

# Irregular spatial prisoner's dilemma and the evolution of cooperation

MINH-QUAN NGUYEN

Gothenburg University

(M1 Project, Erasmus Mundus Masters in Complex Systems)

Supervisor: Prof. Kristian Lindgren

May 22, 2013

---

## Abstract

The prisoner's dilemma is a widely used model to investigate the evolution of cooperation. In this project, we investigate the evolution of cooperation in a non-iterated spatial prisoner's dilemma game. We use an diffusive irregular spatial model which consists of random distribution of players, diffusion and mutation. The interactions of players are local, which means that a player only interact with its neighbours in a radius  $r$ . Using this model, we investigate the conditions for the coexistence between cooperators and defectors. We find out that there is a range of interaction radius that leads to a chaotic spatial coexistence between cooperators and defectors. The frequency of cooperators will go to a quasi-steady state which does not depend on the initial frequency of cooperators. The frequency of cooperators in this quasi-steady state depends strongly on the interaction range and the value of cheating advantage. We consider both deterministic and probabilistic update rule for the strategy and find out that the deterministic rule give more advantage to cooperators. We also study the dynamics of the system in continuous time simulation. The general conclusion is that even in an irregular, chaotic, non-iterated prisoner's dilemma game, within a suitable range of parameters, the spatial effects can give rise to a coexistence between cooperators and defectors in the population.

*Key words:* evolutionary games, spatial model, prisoner's dilemma, cooperation, chaotic.

---

# 1 Introduction

The evolution of cooperation is one of the most discussed problems of evolutionary game theory. It has been studied in many context such as biology, sociology and ecology for many years. To study this problem, the prisoner's dilemma (PD), an evolutionary game introduced by Rapoport and Chammah [11], is often chosen because it gives a simple framework to understand this problem. This game is based on the fact that it pays off to defect. Defectors can take advantage of cooperators. In this game, classical evolutionary game theory showed that defection is the evolutionary stable strategy. A mutation of defector will take over a population of cooperators but a mutation of cooperator can't survive in a population of defectors. This raise an important question: how can cooperation evolve in this situation? Why do we see cooperation in nature?

Many attempts has been made to understand this question. There are five main rules for the evolution of cooperation in the population [10]. The first is kin selection. Natural selection can favour cooperation if the players are genetically related. Another rule is group selection. If the selection process is in group level. Natural selection will favour the group of cooperators. Another powerful mechanism is direct reciprocity. It expresses the idea that if you help me, I will help you and if you cheat, I will punish you. Using the PD game, Axelrod [1] showed that cooperation can evolve in an environment consisting of individuals with selfish motives if the game is repeated. He organized two tournaments to find out the best strategy in a iterated prisoner's dilemma game. The famous simple strategy "tit for tat" won both tournament. The PD game of Axelrod is the PD with memory and complicated strategies. The fourth mechanism is indirect reciprocity. If we consider direct reciprocity to be a battering system when players trade directly with each other, indirect reciprocity is a monetary system. The player can help other players and get a good reputation, if he cheats, he will get a bad reputation. Other players will tends to play with the one who has high reputation. The final mechanisms is spatial reciprocity. It can be generated to network reciprocity. Because the interaction is in space so the players who live near each other will interact more often with each other. Nowak, in his paper [5], showed that spatial effects can promote the coexistence of cooperators and defectors in the population. He showed that a simple spatial model of the prisoner's dilemma can "generate chaotically changing spatial patterns, in which cooperators and defectors both persist indefinitely". His initial model is a lattice model which is a two-dimensional cellular automaton and his result seems to depend on the symmetry of the cellular automaton. Latter he did expand his model to spatial irregularities (random distribution of players) and probabilistic winning [7].

The main purpose of this project is to study this PD game in an irregular spatial model with diffusion of players. We use an irregular spatial model with diffusion to understand the dynamics of the system of cooperators and defectors. We keep the model as simple as possible. There are no memory (one shot game) and no complicated strategy. Each player will only be either a cooperator or a defector and they are randomly distributed in the area which is a square length  $L \times L$ . Each player only interacts with neighbouring players in a radius  $r$ . At each time step, every players will play the PD with its neighbours and then collect scores. Then, each player will look at its neighbours and copy the strategy of

the one with the highest total score. We will consider both deterministic update rule and probabilistic update rule. With deterministic update rule, the player with lower score will always copy the strategy of its neighbour who has highest score. With probabilistic update rule, this player can keep its strategy with a small probability  $1-p$ . We also include mutation. There is a small probability  $m$  that a cooperator become a defector and vice versa. Finally, each player will diffuse by jumping a distance  $d$  from its initial position with a random direction. This will make sure that the space is really irregular and more realistic. We study the dynamics of the system using both synchronous and asynchronous updating.

The structure of this report is as follows: in section 2, we will explain our model in detail. Section 3 is for presenting the results of our study. Finally section 4 is for discussions and conclusions.

## 2 The model

### 2.1 Prisoner's dilemma

The game we use to investigate the evolution of cooperation in spatial model is the prisoner's dilemma. This is a two-player game. Each player will be either a cooperator (C) or a defector (D). The score each one receive after each encounter is depend on this payoff matrix:

	C	D
C	R	S
D	T	P

If both players cooperate, they get a "reward",  $R$  payoff. If one player defects and the other chooses to cooperate, the defector gets the "temptation" payoff  $T$  and the cooperator gets the "sucker's",  $S$  payoff. If both players defect they both get the payoff  $P$ , "punishment".

With  $T > R > P > S$ , we can see the dilemma of this game: because  $R > P$ , mutual cooperation is better than mutual defection but because  $T > R$  and  $P > S$ , defection is the dominant strategy. No matter which strategy the other player choose, it is better for the player to choose to defect. This will lead both player to choose to defect and they will end up in mutual defection which has lower score than mutual cooperation. Mutual defection is a Nash Equilibrium. According to the traditional evolutionary game theory (one shot and non-memory), the evolutionary stable strategy (ESS) is defection. One defectors in a "sea" of cooperators will finally take over the population but one cooperators in a sea of defectors can not survive. Many attempts has been taken to find the conditions in which cooperation can be maintain in this game. Axelrod and Hamilton [1] showed that repeated with memory prisoner's dilemma can lead to cooperation. Nowak [5] showed that a simple spatial model can also give rise to the maintenance of cooperation.

## 2.2 Our model

In our model, we consider a square size  $L \times L$  with boundary condition. Initially, players will be randomly distribution in this area with the average density of one player per unit area, which means that there are  $N = L^2$  players. We will keep this density constant in our program.

Each player will interact with its neighbours who live in a distance  $r$  from this player. The payoffs will be determined by this payoff matrix:

	C	D
C	1	0
D	b	$\epsilon$

with  $b > 1$  and  $\epsilon$  is a small positive number. This is the payoff matrix that Nowak chose in his paper to investigate the spatial chaos in lattice model [6]. This matrix is design to be as simple as possible but still keep essential characteristics of the PD. It will help to reduce the payoff matrix to one parameter  $b$  which represent the temptation of defection. The final payoff of each player is the sum of all payoffs it gets from all interactions.

After each time step, the strategy of every player will be updated according to a deterministic or probabilistic update rule. With the deterministic rule, a player with lower payoff will always copy the strategy of the neighbouring player with the highest payoff. With the probabilistic rule, there is a possibility  $1-p$  that this player still keeps its strategy. This update rule expresses the fact that players want to imitate the strategy of the more successful neighbour. The more successful strategy will spread in the population.

Then each player will defuse by jumping a distance  $d$  from its initial position with random direction. This way we will make sure that the system is really irregular and independent from the initial distribution

Finally, we add mutation into the population. There are very small a probability  $m$  that a player will change its strategy from cooperating to defecting and vice versa. The mutation rate  $m$  will be chosen to be  $10^{-4}$  in this report. We will not change this value.

