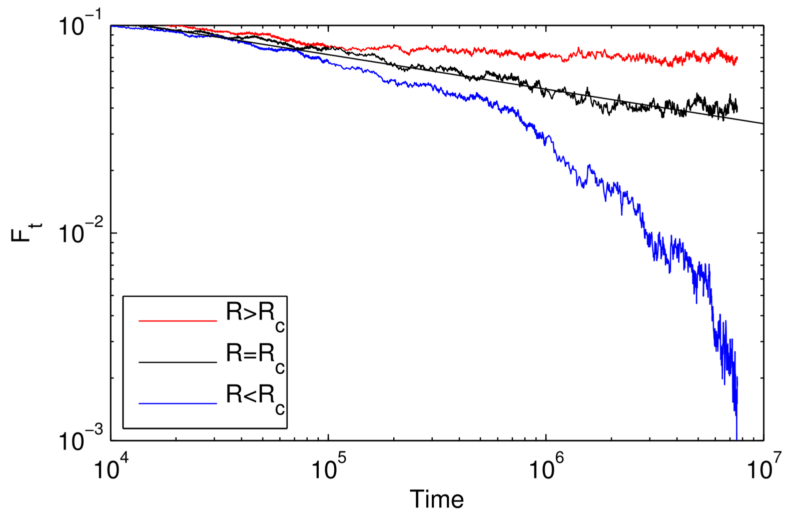

Figure 4 : Turbulence fraction decay

- Figure 4 shows the timeseries of three regimes of the order parameter started from a fully turbulent state
  - $R < R_c$, system tends exponentially to a laminar state with $F_t = 0$
  - $R > R_c$, system saturates at some stationary value of F_t
  - $R = R_c$, we have $F_t \sim t^{-\delta}$ with $\delta = 0.15(2)$

Scaling of the Turbulence Fraction

- Figure 5 shows the scaling of the turbulence fraction F_t near criticality
- We observe $F_t \sim (R - R_c)^\beta$ with $\beta = 0.27(1)$

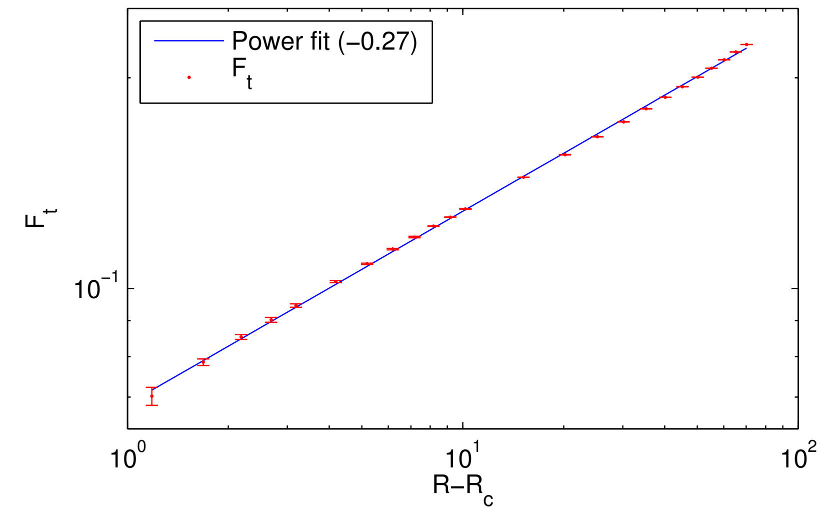

Figure 5 : Turbulence fraction scaling at criticality

- This is consistent with the value observed for Directed Percolation

Laminar Length Distribution

- Inspired by experimental measurements, we also look at the distribution of laminar domains
- Figure 6 shows exponentia decay far from R_c
- At criticality the system undergoes power-law scaling

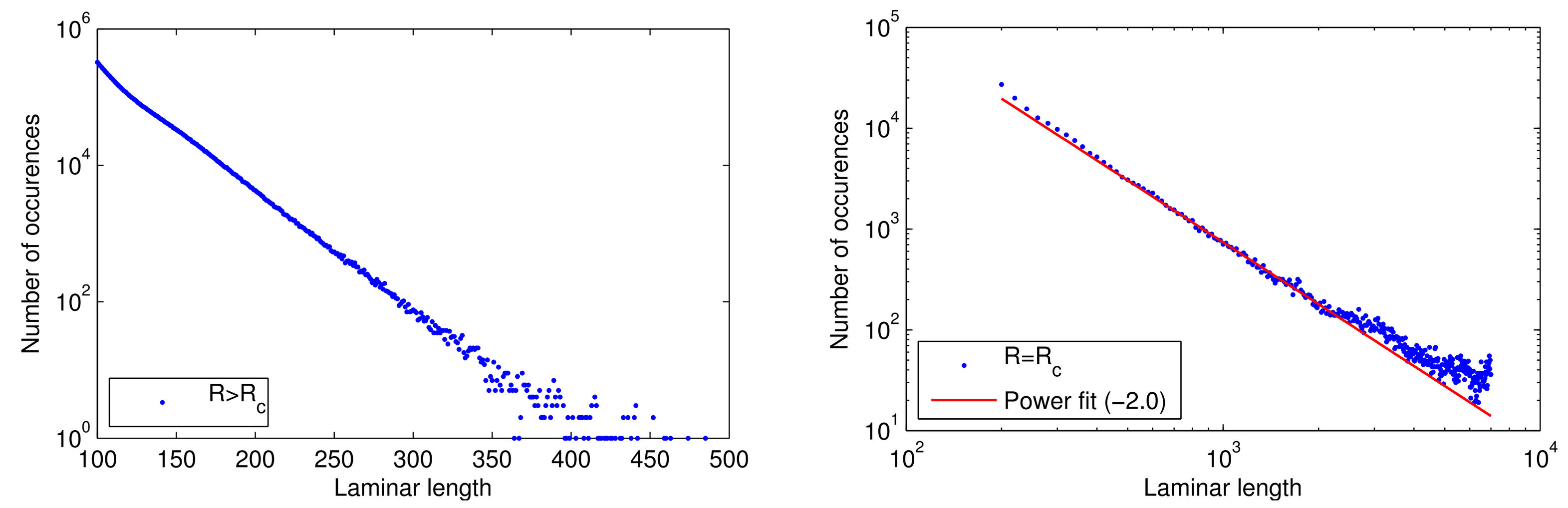

Figure 6 : Histogram of laminar lengths: $R > R_c$ and $R = R_c$

Conclusions and Further Research

- Qualitatively the model exhibits features observed in real pipe flow
- Calculations of the critical exponents β and δ agree with those of the Directed Percolation universality class
- It remains to obtain more quantitatively accurate data to compare with experimental results and DP
- An even greater further challenge is to extend the approach to other shear flows such as plane Couette flow

References

[1] K. Avila, D. Moxey, A. de Lozar, M. Avila, D. Barkley, and B. Hof. The onset of turbulence in pipe flow. *Science*, 333(6039):192–196, 2011.

[2] D. Barkley. Simplifying the complexity of pipe flow. *Phys. Rev. E*, 84:016309, Jul 2011.