Erasmus Mundus Master in Complex Systems

STATISTICAL ANALYSIS OF HIGH FREQUENCY FINANCIAL TIME SERIES: INDIVIDUAL AND COLLECTIVE STOCK DYNAMICS

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Summary

Financial Markets reveal a number of seasonalities related to human rhythms. These periodicities leave a trace in the time series. One of these "intra-day seasonalities" is the well known U-pattern of the volatility of the returns of individual stocks. In the present report, we analyse the intra-day seasonalities of the single and cross sectional (or collective) stock dynamics. In order to do this, we characterize the dynamics of a stock (or a set of stocks) by the evolution of the moments of its returns (and relative prices) during a typical day.

A "single stock intra-day seasonality" is the average behaviour of the moments of the returns (or relative prices) of an average stock in an average day. In the same way, the cross sectional intra-day seasonality is not more than the average behaviour of the moments of an average index (a set of stocks) in an average day. Apart from the well known U-pattern of the volatility of the returns of individual stocks, we also present the intra-day seasonalities for the rest of moments of the returns (and relative prices) of individual (and cross sectional) stocks. For the case of returns, we show that the scale of values that takes the volatility and kurtosis depends on the bin size. Moreover, the inverted U-pattern of the kurtosis "appears" just when we consider a small bin size. This is not the case of the relative prices, in where the intra-day seasonalities are independent of the size of the bin and the index we consider, but characteristic for each index.

When we condition the cross sectional moments to the return of an equiweighted index $|\mu_d|$, we found that the average dispersion of stock returns is an increasing function of $|\mu_d|$. The average cross sectional skewness is an odd function of $|\mu_d|$ and it increases very abruptly for small values of μ_d. Finally, the cross sectional kurtosis decreases with respect to the index return which means that the cross sectional distribution of returns appears to be more Gaussian when its mean is off-centred.

We also include in this report an analysis of the evolution of the average correlation between stocks during a typical day. The largest eigenvalue λ_1 corresponds to the market mode, i.e., all the stocks moving more or less in sync. We show that the average correlation between stocks increases during the day. For smaller eigenvalues, we found that the amplitude of risk factors decreases during the day, as more and more risk is carried by the market factor. Taking as example the S&P 500, we show that the average correlation of the index can be computed by taking a subset of them which means that actually just the more capitalized stocks in the index drive the rest of the stocks.

This work is entirely empirical and it is based on previous works. Some of the results presented here, as far as we know, have not been reported in the literature before.

Abstract

We analyse the intra-day seasonalities of single and cross sectional stock dynamics. In order to do this, we characterize the dynamics of a stock (or a set of stocks) by the evolution of the moments of its returns (and relative prices) during a typical day. We show that these intra-day seasonalities are independent of the size of the bin for relative prices but not for returns. We also include in this report an analysis of the correlation between stocks. We show that the average correlation increases during the day and it can be computed by taking a subset of the stocks that compose an index. Finally, we study the distribution of the cross sectional moments conditioned to the equiweighted index return.

Contents

1 Introduction

Financial Markets reveal a number of seasonalities related to human rhythms. These periodicities leave a trace in the time series [1, 2, 3, 4]. One of these "intra-day seasonalities" is the well known U-pattern of the volatility of the returns of individual stocks [1, 2]. In the present report, we analyse the intra-day seasonalities of single and cross sectional stock dynamics. In order to do this, we characterize the dynamics of a stock (or a set of stocks) by the evolution of the moments of its returns (and relative prices) during a typical day. We also include in this report an analysis of the evolution of the average correlation between stocks and the distribution of the cross sectional moments conditioned to the equiweighted index return.

1.1 Definitions

We have considered two sets of intra-day high frequency time series, the CAC 40 and the S&P 500, during March 2011, from 10 : 00 a.m. to 16 : 00 p.m. The single or collective stock dynamics is characterized by the evolution of the moments of the returns

$$x_\alpha(k,t)_{(1)} = \frac{P_\alpha(k+1,t) - P_\alpha(k,t)}{P_\alpha(k,t)} \tag{1}$$

and the relative prices [5, 6]

$$x_\alpha(k,t)_{(2)} = \frac{P_\alpha(k,t)}{P_\alpha(k_0,t)} - 1 \tag{2}$$

where $P_\alpha(k,t)$ is the price of stock α in bin k and day t.

Below, we present the definitions that we will use for computing these moments [3]. Time averages for a given stock and a given bin are expressed with angled brackets: $< ... >$, whereas averages over the ensemble of stocks for a given bin in a given day are expressed with square brackets [...].

1.2 Single Stock Properties

We characterize the distribution of the stock α in bin k by its four first moments: mean $\mu_\alpha(k)$, standard deviation (volatility) $\sigma_\alpha(k)$, skewness $\zeta_\alpha(k)$ and kurtosis $\kappa_\alpha(k)$

$$\mu_\alpha(k) = \langle x_\alpha(k,t)\rangle \tag{3}$$

$$\sigma^2_\alpha(k) = \left\langle x^2_\alpha(k,t)\right\rangle - \mu^2_\alpha(k) \tag{4}$$

$$\zeta_\alpha(k) = \frac{6}{\sigma_\alpha(k)}(\mu_\alpha(k) - m_\alpha(k)) \tag{5}$$

$$\kappa_\alpha(k) = 24\left(1 - \sqrt{\frac{\pi}{2}}\frac{\langle|x_\alpha(k,t) - \mu_\alpha(k)|\rangle}{\sigma_\alpha(k)}\right) + \zeta^2_\alpha(k) \tag{6}$$

where $m_\alpha(k)$ is the median of all values of the variable x of stock α in bin k.

