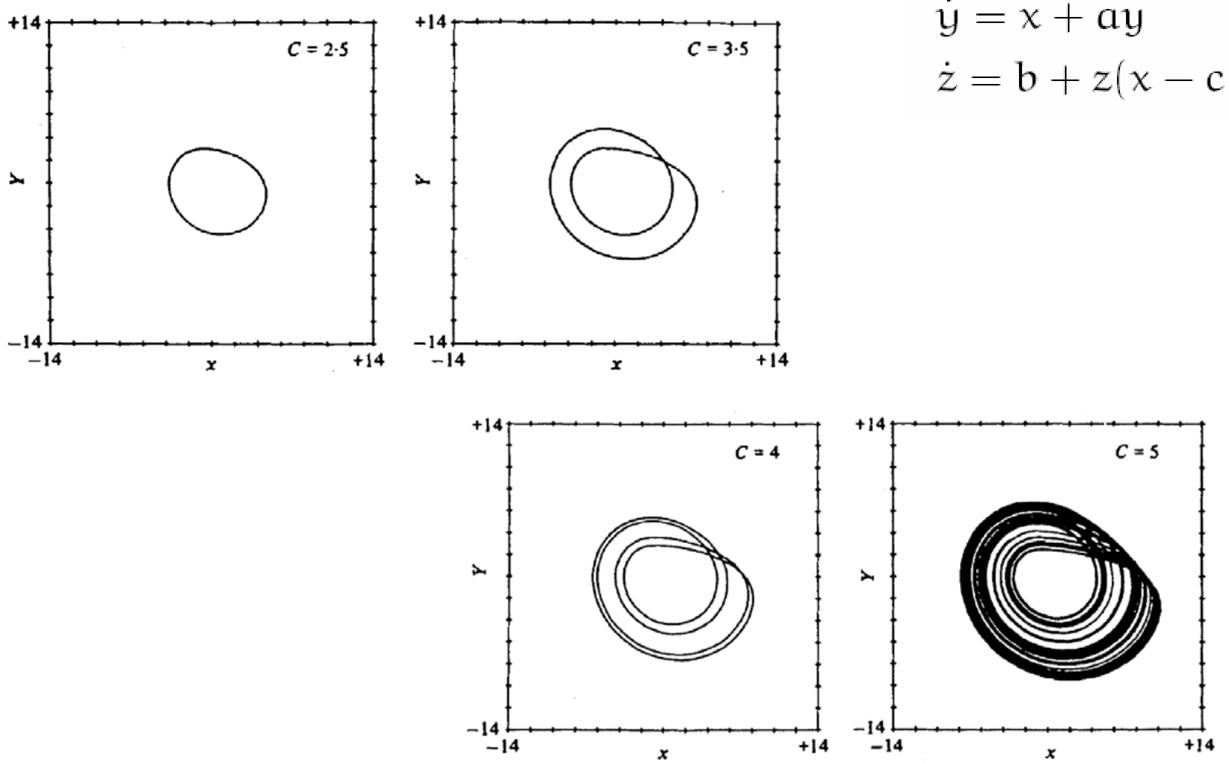
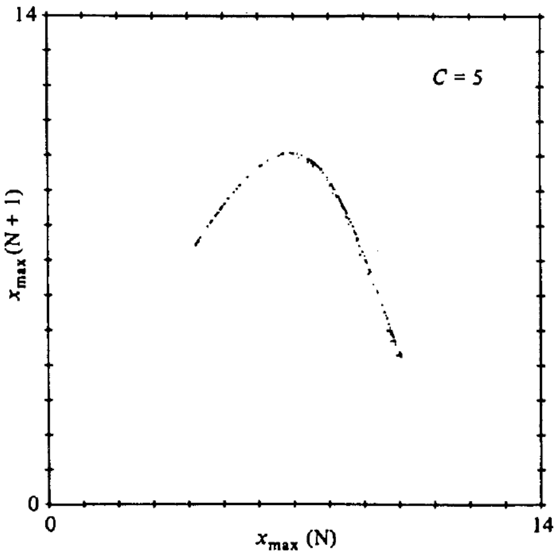
1D maps and continuous dynamics