We will consider both synchronous updating and asynchronous updating. Synchronous

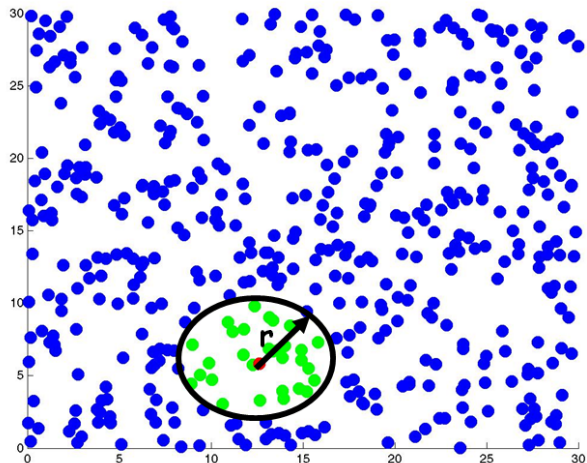


Figure 1: The irregular model. Players are distributed randomly in an square length  $L$ . Each player will interact with all neighbours living in a distance  $r$  from this player.

updating is for discrete time and it corresponds to biological situation where there are no overlapping between each generation. Each interacting state will be followed by a reproduction state. For some situation, this is not correct. There can be overlapping between each generation and the updating rule in this case should be continuous. Some [2] has argued that continuous updating is essential for the mimic of the real world. Others [8] believe that "discrete time is appropriate for many biological situations, and continuous for others". But clearly, asynchronous updating is essential for understanding many problems. So we apply this updating for our model to see what are the changes in the dynamics of the system.

For asynchronous updating, in each time step a random player will be chosen to play the game with its neighbour, we then find all the relevant scores (the score of the player and its neighbour) and then update the strategy of this player. The update rule can be deterministic or chaotic. There are still diffusion and mutation in our model. Now after  $N$  random updates, each player will diffuse and mutate with similar rules for synchronous updating.

The questions we want to answer with this model are:

- Can there be a spatial coexistence of cooperators and defectors in this irregular spatial model? What is its condition? Under what conditions will defector take over the population?
- Can a small group of cooperators invade a population of defectors and under what condition?
- What is evolution through time of the frequency of cooperators? What is its dependence on the size of the domain  $L$ , the interaction radius  $r$  and diffusion distance  $d$ ? What is the difference between deterministic and stochastic update rule.
- What happens if we use asynchronous updating? Can cooperators still survive?

## 3 Results

### 3.1 The evolution of cooperation with deterministic update rule

In this section, we will investigate the evolution of cooperation with deterministic rule. Player will always copy the strategy of the more successful player. In another way, the most successful player will always spread its strategy to its neighbour with probability  $p=1$ .

#### 3.1.1 Defector invasion and cooperators invasion

Cooperation is prone to be invaded by defectors. A single cooperator in a sea of defectors can't survive but a single defector in a sea of cooperators will spread its strategy. There

are only three mechanisms that a cooperator can survive if it lives near a defector. First, the cooperator's payoff from its interaction with other cooperators can offset the costs of cooperation with the defector. Second, the cooperator can get more payoff by simply interacting with more neighbours than its neighbouring defectors. And finally the best player in the neighbourhood of this player is a cooperator. For the stochastic update rule, the cooperators can also survive because of the stochastic update rule. The only way that a cooperator can spread its strategy to a neighbouring defectors is by having greater score by interacting with other cooperators and the defector have to interact with other defectors. This only happens at the boundary of clusters of defectors and cooperators. These mechanisms come from the local interaction mechanism.

The standard procedure for studying evolutionary game theory is by studying invasion: can a mutation spread its strategy? First we will study the invasion of defector in a sea of cooperators. We initialize the population with all cooperators, then we introduce a defector to the population (or just running the program and wait for mutation to occur (the mutation rate is  $10^{-4}$ )). This defector will quickly spread its strategy by exploiting other cooperators. The frequency of the remaining cooperators depend strongly on the interaction radius (see Fig 2). When the interaction radius  $r$  is small ( $r=1$ ), the group of interacting players is small. Because of the influx of defectors connected to fluctuations in the diffusion, cooperators will interact with defectors and the population will be taken by defectors. With no diffusion, the smaller the interaction range, the higher the frequency of cooperators. When  $r$  is larger, the clusters of players become larger. Groups of cooperators can survive and evolve. The population quickly go to a quasi-steady state, in which there is a spatial coexistence of cooperators and defectors. These coexistence is very chaotic (see Fig 3). Groups of cooperators and defectors evolve and disappear. The larger the interaction, the lower the average frequency of cooperators in the population. This is because when  $r$  is large, the local mechanisms which help the cooperators become weaker. The cluster become bigger so the system become more like a system with global interaction. Furthermore, when  $r$  is larger, the system also become more chaotic. Groups of cooperators and defectors move very fast in the area. The population can go to fixation because of random fluctuation in finite population (Fig 2,  $r=8$ ).

We know that one cooperators will quickly die out in a population of defectors. But can a group of cooperators survive in or invades a population of defectors? We initially put a group of cooperators in a square size  $4 \times 4$  in a middle of a sea of defectors. These group of cooperators can invade the defectors (see figure 4 for snap-shot of the spatial distribution). The frequency of cooperators will go to a quasi-steady state which is the same as when we initialize the population with all cooperators. This quasi-steady state is independent from the initial frequency of cooperators (Fig 5).

### 3.1.2 The quasi-steady state

From the previous section, we see that there is a quasi-steady in which the frequency of cooperators fluctuate around, regardless of the initial frequency of cooperators. But What is the characteristic of this quasi-steady state? Is there only a quasi-steady state