1.3 Cross Sectional Stock Properties

The cross sectional distributions (i.e. the dispersion of the values of the variable x of the N stocks for a given bin k in a given day t) are also characterized by the four first moments

$$\mu_d(k,t) = [x_\alpha(k,t)] \tag{7}$$
$$\sigma_d^2(k,t) = \left[x_\alpha^2(k,t)\right] - \mu_d^2(k,t) \tag{8}$$
$$\zeta_d(k,t) = \frac{6}{\sigma_d(k,t)}(\mu_d(k,t) - m_d(k,t)) \tag{9}$$
$$\kappa_d(k) = 24\left(1 - \sqrt{\frac{\pi}{2}}\frac{[|x_\alpha(k,t) - \mu_\alpha(k)|]}{\sigma_d(k)}\right) \tag{10}$$

where $m_d(k,t)$ is the median of all the N values of the variable x for a given (k,t). $\mu_d(k,t)$ can be seen as the return of an index equiweighted on all stocks.

2 Intra-day Seasonalities for Returns

2.1 Single Stock Intra-day Seasonalities

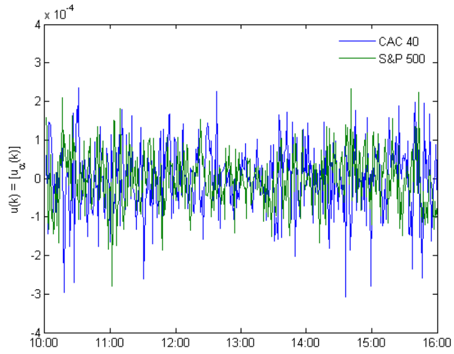
Figure 1 shows the stock average of the single stock mean $[\mu_\alpha(k)]$, volatility $[\sigma_\alpha(k)]$, skewness $[\zeta_\alpha(k)]$ and kurtosis $[\kappa_\alpha(k)]$ for the CAC 40 (blue) and the S&P 500 (green), and $T = 1$ minute bin. As can be seen in figure 1.a., the mean tends to be small (in the order of 10^{-4}) and noisy around zero. As expected, the average volatility reveals a U-shaped pattern (figure 1.b.). The average volatility (for a typical stock, in a typical day) is high at the opening of the day, decreases during the day and increases again at the end of the day. The average skewness (figure 1.c.) is also noisy around zero and the average kurtosis exhibits an inverted U-pattern (figure 1.d.). It increases from around 2 at the beginning of the day to around 4 at mid day, and decreases again during the rest of the day.

2.2 Cross Sectional Intra-day Seasonalities

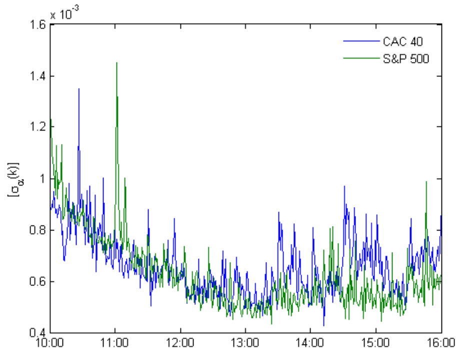
As the time average of the cross sectional mean is equal to the stock average of the single stock mean, the result we show in figure 2.a is exactly the same as the one shown in figure 1.a. The time average of the cross sectional volatility $< \sigma_d(k,t) >$ (figure 2.b) reveals a U-shaped pattern very similar to the stock average volatility, but less noisy (i.e. with less pronounced peaks). The dispersion of stocks is stronger at the beginning of the day and decreases as the day proceeds. The average skewness $< \zeta_d(k,t) >$ is noisy around zero without any particular pattern (figure 2.c). The cross sectional kurtosis $< \kappa_d(k) >$ also exhibits an inverted U-pattern as in the case of the single stock kurtosis. It increases from around 2.5 at the beginning of the day to around 4.5 at mid day, and decreases again during the rest of the day. This means that at the beginning of the day the cross sectional distribution of returns is on average closer to Gaussian.

2.3 U-Pattern Volatilities

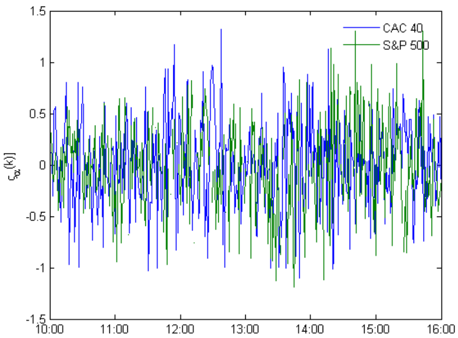
In figure 3, we compare the stock average of single stock volatility $[\sigma_\alpha(k)]$ (black), the time average of the cross sectional volatility $< \sigma_d(k,t) >$ (red) and the average absolute value of the equiweighted index return $< |\mu_d| >$ (blue) for the CAC 40, and $T = 1$ (left) and $T = 5$ minute bin (right). Similar results were obtained for the S&P 500. As can be seen, the average absolute value of the equiweighted index return also exhibits a U-shaped pattern and it is a proxy for the index volatility. One thing that results interesting to observe is that the values of these "volatilities" actually depends of the



