PERCOLATION

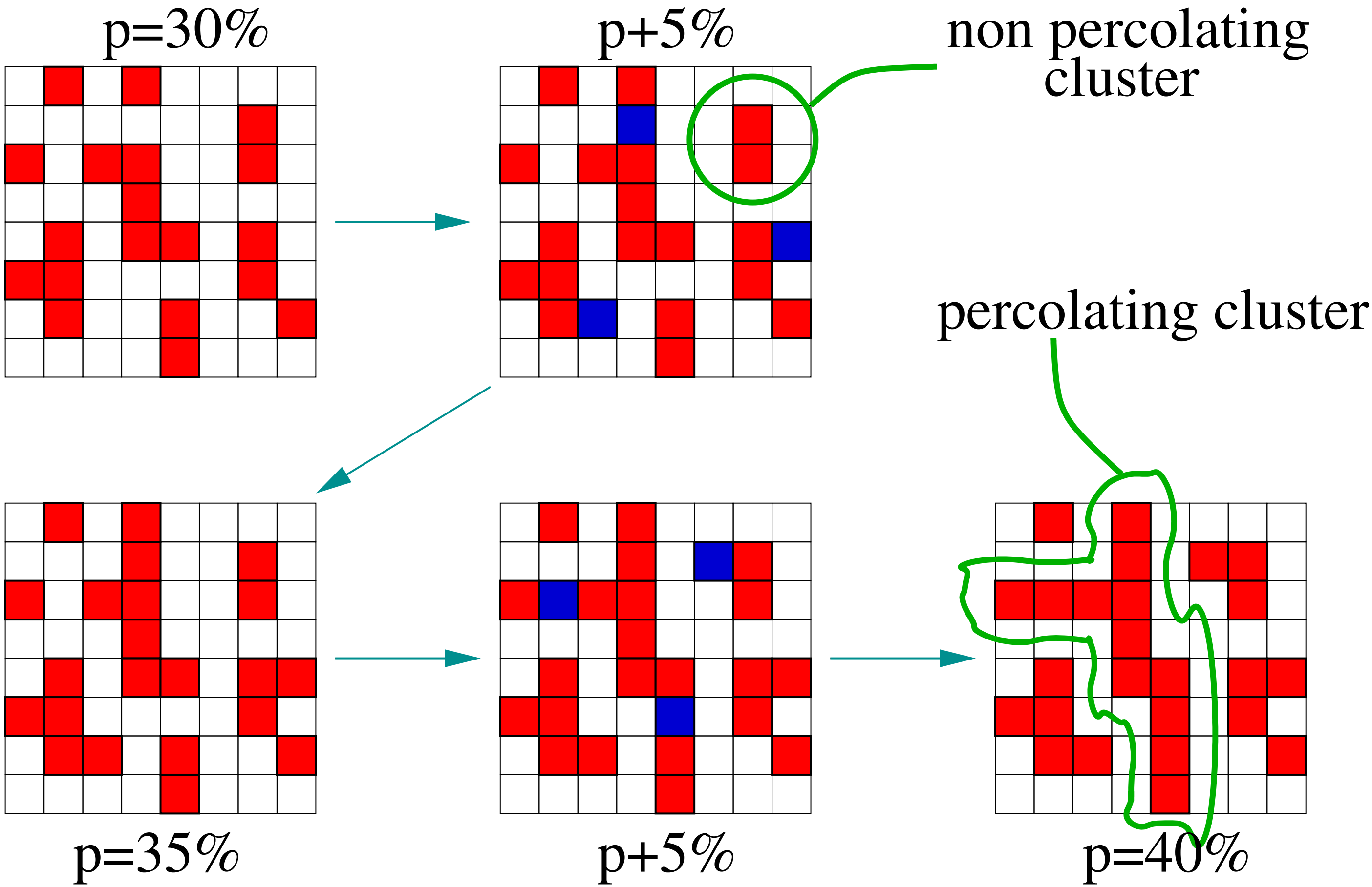
Mario Nicodemi

Complexity Science & Theor. Phys., University of Warwick"