Rossler system

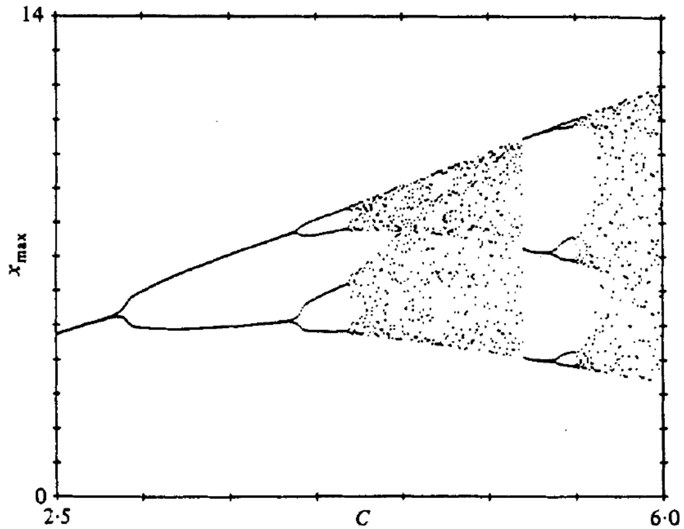
$$\dot{x} = -y - z$$
$$\dot{y} = x + ay$$
$$\dot{z} = b + z(x - c)$$



Local maxima (like in Lorenz map)



Orbit diagram



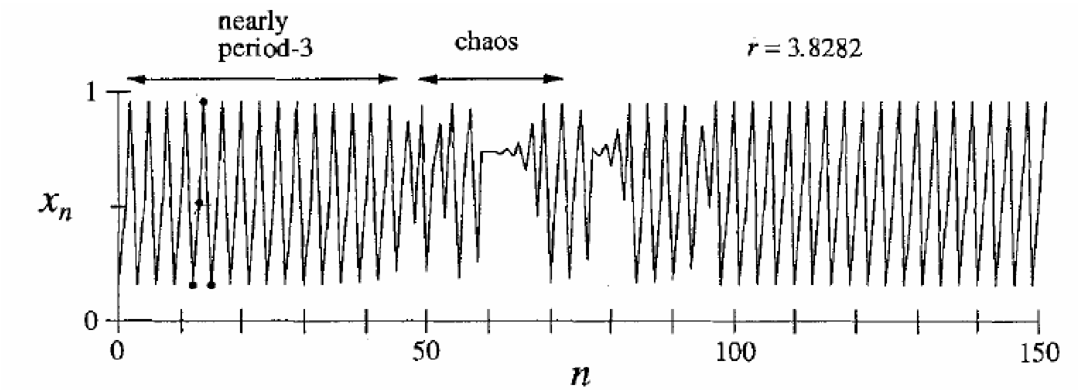
Routes to chaos

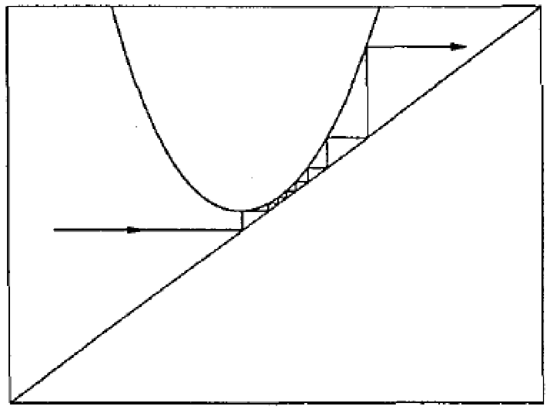
Period-doubling

$r_1 = 3$	period 2
$r_2 = 3.449...$	4
$r_3 = 3.54409...$	8
$r_4 = 3.5644...$	16
$r_5 = 3.568759...$	32
$...$	
$r_\infty = 3.569946...$	∞

$$\lim_{n\to\infty} \frac{r_n - r_{n-1}}{r_{n+1} - r_n} = \delta \approx 4.66920$$