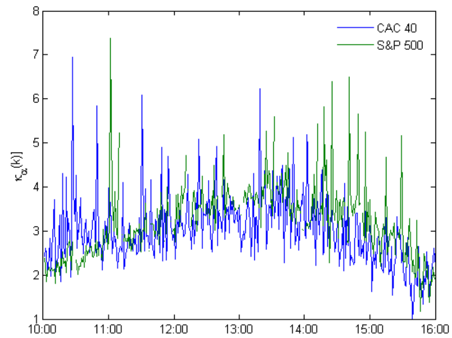
(a) MEAN



(b) VOLATILITY



(c) SKEWNESS



(d) KURTOSIS

Figure 1: Single Stock Intra-day Seasonalities: Stock average of the single stock mean, volatility, skewness and kurtosis for the CAC 40 (blue) and the S&P 500 (green), and $T = 1$ minute bin.

size of the bin that we consider. For $T = 5$ minute bin, the volatilities double the values found for $T = 1$ minute bin (we will discuss again this result in the next sections).

2.4 Intra-day Seasonality in Inter Stock Correlation

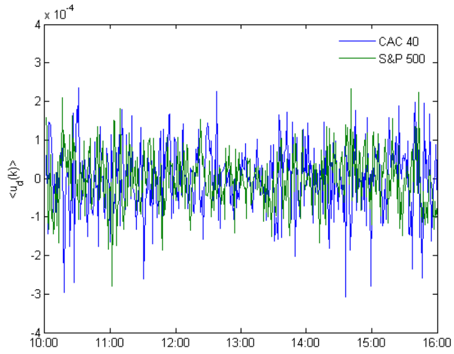
In order to compute the inter correlation between stocks, we first normalize the returns ($\eta_\alpha(k,t) = x_\alpha(k,t)_{(1)}$) by the dispersion of the corresponding bin [3] i.e.,

$$\widehat{\eta}_\alpha(k,t) = \eta_\alpha(k,t)/\sigma_d(k,t) \tag{11}$$

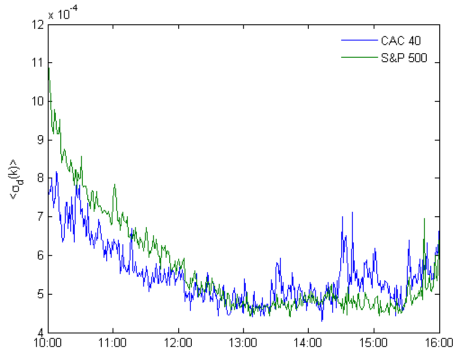
The $N \times N$ correlation matrix for a given bin k would be given by

$$C_{\alpha\beta}(k) = \frac{\langle \widehat{\eta}_\alpha(k,t)\widehat{\eta}_\beta(k,t)\rangle - \langle \widehat{\eta}_\alpha(k,t)\rangle \langle \widehat{\eta}_\beta(k,t)\rangle}{\sigma_\alpha(k)\sigma_\beta(k)} \tag{12}$$

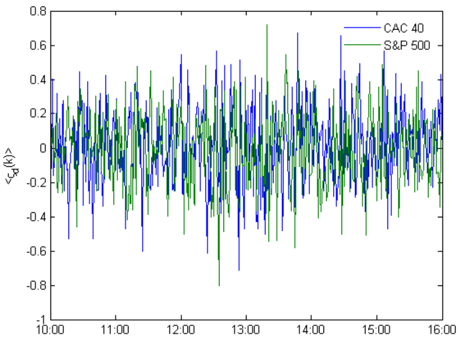
The largest eigenvalues and eigenvectors of $C_{\alpha\beta}(k)$ characterize the correlation structure of stock returns. The structure of large eigenvectors reflects the existence of economic sectors of activity [3, 7, 8, 9, 10]. The largest eigenvalue, in particular, corresponds to the market mode, i.e., all the stocks moving more or less in sync and can be seen as a measure of the average correlation between stocks (figure 4.a). The average correlation increases during the day from a value around 0.35 to a value around 0.45 when the



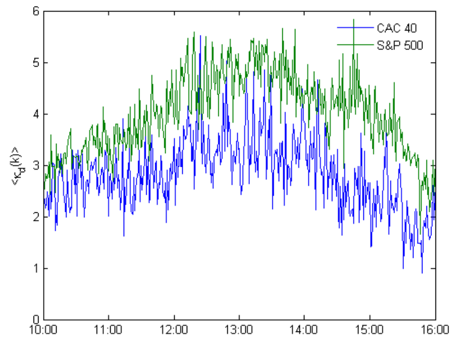
(a) MEAN



(b) VOLATILITY

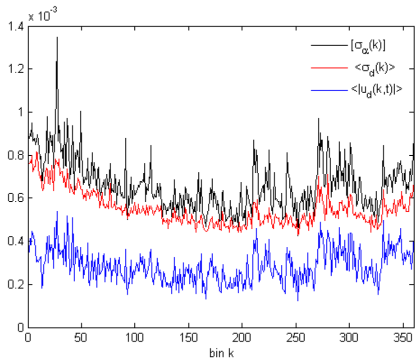


(c) SKEWNESS

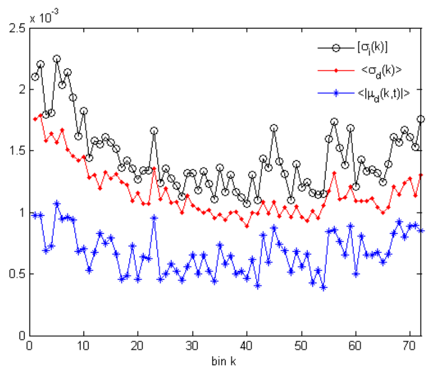


(d) KURTOSIS

Figure 2: Cross Sectional Intraday Seasonalities: Time average of the cross sectional mean, volatility, skewness and kurtosis for the CAC 40 (blue) and the S&P 500 (green), and $T = 1$ minute bin.