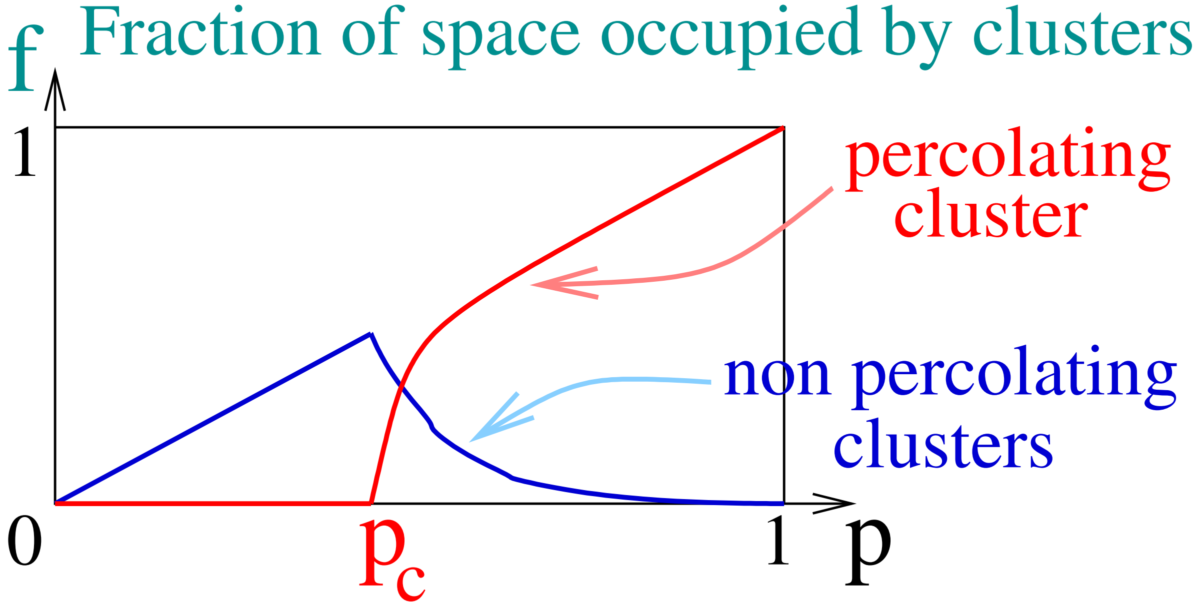
Topic:

- Basic concepts in **percolation theory**;