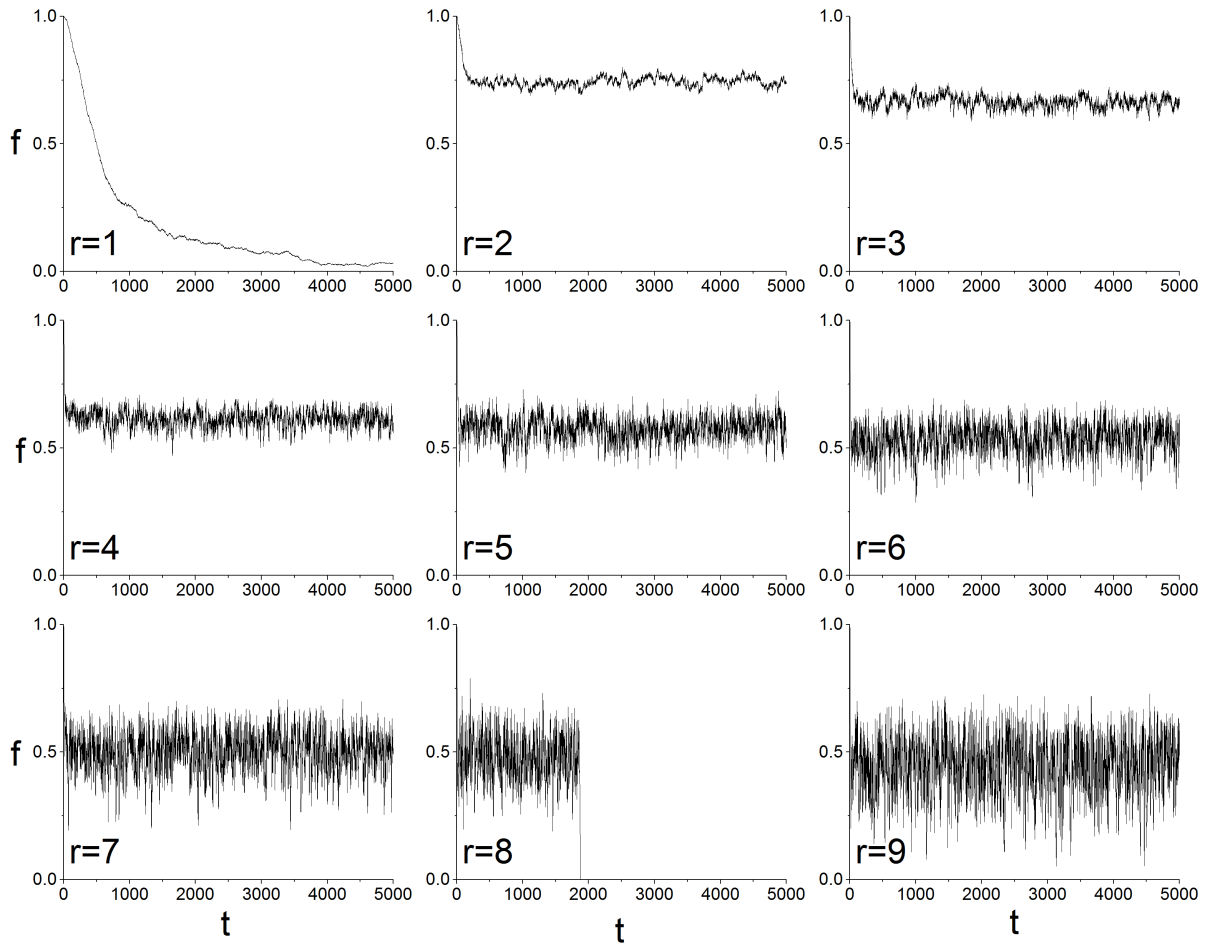


Figure 2: The frequency of cooperators  $f$  as a function of time  $t$  with deterministic update rule with the interaction radius  $r$  from 1 to 9. The size of the domain is  $L=100$  and the value of  $b$  is 1.61. We can see that the frequency go to a quasi-steady state and fluctuates around this value. For small  $r$  ( $r=1$ ), defectors can take over the population because of the diffusion. When  $r$  increase, there is a spatial coexistence of cooperators and defectors. The larger the interaction radius, the smaller the frequency and the larger the fluctuation. When  $r$  become too large, defectors will finally take over the population because of random fluctuation in finite population. Because of the mutation, there can't be fixation with all cooperators but there can still be fixation with all defectors because one cooperator can never survive in a population of defectors. Cooperators can go extinct because of the random fluctuation in finite population.

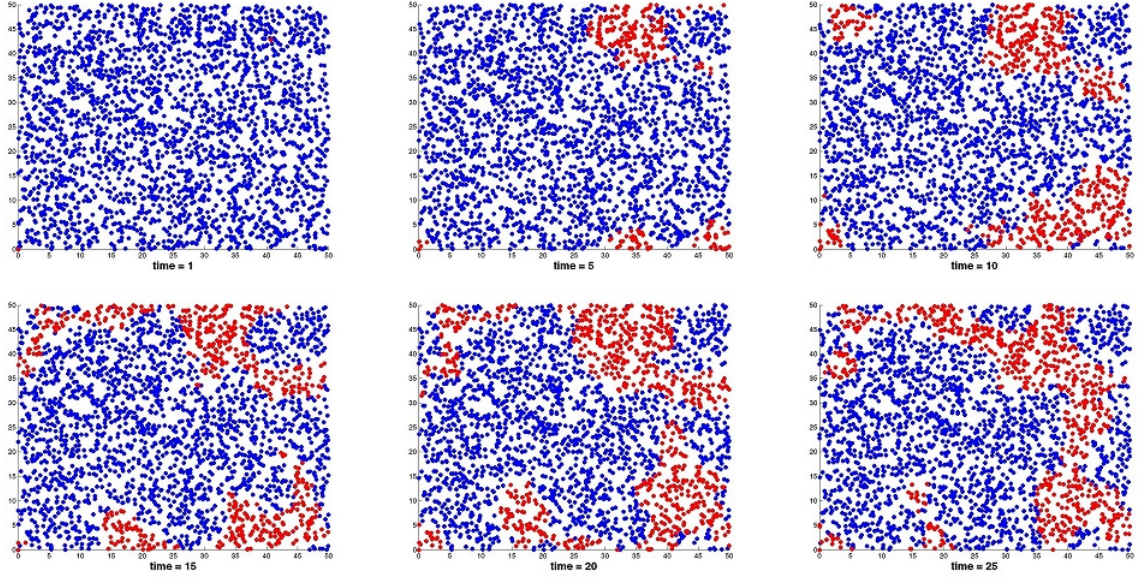


Figure 3: Spatial distribution of cooperators (blue) and defectors (red) on the domain of size  $L \times L$ ,  $L=50$  at time  $t=1,5,10,15,20,25$  with the interaction radius  $r=4$ . Initially all of the population are cooperators. At large value of interaction radius, the movement of groups of cooperators and defectors is very fast compare with the diffusion rate.

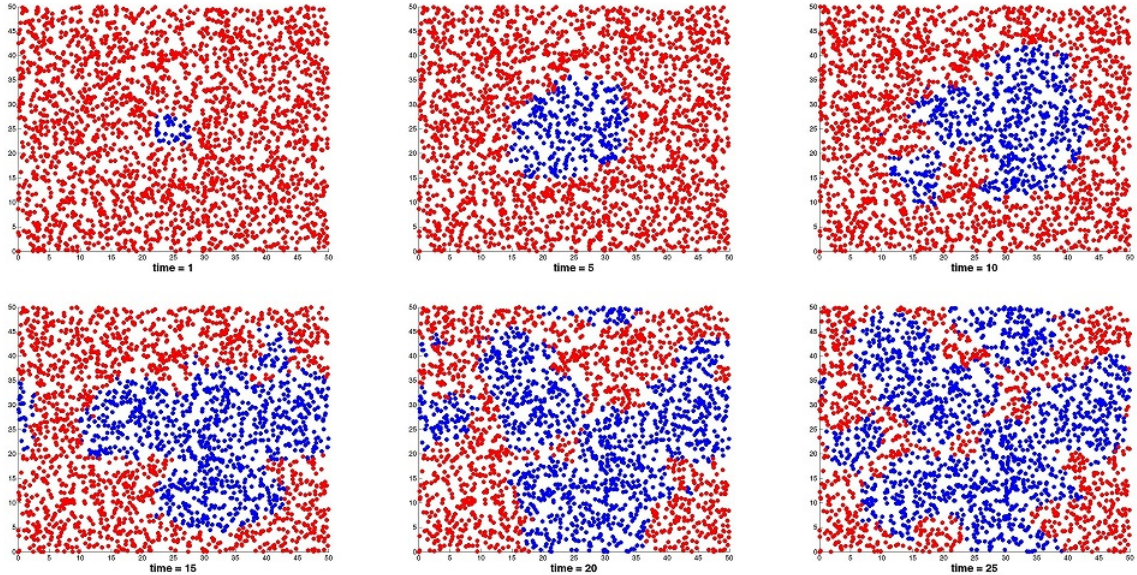


Figure 4: Spatial distribution of cooperators (blue) and defectors (red) on the domain of size  $L \times L$ ,  $L=50$  at time  $t=1,5,10,15,20,25$  with the interaction radius  $r=4$ . Initially we put a group of cooperators in a square size  $4 \times 4$  in the middle of a defectors population. This group will invade the population of defectors and the frequency of these cooperators will go to the quasi-steady state which is the same as in the case when we initialize the population with all cooperators.



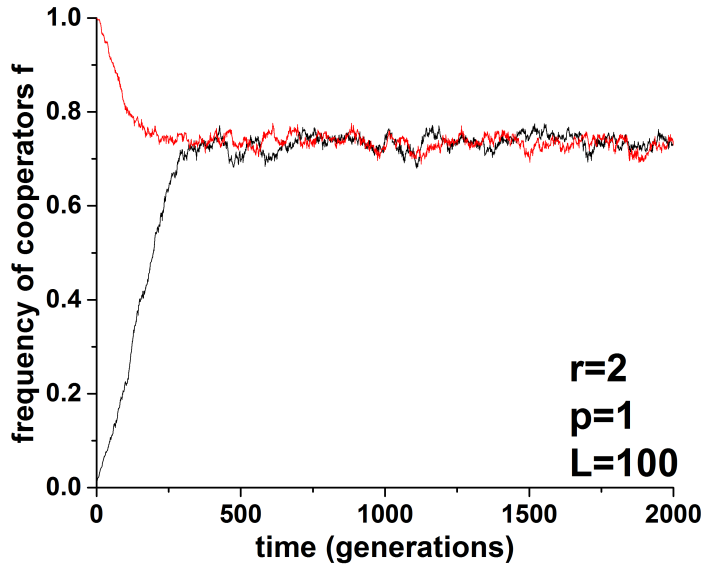


Figure 5: The frequency of cooperators  $f$  vs time on the domain of size  $L \times L$ ,  $L=100$ . The interaction radius is  $r=2$ . Two lines is for two different initial frequencies of cooperators. We can see that the frequency of cooperators will go to a quasi-steady state which is independent from initial condition.

or many quasi-steady states for one set of parameters? What is the dependence of it on the parameters of the system?