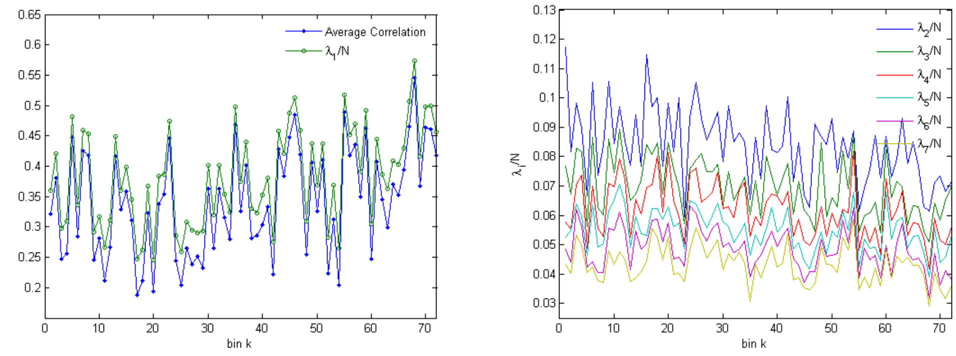
(a) $T = 1$ minute bin



(b) $T = 5$ minute bin

Figure 3: U-Pattern Volatilities: Stock average of single stock volatility $[\sigma_\alpha(k)]$ (black), time average of the cross sectional volatility $< \sigma_d(k,t) >$ (red) and the average absolute value of the equiweighted index return $< |\mu_d| >$ (blue) for the CAC 40.

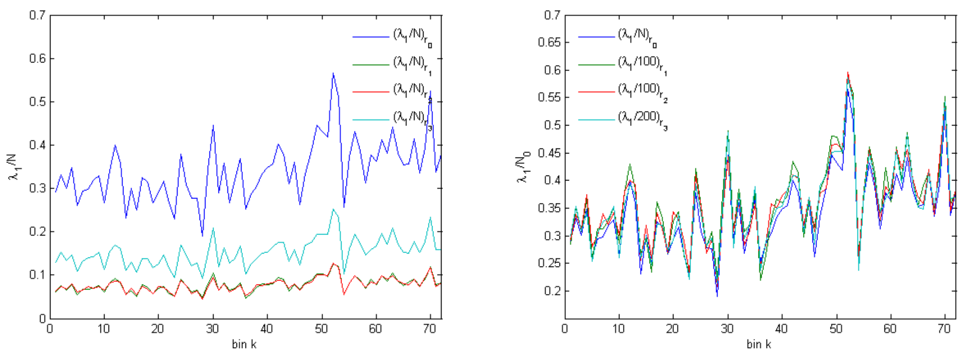
market closes. For the case of smaller eigenvalues, what we can see is that the amplitude of risk factors decreases during the day (figure 4.b), as more and more risk is carried by the market factor (figure 4.a). For the case of the S&P 500, in order to simplify the



(a) (b)

Figure 4: Largest eigenvalues structure for the CAC 40, $T = 5$ minute bin. (a) Average correlation between stocks (blue) and top eigenvalue λ_1/N (green) of the correlation matrix $C_{\alpha\beta}(k)$. (b) Smaller eigenvalues.

computations of the N^2 correlation matrices for each k bin, we computed the correlation matrix $C_{\alpha\beta}$ for 4 different sets of stocks: r_0: composed by the 100 first stocks of the S&P 500; $r_{1,2}$: composed by 100 stocks picked randomly; and r_3: composed by 200 stocks picked randomly. Figure 5.a shows $\frac{\lambda_1}{N}$ as function of the k bins. Although the values of the eigenvalues seem to be out of scale, it can be seen clearly that the average correlation increases during the day. This scale conflict is solved by normalizing the value of the top eigenvalue not by N but by the sample size N_0 (i.e. 100 or 200) (Figure 5.b). As can be seen the average correlation of the index can be computed by taking a subset of them which means that actually just the more capitalized stocks in the index drive the rest of stocks. As far as we know this has not been reported before in the literature.

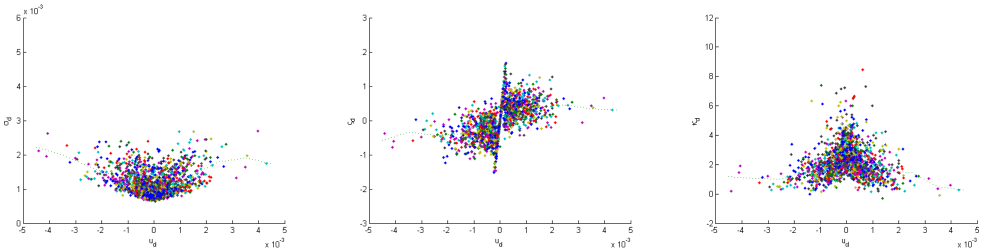


(a) λ_1/N (b) λ_1/N_0

Figure 5: Top eigenvalue λ_1/N (a) and λ_1/N_0 (b) for the S&P 500 for 4 different sets of stocks: r_0 (blue), r_1 (green), r_2 (red) and r_3(clear blue). $T = 5$ minute bin.