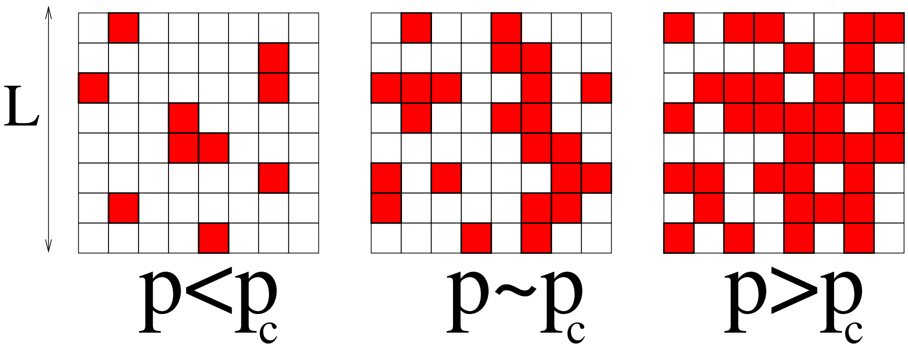
□ Percolation



□ **Normal and percolanting clusters**



☐ Critical point

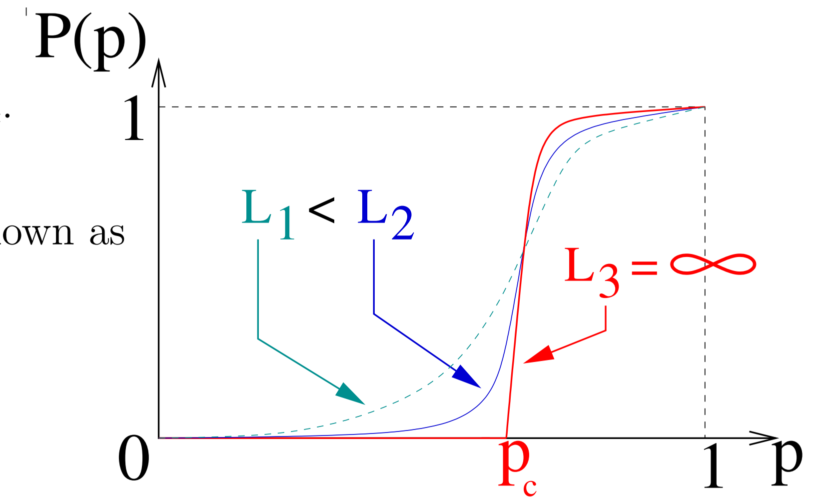


The fraction of filled sites is named p.

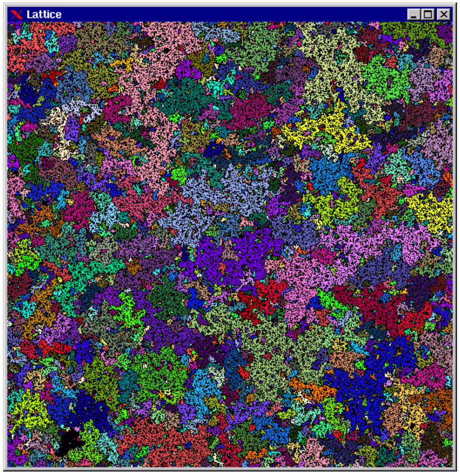
The *percolation probability*, $P(p)$, is defined as the probability that a "cluster" of filled sites spans the whole lattice.

If $L \longrightarrow \infty$, $P(p) = 0 \ \forall p < p_c$.

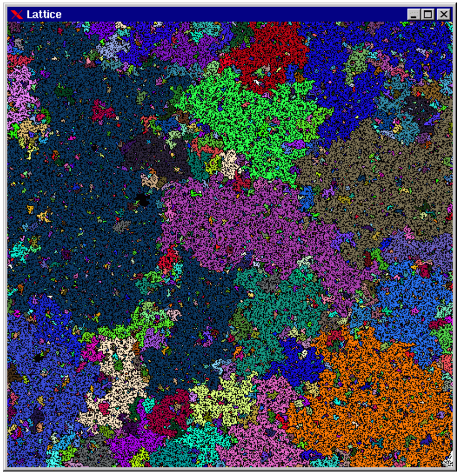
The critical threshold p_c is known as **percolation point**.