First, at small level of diffusion  $d$ ,  $d=0.1$ , we find that the system really go to a quasi-steady state and there seem to be only one quasi-steady state for each set of parameter. The frequency of cooperators will go to this quasi-steady state, regardless of the initial frequency of cooperators (Fig 5).

This quasi-steady states does not depend on the domain size/population size (Fig. 6). Remember that we keep the density of player to be one player per unit area. The population size is equal to  $L^2$ . It is only that when  $L$  is low, the fluctuation is large. This fluctuation can cause the extinction of cooperators.

In figure 7, we plot the average of the frequency of cooperators at the quasi-steady state vs the interaction range  $r$ . The average frequency decrease when the interaction range increase. The larger the interaction range, the larger the fluctuation. At large  $r$ , the groups of defectors or cooperators travel in the area very fast. The local effects reduce and this is why the average frequency of cooperators will decrease.

In figure 8, we plot the average of the frequency of cooperators at the quasi-steady state vs the cheating advantage value  $b$ . When cheating is rewarded by larger score, the frequency of cooperators decrease. This is easy to understand. At  $b$  larger than 1.75, we see that the frequency of cooperators for  $r=2$  is smaller than the frequency for  $r=4$ . It is because that at  $r=2$  at these values of  $b$ , the behaviour of the system is very similar to itself with  $r=1$ . The system is static and the diffusion is the main cause of the extinction of cooperator.

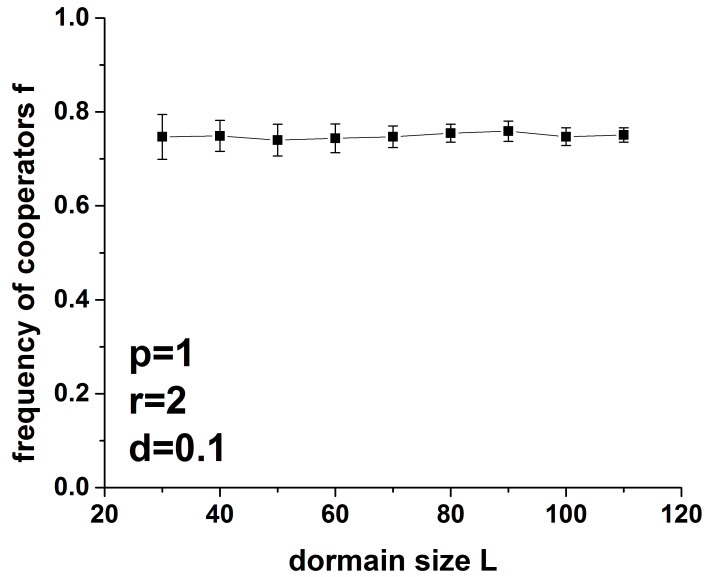


Figure 6: The average of the frequency of cooperators  $f$  in the quasi-steady state vs the domain size  $L$ . The population size is  $L^2$ . The interaction range  $r$  is 2. The average value of the frequency does not depend on the population size. It is only that when the population size is small, the fluctuation is large.

Now things will become more complicated at large value of diffusion. At small value of interaction range and small diffusion, the behaviour of the system is normal. There is still only a quasi-steady state and the dependence of this quasi-steady state on the diffusion length  $d$  is in figure 9. When  $r$  is small, the frequency of cooperators depends strong on diffusion. At large  $r$ , the dependence on diffusion decrease. Diffusion reduce the spatial effects that give advantage to cooperators. The larger the diffusion, the smaller the frequency of cooperators.

Now, at larger value of interaction range ( $r=4, r=6$ . Fig 10). The quasi-steady state degenerates. There are many quasi-steady state that the system can stay. The frequency of cooperators stay in this value for a period of time and it can jump to another state (Fig 10). There is one usual steady-state at  $f$  around 0.5. But there are some steady state at small  $f$ . These steady state has small value of  $f$  and small fluctuation. The system is quite static at this steady state. The frequency of cooperators can jump between these steady states. We can not see this at  $r=8$ . At  $r=8$  the system just fluctuate around the "normal" steady state or go extinct. We believe that it is because of the size of population. If the population size is larger, we can see this kind of behaviour at  $r=8$ . We still don't understand what is happening at this situation.

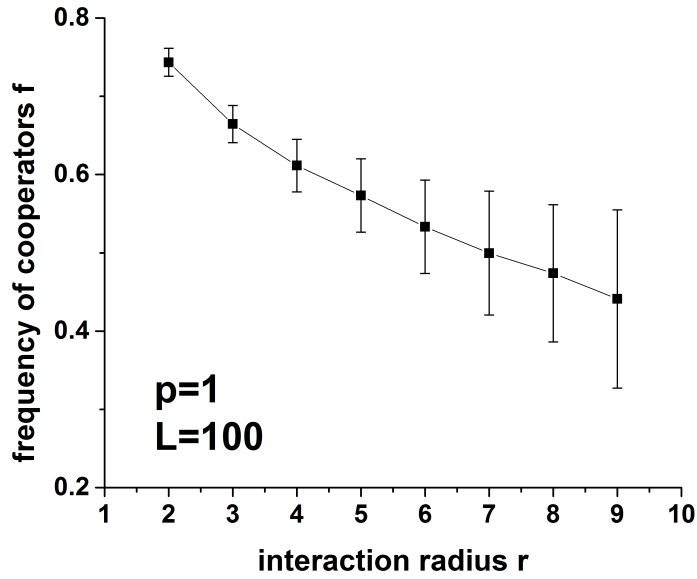


Figure 7: The average of the frequency of cooperators  $f$  in the quasi-steady state vs the interaction range  $r$ . The vertical bar is the standard deviation. We can see that the larger the interaction range, the lower the frequency of cooperators and the larger the fluctuation.

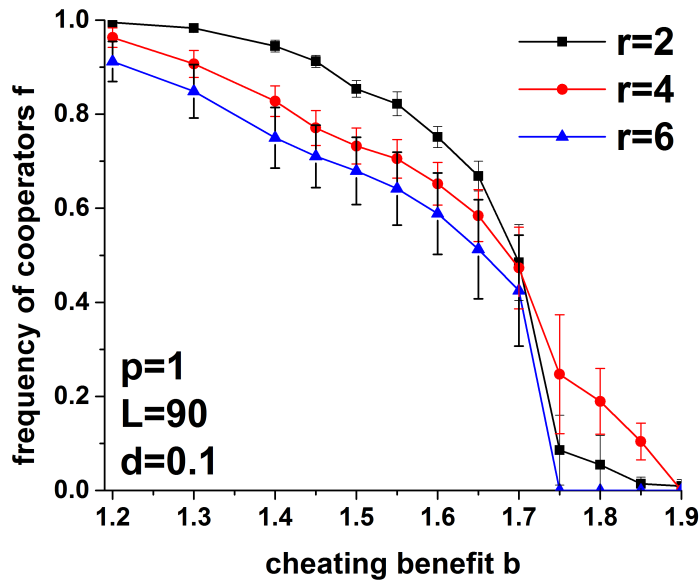


Figure 8: The average of the frequency of cooperators  $f$  in the quasi-steady state vs the cheating advantage value  $b$ . The vertical bar is the standard deviation. The frequency of cooperators depends strongly on the value of  $b$ . When the advantage of defection increase, the frequency of cooperators decrease. The fluctuation also increase. The system becomes more chaotic at large value of  $b$ .