Intermittency

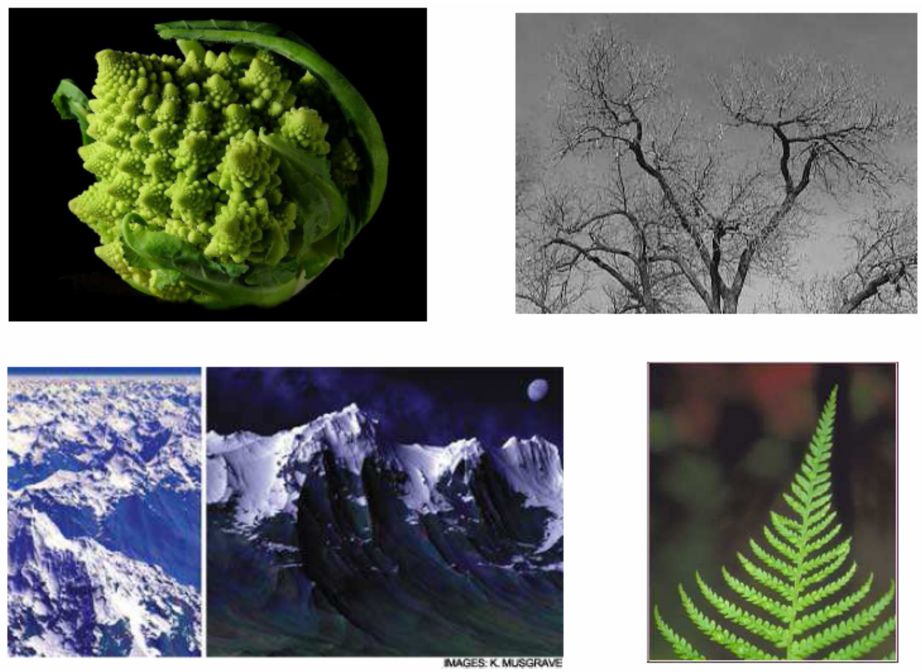




Nearly periodic motion interrupted
by occasional irregular bursts

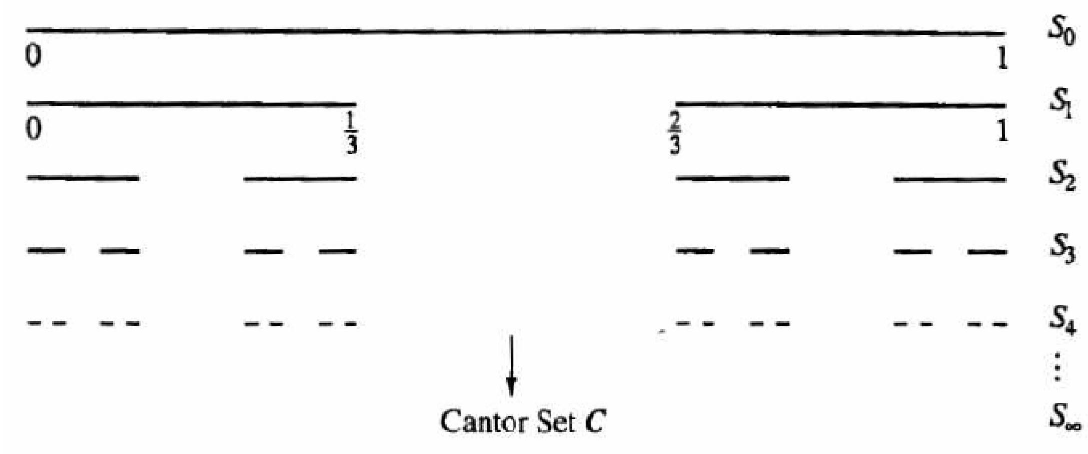
Fractals and fractal dimensions

A fractal is a complex geometric object with fine structure at arbitrarily small scales, perhaps with some degree of self-similarity