2.5 Cross Sectional Moments Conditioned on the Index Return

In figure 6 we present the results we found when we condition the cross sectional moments to the return of an equiweighted index $\mu_d(k,t)$. These results agree with previous works with daily and intra-day returns [3, 10, 11, 12, 13]. Each color represents one day in our period, and each point represents the value of the pair cross sectional moment - mean in a particular bin and day. The average dispersion of stock returns is an increasing function of the absolute value of the equiweighted index return (figure 6.a). The average cross sectional skewness is an odd function of $\mu_d(k,t)$ (figure 6.b). The skewness increases very abruptly for small μ_d and saturates for larger values of the index return. Finally, the cross sectional kurtosis shows a decreasing behaviour with respect to the index return (figure 6.c) which means that the cross sectional distribution of returns appears to be more Gaussian when its mean is off-centred.



(a) (b) (c)

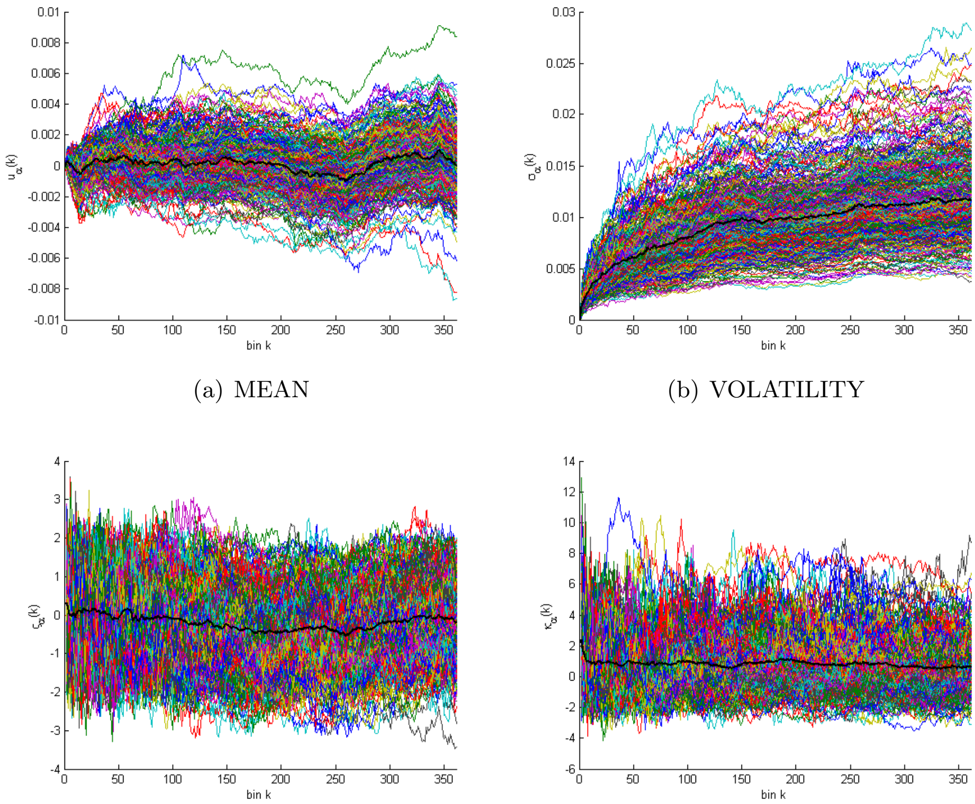
Figure 6: Cross sectional volatility (a), skewness (b) and kurtosis (c) conditioned to the index return (equiweighted on all stocks) for the S&P 500. $T = 5$ minute bin.

3 Intra-day Seasonalities for Relative Prices

As far as we know, the following analysis has not been reported before in the literature. We will report only the results we found for the S&P 500 and $T = 1$ minute bin. Similar behaviour was found also for the CAC 40 and for $T = 5$ minute bin. We will see how in the case of the relative prices these intra-day seasonalities are independent of the size of the bin, also independent of the index we consider, but characteristic for each index. As we will see in following sections, this is not the case for the returns.

3.1 Single Stock Intra-day Seasonalities

Each path in figure 7 represents the evolution of a particular moment of one of the stocks that compose the S&P 500 (i.e. one path, one stock moment). The stock average of the single stock mean $[\mu_\alpha(k)]$, volatility $[\sigma_\alpha(k)]$, skewness $[\zeta_\alpha(k)]$ and kurtosis $[\kappa_\alpha(k)]$ of the S&P 500 are shown in black. The stock average of the single stock mean varies around zero. The average volatility increases logarithmically with respect to the time. The values for the skewness vary between $[-3, 3]$ with an average value of zero. The single stock kurtosis for the S&P 500 takes values between $[-2, 6]$ with an average value of one. It can be seen that the stock average of the single stock kurtosis starts from a value around 2 in the very beginning of the day and decreases quickly to the mean value 1 in the first minutes of the day.