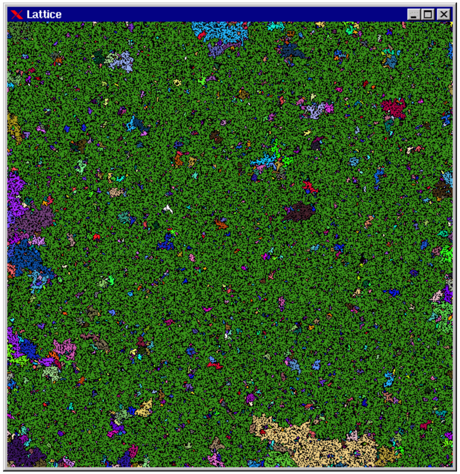
□ Percolation threshold



p=0.57



p=0.59

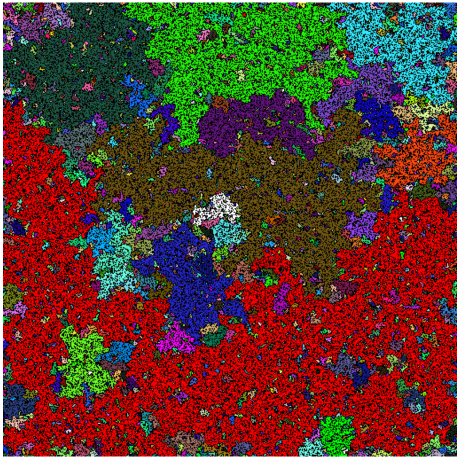


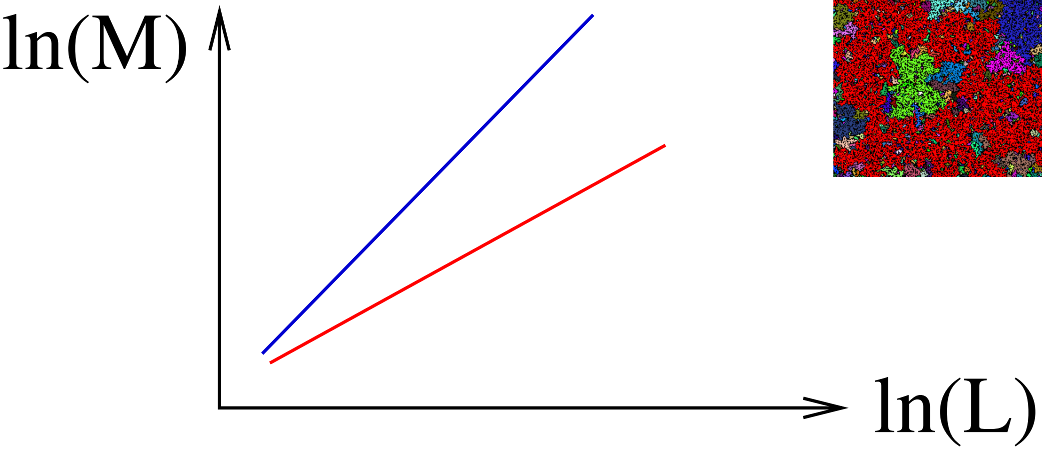
p=0.61

☐ Percolanting cluster

The "mass", M, of percolantion cluster is defined as the total number of sites belonging to it, or, equiv., its area. M scales as:

- if $p > p_c$: $M \sim L^2$
- if $p = p_c$: $M \sim L^D$ (e.g. $D \simeq 1.89$ in $d = 2$)
- if $p < p_c$: $M = 0$



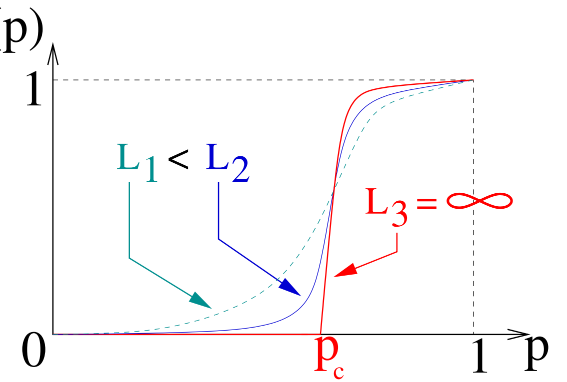


□ Summarising

- The percolation probability $P(p)$ is

  - for $p < p_c$: $P(p) = 0$
  - for $p \geq p_c$: $P(p) = (p - p_c)^\beta$

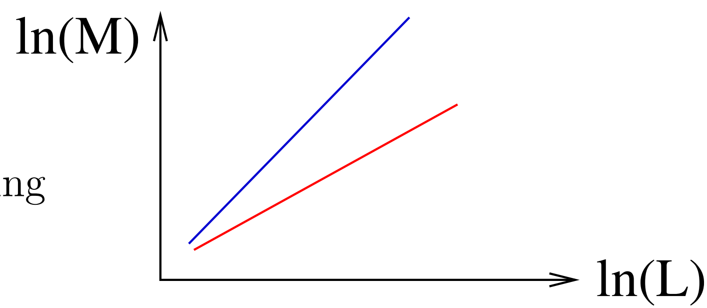
  E.g., in $d = 2$ we have $\beta \simeq 0.14$, and on a square lattice $p_c \simeq 0.593$.