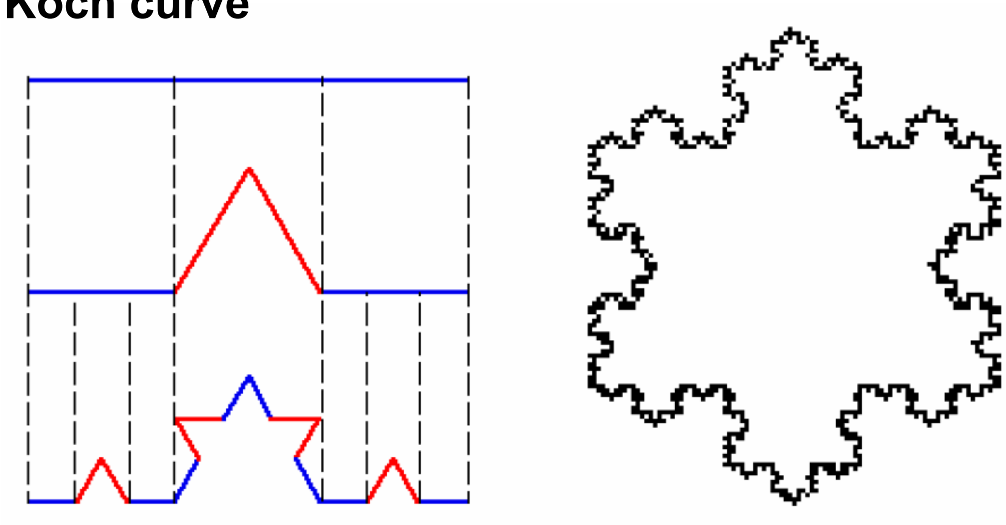
Fractals in nature

Cantor set



1. C has structure at arbitrarily small scales.
2. C is self-similar eg. the left half of S_2 is a scaled version of S_1.
3. C has noninteger dimension ($\ln 2/\ln 3 \approx 0.63$).

Koch curve



$$L_n = (4/3)^n L_0 \rightarrow \infty \text{ as } n \rightarrow \infty$$

Similarity dimension (for self-similar fractals)

Suppose that a self-similar set is composed of *m* copies of itself scaled down by a factor of *r*

$$d = \frac{\ln m}{\ln r}$$

Box dimension

Let S be a subset of $\mathbb{R}^D$

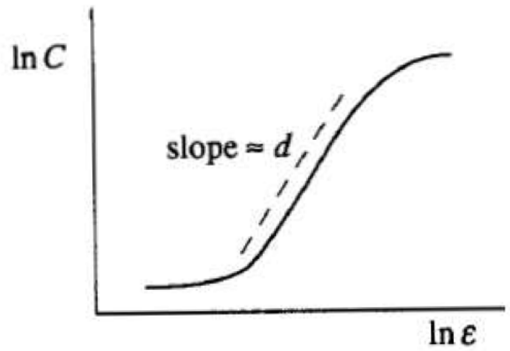
let $N(\epsilon)$ be the minimum number of D-dimensional boxes of side ϵ needed to cover S

$$d_{\text{box}} = \lim_{\epsilon \to 0} \frac{\ln N(\epsilon)}{\ln(1/\epsilon)}$$

Correlation dimension

$$N_x(\epsilon) \propto \epsilon^d$$

average $N_x(\epsilon)$ over many x $\quad C(\epsilon) \propto \epsilon^d$



For Lorenz attractor (for fixed parameters) $d = 2.05 \pm 0.01$

The fractal dimension is a statistical quantity that gives an indication of how completely a fractal appears to fill space