(a) MEAN (b) VOLATILITY

(c) SKEWNESS (d) KURTOSIS

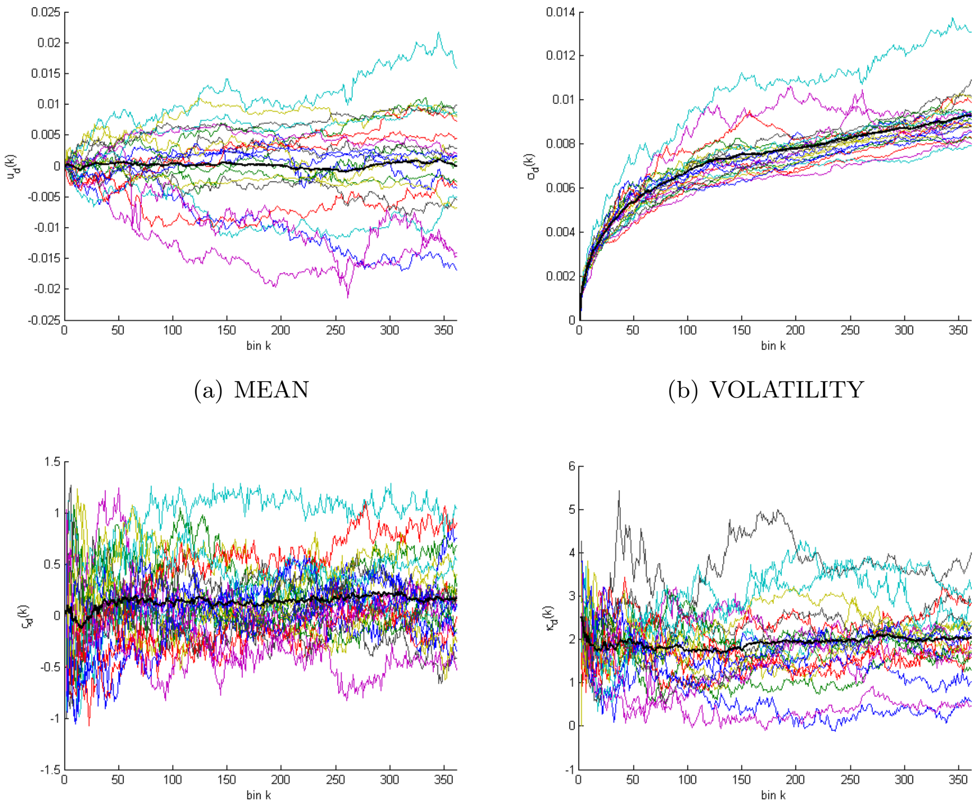
Figure 7: Single Stock Intra-day Seasonalities: Stock average of the single stock mean, volatility, skewness and kurtosis for the S&P 500 (black). $T = 1$ minute bin.

3.2 Cross Sectional Intra-day Seasonalities

Each path in figure 8 represents the evolution of a particular (cross sectional) moment for one of the days of the period under analysis (i.e. one path, one day moment). As in the case of the single stock volatility, the cross sectional dispersion $< \sigma_d(k) >$ increases logarithmically with respect to the time (figure 8.b). The cross sectional skewness $< \zeta_d(k) >$ takes values in the interval $[-1, 1]$ with an average value of zero (figure 8.c). The average kurtosis $< \kappa_d(k) >$ starts from a value around 2.5 in the very beginning of the day and decreases quickly to the mean value 2 in the first minutes of the day (figure 8.d).

3.3 C-Pattern Volatilities

In figure 9 we show a comparative plot between the stock average of single stock volatility $[\sigma_\alpha(k)]$, the time average of the cross sectional volatility $< \sigma_d(k, t) >$ and the average absolute value of the cross sectional mean $< |\mu_d| >$ for the relative prices of the S&P 500, and $T = 1$ and $T = 5$ minute bin. As can be seen these three measures exhibit the same intra-day pattern. But more important is to notice that this intra-day seasonality is independent of the size of the bin, also independent of the index we consider, but characteristic for each index (see inset). As far as we know, this have not been reported



(a) MEAN (b) VOLATILITY

(c) SKEWNESS (d) KURTOSIS

Figure 8: Cross Sectional Intra-day Seasonalities: Time average of the cross sectional mean, volatility, skewness and kurtosis for the S&P 500 (black). $T = 1$ minute bin.

in the literature before.

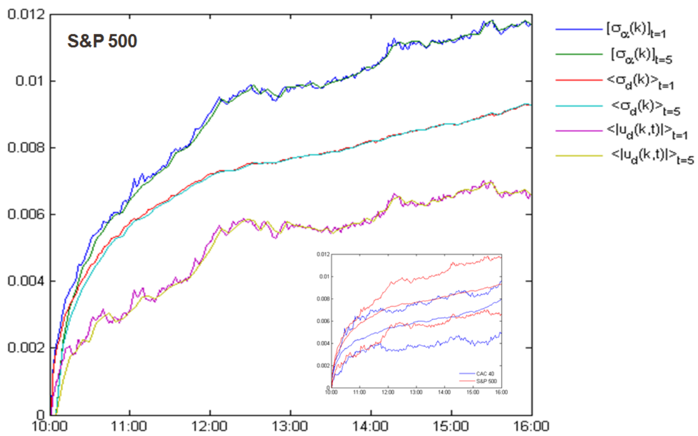
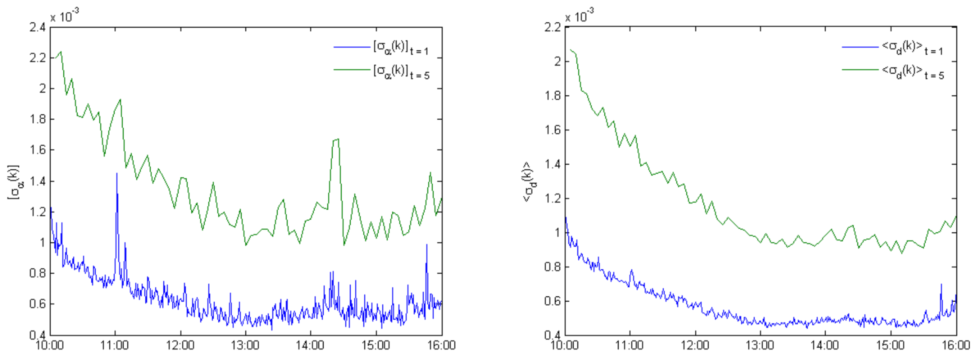