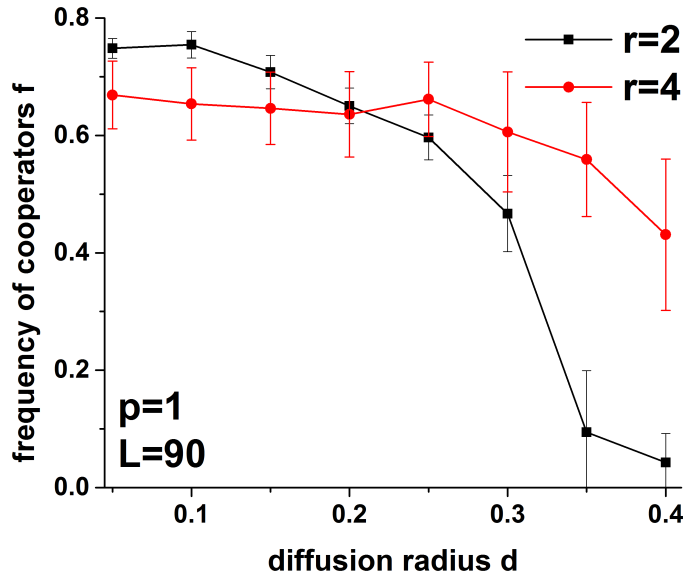


Figure 9: The average of the frequency of cooperators  $f$  in the quasi-steady state vs the diffusion radius  $d$  at small value of interaction and diffusion. The vertical bar is the standard deviation. Diffusion reduce the spatial effects that give advantage to cooperators so the frequency of cooperators decrease. The effect of diffusion is larger at small interaction range. At large level of diffusion, the frequency of cooperators at large interaction range can be larger than the frequency at small interaction range.

### 3.2 The evolution of cooperation with stochastic update rule

Now, we introduce some stochasticity into the update rule. In this case, even if the player has a smaller total score than its neighbours, there is a probability  $1-p$  that it still keep its strategy. It means that the probability of spreading the gene of the superior neighbour is  $p$ .

As the previous section, first, we study the evasion of defectors. In Fig. 11 we plot the evolution of the frequency of cooperators vs time. We see that life is harder for the cooperators. This stochastic update rule reduce the advantage of spatial effects for cooperators. The frequency of cooperators is lower in this case. Stochastic update rule favours defectors. When  $r$  is larger than 3, the cooperators die out quickly. In Fig. 12 we plot the frequency of cooperators vs time for different value of  $p$ . We see that the more stochastic the update rule (the lower the value of  $p$ ) is, the lower the frequency of cooperators at the quasi-steady state. It is clear that the stochastic update rule reduce the spatial segregation that gives advantage to cooperators. The deterministic update rule give more advantage to cooperators.

Similar to the deterministic update rule, in this case, a group of cooperators can also invade a population of defectors. The frequency of cooperators will go up to the quasi-steady state which is the same as when we begin with a population of only cooperators (Fig. 13). The quasi-steady state of the system does not depend on the initial condition.

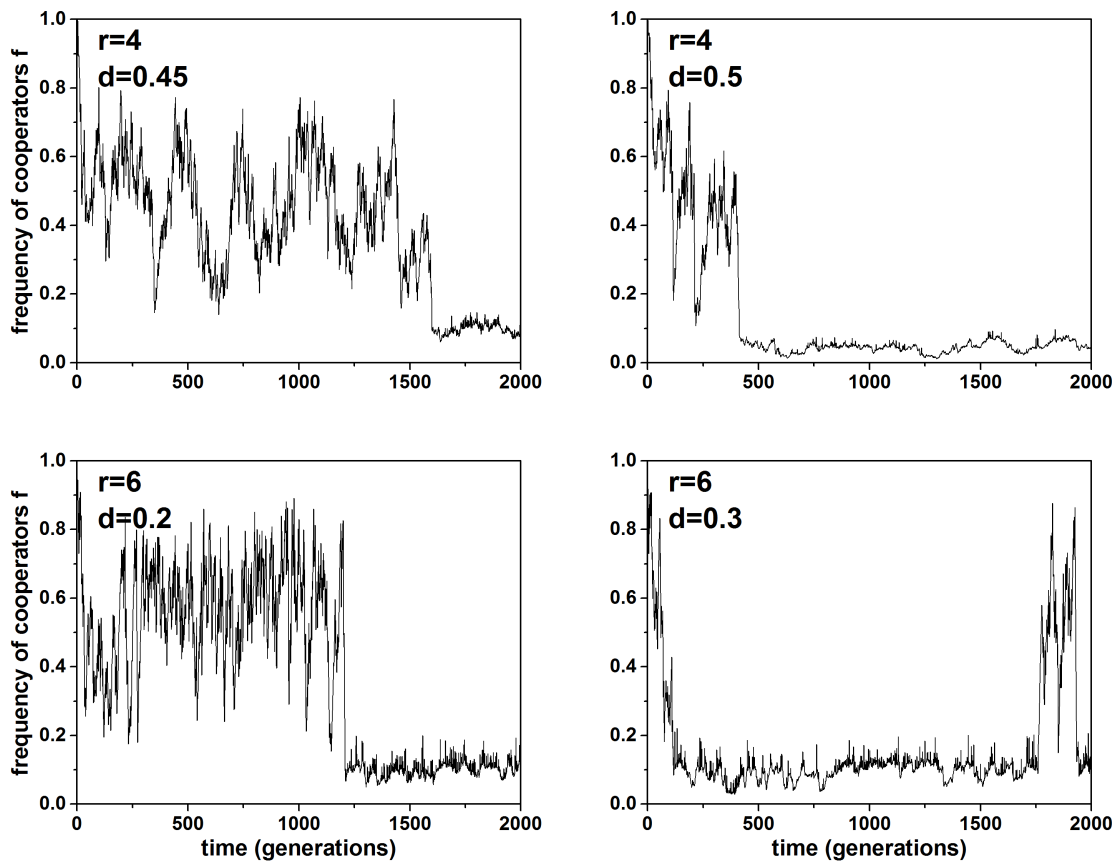


Figure 10: The frequency of cooperators  $f$  vs time at large interaction range and diffusion radius. The evolution of frequency become more complicated. The quasi-steady state degenerate into basically two level of . Now beside the "normal" steady state around  $f=0.5$  there are other quasi-steady states at small  $f$ . The system can jump between these steady states. At larger  $r$  ( $r=8$ ), we can not see this.

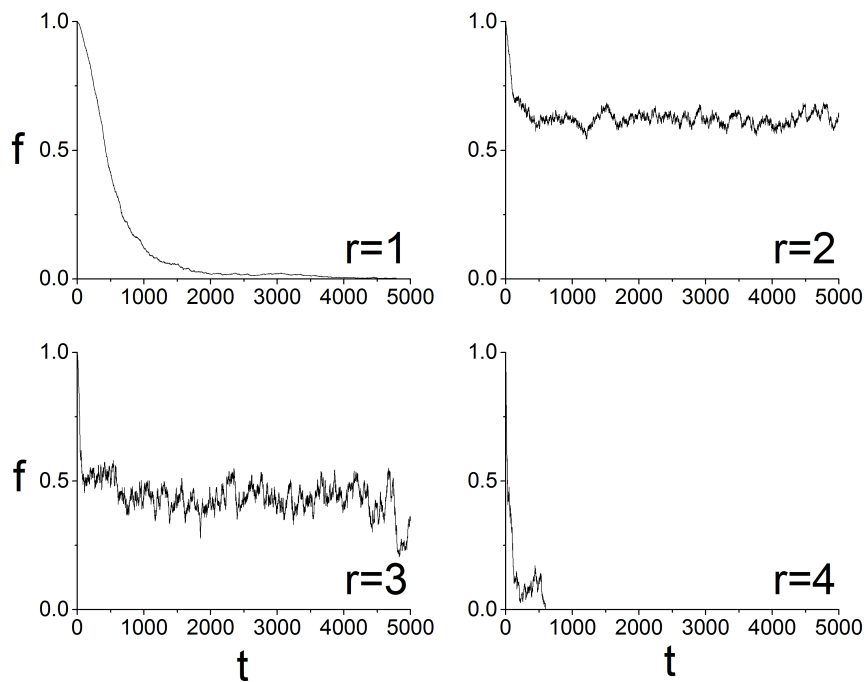


Figure 11: The frequency of cooperators  $f$  as a function of time  $t$  with stochastic update rule with  $p=0.9$ . The interaction radius  $r$  from 1 to 4. The size of the domain is  $L=100$ . The behaviour of the system is similar to the case when we use deterministic update rule but life is harder for the cooperators now. The frequency of cooperators is lower. This stochastic update rule reduce the clustering advantage of cooperation.