- The number of sites, $M(L)$ (Mass), of the largest cluster (which for $p \geq p_c$ is the percolating one), in the limit $L \to \infty$ is:

  - per $p < p_c$: $M(L) \sim \ln L$
  - per $p = p_c$: $M(L) \sim L^D$
  - per $p > p_c$: $M(L) \sim L^d$

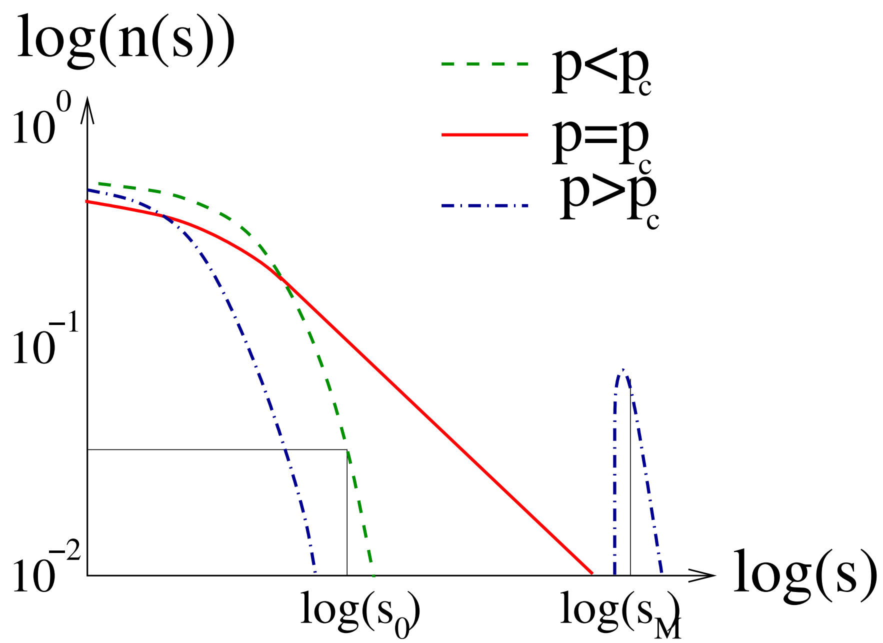
  Note that for $p = p_c$, the percolating cluster is a *fractal*. E.g., in $d = 2$ it is $D \simeq 1.89$.

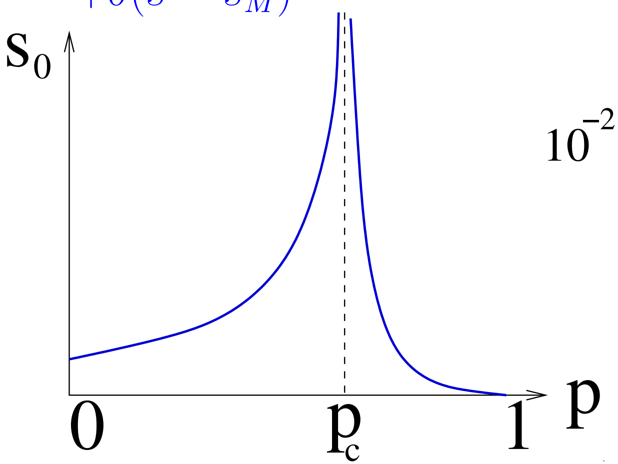


□ Cluster size distribution

The probability, to find a cluster with *size* s (i.e., with s sites) is:

- for $p < p_c$:
  $n(s) \sim s^{-\tau} \exp(-s/s_0)$
- for $p = p_c$:
  $n(s) \sim s^{-\tau}$
  with $\tau = (d + D)/D$
- for $p > p_c$:
  $n(s) \sim s^{-\tau} \exp(-s/s_0) +$
  $+\delta(s - s_M)$





☐ Scale properties

- Percolating cluster is characterised at p_c by a very important feature, named *scale invariance*:

  > if you take a subset, S', of the percolating cluster S, and blow up with a scale factor r (i.e., multiply its points coordinates by the factor r) you get a set rS which is statistically congruent ("equal") to the original set S.

  For such a reason, the percolating cluster is named **self similar**.