Figure 9: Stock average of the single stock volatility $[\sigma_\alpha(k)]$, time average of the cross sectional volatility $< \sigma_d(k,t) >$ and the average absolute value of the cross sectional mean $< |\mu_d| >$ for the relative prices of the S&P 500, and for $T = 1$ and $T = 5$ minute bin. Inset: CAC 40 (blue) and S&P 500 (red).

4 Intra-day Patterns and Bin Size

As we saw in the last section, the volatilities for the relative prices exhibit the same kind of intra-day pattern (figure 9). This intra-day seasonality is independent of the size of the bin, and the index we consider, but characteristic for each index (inset figure 9). Actually, this is not true in the case of the returns. If we consider the odd moments (mean and skewness) of the returns of the stocks, the behaviour is basically the same (noisy around zero) independently if we take 1 or 5 minute bin. But for the case of the even moments, although the behaviour of the moments is the same (i.e., U and inverted U-patterns), the values of the volatility and kurtosis depends on the bin size (figures 10 and 11). In the case of the kurtosis, apart from the same fact found for the volatilities,



(a) (b)

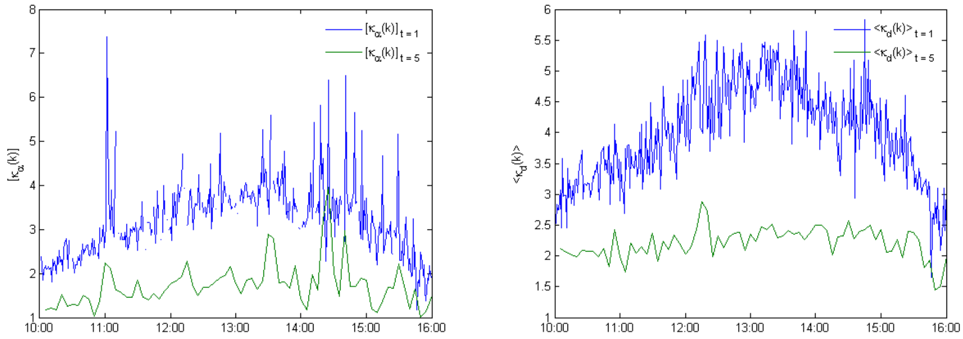
Figure 10: Stock average of the single stock volatility $[\sigma_\alpha(k)]$(left) and time average of the cross sectional volatility (right) $< \sigma_d(k,t) >$ for the S&P 500. $T = 1$ (blue) and $T = 5$ (green) minute bin.

also its behaviour depends on the bin size. The inverted U-pattern we showed (figures 1.d and 2.d) is evident just when we consider $T = 1$ minute bin (figure 11). As far as we know this dependence on the bin size of the intraday seasonalities have not been reported in the literature before.

5 Discussion

In the present report, we have analysed the intra-day seasonalities of the single and cross sectional (or collective) stock dynamics. In order to do this, we characterized the dynamics of a stock (or a set of stocks) by the evolution of the moments of its returns (and relative prices) during a typical day. What we have called “single stock intra-day seasonality” is the average behaviour of the moments of the returns (or relative prices) of an average stock in an average day. In the same way, the cross sectional intra-day seasonality is not more than the average behaviour of the moments of an average index (a set of stocks) in an average day. Apart from the well known U-pattern of the volatility of the returns of individual stocks, we have also presented the intra-day seasonalities for the rest of moments of the returns (and relative prices) of individual (and cross sectional) stocks.

In the case of the single stock intra-day seasonalities, the odd moments (mean and skewness) of the returns were found to be small and noisy around zero and, as expected,



(a) (b)

Figure 11: Stock average of the single stock kurtosis $[\kappa_\alpha(k)]$(left), and time average of the cross sectional kurtosis $< \kappa_d(k,t) >$ (right) for the S&P 500. $T = 1$ (blue) and $T = 5$ (green) minute bin.

the average volatility revealed a U-shaped pattern mean while the average kurtosis exhibit an inverted U-pattern. Similar behaviour was observed for the cross sectional moments. Moreover, the average absolute value of the equiweighted index return also exhibits a U-shaped pattern and it is a proxy for the index volatility (figure 3).

It is important to remark that the values of the volatilities and kurtosis depend actually on the bin size (figures 10 and 11). In the case of the kurtosis also its behaviour depends on the bin size and the inverted U-pattern is evident just when we consider a small bin size (figure 11). This is not the case of the relative prices, in where the intra-day seasonalities are independent of the size of the bin, also independent of the index we consider, but characteristic for each index (figure 9).