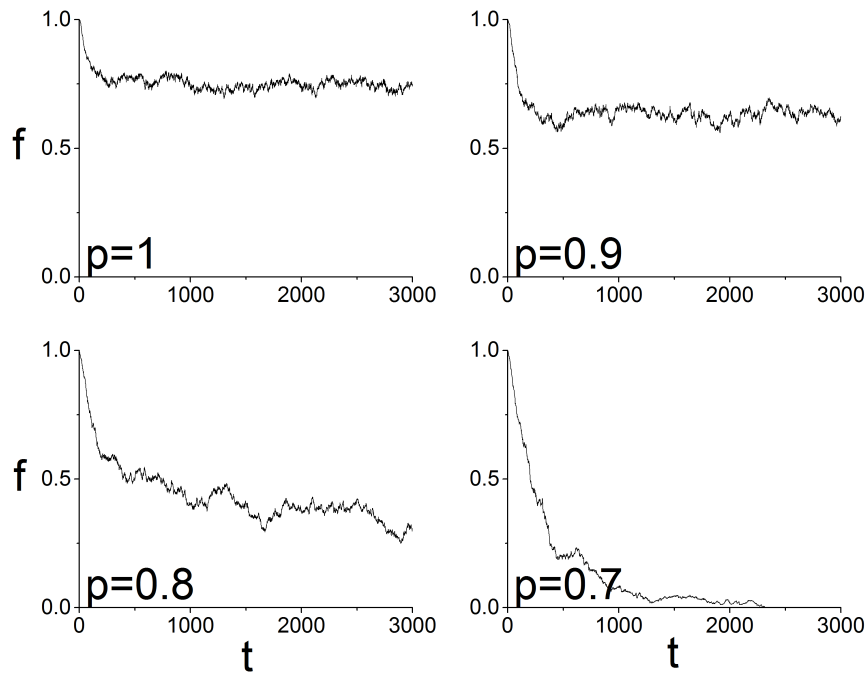


Figure 12: The frequency of cooperators vs time with different values of  $p$  on the domain of size  $L \times L$ ,  $L=100$ . The interaction radius is  $r=2$ . The larger the value of  $p$ , the lower the frequency of cooperators in the population. The stochastic update rule destroy the clustering advantage of cooperation. If the value of  $p$  is too small ( $p < 0.7$ ) defectors can take over the population.

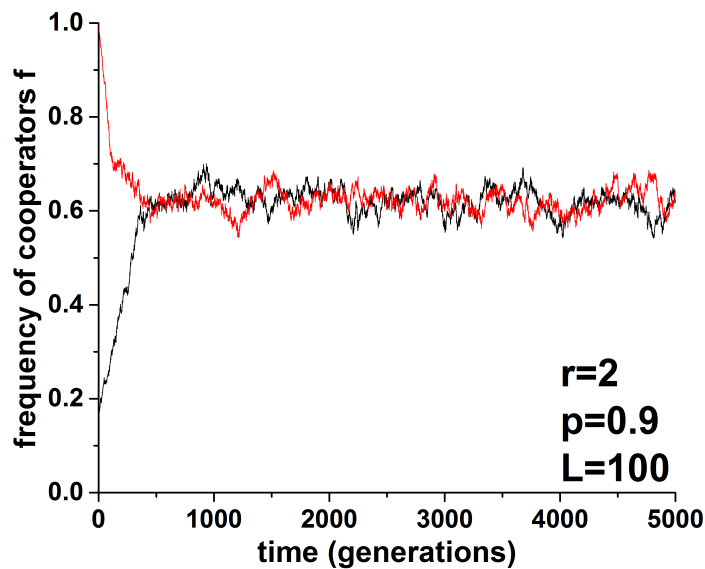


Figure 13: The frequency of cooperators  $f$  vs time on the domain of size  $L \times L$ ,  $L=90$ . The interaction radius is  $r=2$ . Two lines is for two different initial frequency of cooperators. We can see that, as in the case when we use deterministic update, the frequency of cooperators will go to a quasi-steady state which is independent from initial condition. It is only that this quasi-steady state has lower frequency of cooperators.

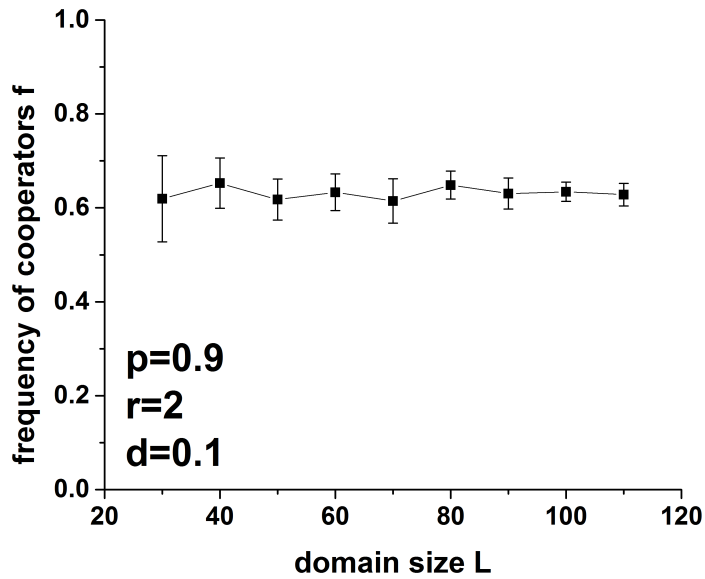


Figure 14: The average of the frequency of cooperators  $f$  in the quasi-steady state vs the domain size  $L$ . The population size is  $L^2$ . The average value of the frequency does not depend on the population size. It is only that when the population size is small, the fluctuation is large. The fluctuation is larger when we use stochastic update rule.

### 3.2.1 The quasi-steady state

The same as in the previous section with deterministic update rule, at small level of diffusion, there is a quasi-steady state that does not depend on the initial condition and the population size (the domain size) (Fig. 14). The average frequency depends strongly on the cheating advantage  $b$  (Fig. 15). The frequency now depends very strongly in the interaction range  $r$ . At large value of interaction range, it is very difficult for cooperators to survive. Clearly, stochastic update rule does not favour cooperators. In figure 16, I plot the frequency of cooperators vs the diffusion distance  $d$ . The behaviour of the system is very similar to the system with deterministic update. We can't not see the degeneration of steady states in this case.



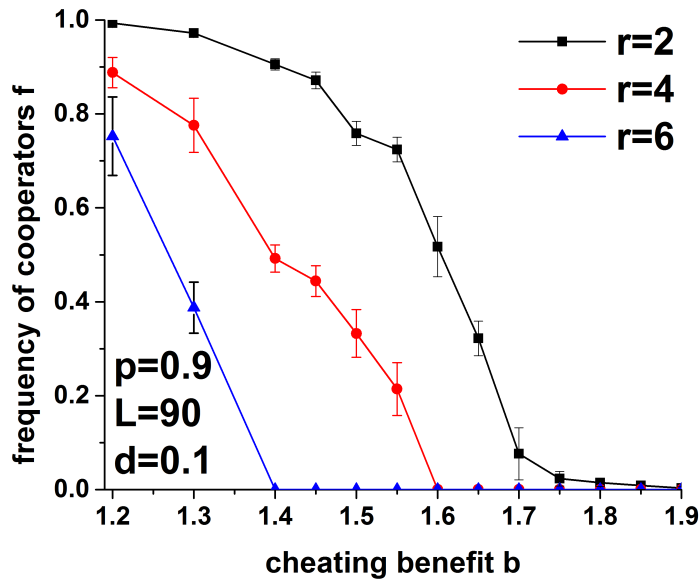


Figure 15: The average of the frequency of cooperators  $f$  in the quasi-steady state vs the cheating advantage value  $b$ . The vertical bar is the standard deviation. We can see that the larger the interaction range, the lower the frequency of cooperators. It is very hard to maintain cooperation at large interaction range.