The single stock dynamics of the relative prices exhibits an average mean around zero and a skewness varying between $[-3, 3]$ with an average value of zero. The average volatility increases logarithmically with respect to the time and the kurtosis takes values between $[-2, 6]$ with an average value of one. We observed a similar behaviour in the cross sectional dynamics but with the skewness taking values in the interval $[-1, 1]$ with an average value equal to zero and the kurtosis with an average value equal to 2.

We also included in this report an analysis of the average correlation between stocks. The largest eigenvalue λ_1 corresponds to the market mode, i.e., all the stocks moving more or less in sync. This measure of the average correlation between stocks (figure 4.a) increases during the day. For smaller eigenvalues, what we can see is that the amplitude of risk factors decreases during the day (figure 4.b), as more and more risk is carried by the market factor (figure 4.a). We also showed that the average correlation of an index can be computed by taking a subset of them which means that actually just the more capitalized stocks in the index drive the rest of the stocks (figure 5).

When we conditioned the cross sectional moments to the return of an equiweighted index we found that, the average dispersion of stock returns is an increasing function of $|\mu_d|$ (figure 6.a). The average cross sectional skewness is an odd function of $|\mu_d|$ (figure 6.b). It increases very abruptly for small μ_d and saturates for larger values. Finally, the cross sectional kurtosis decreases with respect to the index return (figure 7.c) which means that the cross sectional distribution of returns appears to be more Gaussian when its mean is off-centred.

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A Appendix: Data Manipulation

We considered two sets of intra-day high frequency financial time series corresponding to the CAC 40 and the S&P 500. The main reasons why we chose these two indexes are: The number of stocks that compose them ($N_1 = 40$ and $N_2 = 500$, obviously), the gap between the markets (because of different time zones in Europe and America), and the different range of stock prices (between 5 and 600 USD for the S&P 500 and between 5 and 145 EU for the CAC 40). For each day D_k, $k = 1, ..., 22$, of our period of analysis (March 2011), we count with the evolution of the prices of each stock during that day from $10:00$ a.m. to $16:00$ p.m.

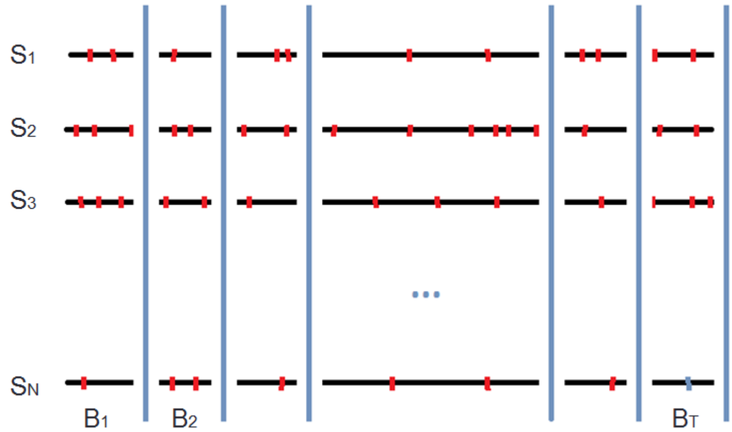

Figure 12: Intraday asynchronous financial time series. S_i are the stocks and B_j are the bins. The asynchronous prices are show in red an the bin limit in blue.

As the changes in prices are not synchronous between different stocks (as can be seen in figure 12), we manipulated our original data in order to construct a new homogeneous matrix $P_D^{(k)}$ of bin prices. In order to do this, we took the last price of that stock just before the bin limit in that particular (k) day. In this report we have worked with $T = 1$ and $T = 5$ minute bin.

$$P_D^{(k)} = \begin{pmatrix} P_{11}^{(k)} & P_{12}^{(k)} & \dots & \dots & \dots & P_{1T}^{(k)} \\ P_{21}^{(k)} & P_{22}^{(k)} & \dots & \dots & \dots & P_{2T}^{(k)} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & P_{ij}^{(k)} & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ P_{N1}^{(k)} & P_{N2}^{(k)} & \dots & \dots & .. & P_{NT}^{(k)} \end{pmatrix} \tag{13}$$

Each row in the matrix above represents the evolution of the prices of a particular stock as function of the bins. For example, the element $(P_D)_{ij}^{(k)}$, represents the price for the day (k) of stock i in the bin j.

In a similar way, we can construct a matrix $P_S^{(i)}$ for each of the $i = 1, ..., N_{1,2}$ stocks

$$P_S^{(i)} = \begin{pmatrix} P_{11}^{(i)} & P_{12}^{(i)} & ... & ... & ... & P_{1T}^{(i)} \\ P_{21}^{(i)} & P_{22}^{(i)} & ... & ... & ... & P_{2T}^{(i)} \\ ... & ... & ... & ... & ... & ... \\ ... & ... & ... & P_{kj}^{(i)} & ... & ... \\ ... & ... & ... & ... & ... & ... \\ P_{D1}^{(i)} & P_{D2}^{(i)} & ... & ... & .. & P_{DT}^{(i)} \end{pmatrix} \tag{14}$$

where $(P_S)_{kj}^{(i)}$ is the price of the stock (i) in the day k and in the bin j.

Once we have constructed these matrices, we can transform these prices into a new variable $x_\alpha(k,t)$ of returns, log-returns, cumulative returns, relative prices, or any other definition. $x_\alpha(k,t)$ is the value of the variable x for the stock α in the bin k and in the day t. In the present analysis we have worked with returns and relative prices [5, 6].