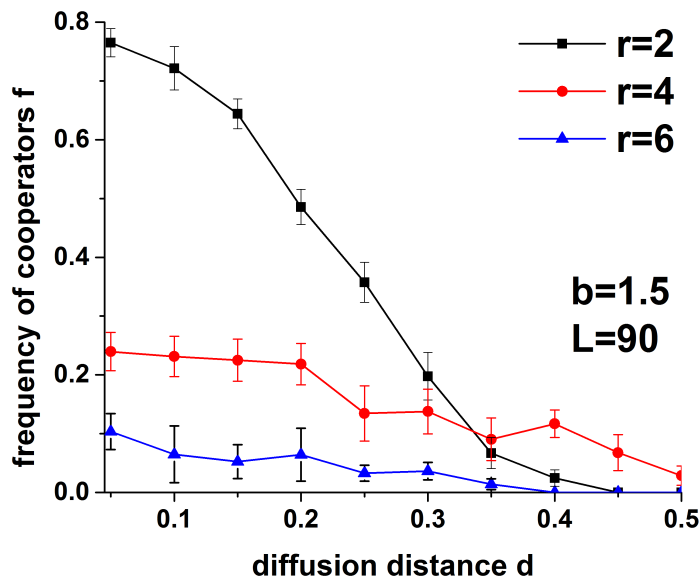


Figure 16: The average of the frequency of cooperators  $f$  in the quasi-steady state vs the diffusion radius  $d$ . The vertical bar is the standard deviation. The size of the domain is  $L=90$ . Diffusion reduce the spatial effects that give advantage to cooperators. The effect of diffusion is larger at small interaction range. The behaviour of the system is very similar to the case with deterministic update.

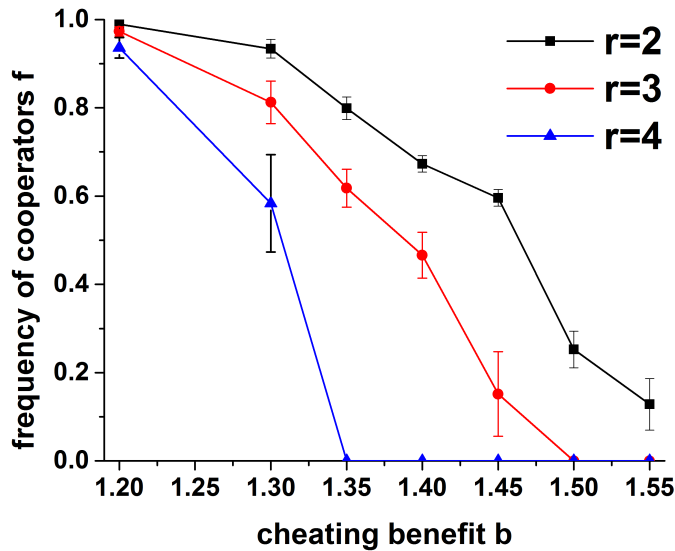


Figure 17: The average of the frequency of cooperators  $f$  in the quasi-steady state vs the cheating advantage value  $b$ . The vertical bar is the standard deviation. We can see the qualitative similarity between the result of synchronous and asynchronous update. Now the frequency reduces when  $b$  increase and it depends strongly on the interaction range.

### 3.3 Continuous time

First we study the system with deterministic update rule. using the same parameter as in synchronous updating case we find out that eventually, cooperators will die out. This seem to suggest that there is a huge difference between discrete and continuous time. The cooperators can't survive in a continuous model. But when we look at the dynamics of the system with different values of  $b$  we can see a qualitative similarity between continuous and discrete time model.

Fig. 17 is the plot of the frequency of cooperators vs  $b$  with deterministic update. We can see that at  $b$  larger than 1.55, cooperators can not survive. The shape of the graph look very similar to the graph we get when we use synchronous update. It is very clear that synchronous update destroy cooperators. Because of this scheme of updating, the group of cooperators can not move together in the space and this is not good for the cooperators. The frequency depends very strongly on interaction range.

The figure 18, we summarize the result using the color coding.

## 4 Conclusions

Using this stochastic spatial model for one-shot prisoner's dilemma game, we have observed the coexistence through space time chaos of cooperators and defectors and the invasion of a group of cooperators in a sea of defectors. These phenomena has no correspondence in the traditional game model when all pairwise interaction are included.

The local interaction give advantage to cooperators. Spatial aggregation enables more interaction with others cooperators in the cluster and reduce the exploitation by defectors. Clusters of cooperators can resist the attack of defectors.

We have shown that there is a range of interaction range  $r$  that help to maintain the existence of cooperators in the population. If the interaction range is too small, there is not enough interaction between cooperators to help them overcome the exploitation of defectors and because of diffusion, cooperators will slowly meet other defectors and will slowly become extinct. When the interaction range increase, the clustering effect, which give the advantage to cooperators, will become weaker and the frequency of cooperators will reduce. At large interaction, both groups of cooperators or defectors can travel quickly in the space and the fluctuation become large. Even though the evolution dynamic does not favour cooperation, cooperators and defectors can coexist for a very long time. Because of the fluctuation in finite population there is a probability that the system can go to fixation but this may take a very long time to reach. There can be only fixation of defectors because of mutation. Mutation can prevent the fixation of cooperators but it can not prevent the fixation of defectors. One cooperator can never survive in a population of defectors.

A group of cooperators in a favourable condition can invade a population of defectors. At small value of diffusion, The percentage of cooperators will fluctuate around a quasi-steady state, which is independent of the initial frequency of cooperators. This quasi-steady state does not depend on the population size. It depends strongly on the cheating advantage  $b$  and the interaction radius  $r$ . When  $r$  is small, the clusters of cooperators is small and diffusion can cause the extinction of cooperators by slowly make cooperators meet defectors. When  $r$  is too large, the spatial effects that give advantage to cooperators reduce, the frequency of cooperators reduce, the fluctuation is large and the dynamics of the system become more chaotic. At large diffusion and large interaction range, the quasi-steady state degenerate. There are many quasi-steady states and the frequency of cooperators can jump between these quasi-steady states.

Diffusion reduces the spatial segregation which gives advantage to cooperators. It makes the system more homogeneous. The larger the diffusion, the lower the frequency of cooperators. Diffusion has a very strong effect at small value of  $r$ . At  $r$  small, diffusion is the main cause of the extinction of cooperators. At larger  $r$ , the effect of diffusion reduce. At large value of diffusion and interaction range, the system become very complicated. The quasi-steady state degenerates. The frequency of cooperators can stay jump between these quasi-steady states.

Deterministic update rule favour cooperators. The frequency of cooperators will reduce if we use stochastic update rule. Stochastic update rule. When the interaction range  $r$  is large, stochastic update rule has a tremendous effect on the dynamics of the system (Fig 6). It is very difficult to maintain cooperation at large value of  $r$  if we use stochastic update rule.

When we go from synchronous updating (discreet time) to asynchronous updating (continuous time), it is more difficult to maintain cooperation. Asynchronous update cause the mixing of cooperators and defectors in case when there are only one group if we

use synchronous update. This will reduce the spatial segregation and reduce the frequency of cooperators. But there are still a range of the cheating advantage parameter  $b$  and the interaction range  $r$  that give rise to a chaotic spatial coexistence between cooperators and defectors. The frequency of cooperators depends very strong on interaction range. Cooperation can still be maintain in this case.

The result we get here is very similar to Nowak's result. Without diffusion, he found out that the final frequency will depend on the initial condition. With small diffusion level, we find the quasi-steady state around which the frequency of cooperators will fluctuate. We investigate this quasi-steady state and study the dependence of this quasi-steady state on the parameters of the system:  $r, b, d$ . But the conclusion is the same. The spatial effect can help to maintain cooperation in the population.

In conclusion, in this project, we characterise the conditions in which there are a coexistence between cooperators and defectors in a diffusive spatial prisoner's dilemma game. We extent the spatial model of Nowak to include irregular space, diffusion of players and mutation. We found out that local interaction in this chaotic model can still give rise to the coexistence between cooperators and defectors in a suitable range of the cheating advantage  $b$  and interaction range  $r$ . Spatial effects promote the variety of strategy in the population.

## Acknowledgements

I would like to express my gratitude to my supervisor, Prof. Kristian Lindgren, for his idea, his valuable comments and support for this project. I am also very grateful to Oskar Lindgren for his comments and advices during the development of this project.

## References

- [1] Robert Axelrod. The evolution of cooperation. Basic Books, New York, 1984.
- [2] Hubermann, B.A. and Glance, N.S. Evolutionary games and computer simulations. PNAS, 90 (1993).
- [3] Philipp Langer, Martin A. Nowak, Christoph Hauert. Spatial invasion of cooperation. Journal of Theoretical Biology 250 634-641, 2008.
- [4] K. Lindgren and M. G. Nordahl. Evolutionary dynamics of spatial games. Physica D, 75: 292309, 1994.
- [5] Martin A. Nowak and Robert M. May. Evolutionary games and spatial chaos. Nature, 359: 826829, October 1992.
- [6] M. A. Nowak and Robert M. May. The spatial dilemmas of evolution. International Journal of Bifurcation and Chaos, 3(1):3578, 1993.

- [7] Martin A. Nowak, Sebastian Bonhoeffer and Robert M. May. More spatial games. *International Journal of Bifurcation and Chaos*, Vol. 4, No.1 33-56, 1994.
- [8] Martin A. Nowak, Sebastian Bonhoeffer and Robert M. May. Spatial games and evolution of cooperation.
- [9] Martin A. Nowak. *Evolutionary dynamics, exploring the equation of life*. Harvard University Press, 1994.
- [10] Martin A. Nowak. Five Rules for the Evolution of Cooperation. *Science* Vol. 314 8, 2006
- [11] A. Rapoport and A. Chammah. *Prisoners dilemma*. University of Michigan Press, Ann Arbor, 1965.
- [12] Frank Schweitzer, Laxmidhar Behera, Heinz Muhlenbein. Evolution of cooperation in a spatial prisoner's dilemma. *Advances in Complex Systems*, vol.5, no 2-3, pp. 269-299, 2002.
- [13] J. Maynard Smith. *Evolution and the theory of games*. Cambridge University Press, UK, 1982.

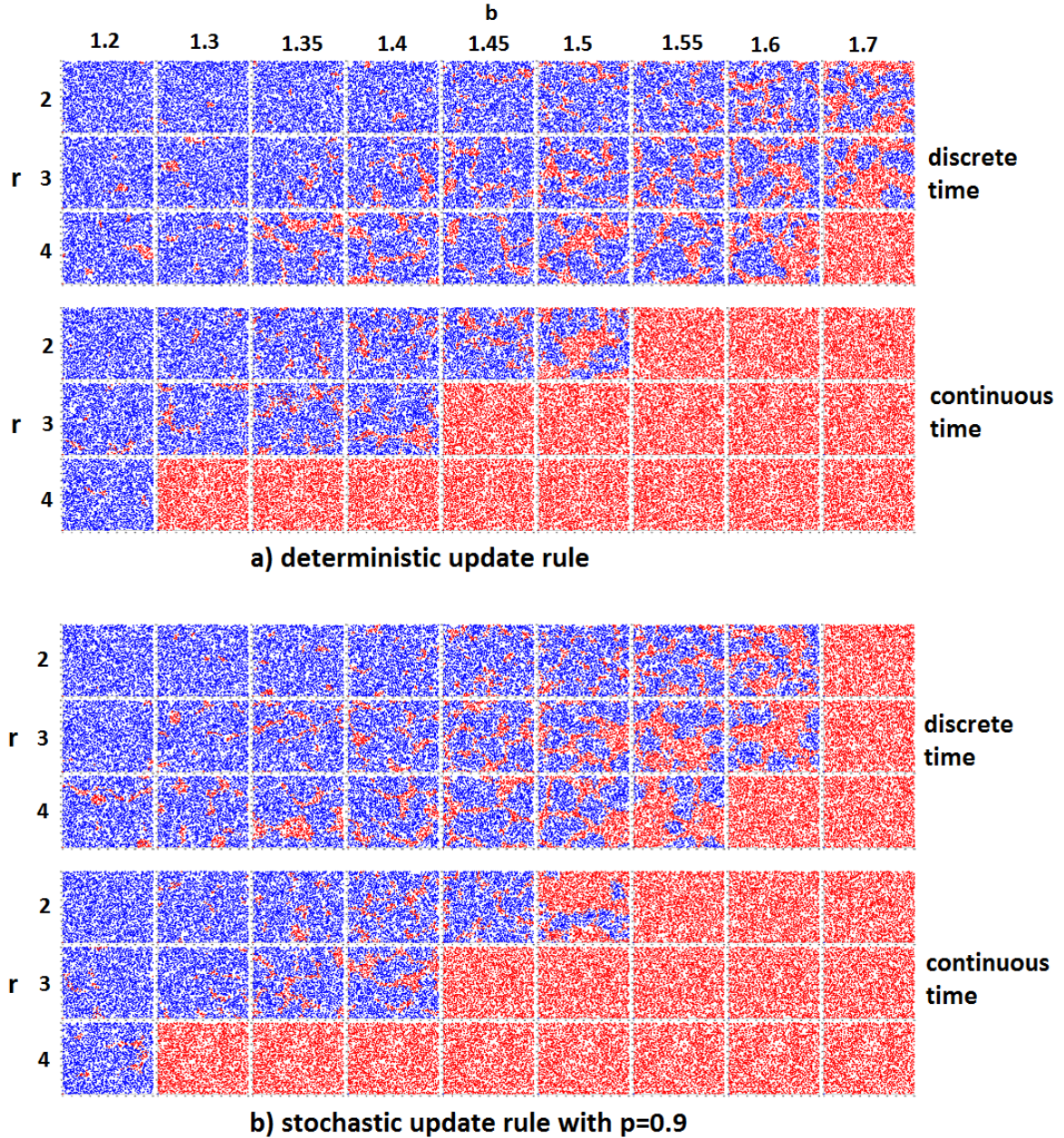


Figure 18: The comparison of spatial distribution of cooperators (blue) and defectors (red) at time  $t=2000$  in discrete time and continuous time simulation. The part (a) (upper) is for deterministic update rule and the part (b) (lower) is for stochastic update rule with  $p=0.9$ . The domain is of size  $L \times L$ ,  $L=50$ . We use different interaction radius  $r=(2,3,4)$ . Mutation rate  $m$  is  $10^{-4}$ . For each part the upper is for discrete time simulation and the lower is for continuous time simulation. The row show the value of interaction range  $r$ . For continuous time simulation, one unit of time is the time it takes to update the strategy of  $N$  random players. The column show the parameter  $b$  which is the payoff that a defector gets when it interacts with a cooperator. There are eight values of  $b$ :  $b=(1.2,1.3,1.35,1.4,1.45,1.5,1.55,1.6,1.7)$ . Although the quantitative behaviour of discrete time and continuous time simulation are different, we see that there is no cooperation at  $b=1.6$  with any value of  $r$  in continuous time simulation, there is a qualitative similarity between discrete time and continuous time. The dynamics of the system is shifted to the right when we go from discrete to continuous time simulation. The essential conclusion about the existence of chaotic spatial coexistence between cooperators and defectors is still true for continuous time.