Problem Sheet 2

CO904
2011 Autumn

Deadline: Nov 21 (Mon) 10:00am

Marks will be given not only for the technical correctness, but also for how informative, concise and clear your answer is.

1. Calculate the Legendre transform $f^*(p)$ of the following functions. For each case, calculate $f^{**}(y)$ as well to verify that the Legendre transform is its own inverse.
   (a) $f(x) = x^{1/3}$, for $x \geq 0$
   (b) $f(x) = \sin(x)$. Also specify a suitable range of x.
   (c) $f(x) = \frac{1}{2}|x|$, for $x \in \mathbb{R}$. Relax the requirement that $f(\cdot)$ has to be *strictly* convex or concave, and use a bit your imagination.

2. Suppose X is a random variable, and $g(X)$ is its function. Using our standard notation for the Maximum Entropy framework, express
$$\frac{\partial \langle g(X) \rangle}{\partial \lambda_k}$$
in terms of $\mathrm{Cov}(g(X), f_k(X))$.

3. *(computational exercise)* Using the hints given in class, write a code in a compiled programming language (e.g. C, C++, Fortran) to simulate the ballistic deposition model. Start from a one-dimensional discrete lattice of length L with periodic boundary conditions, which has height zero initially: $h_x = 0$ for $x = 0, 1, \ldots, L-1$. In a growth step a block is released above a randomly selected position x, and as it falls down, it is glued to the first neighbouring block it encounters. The height h_x of the surface is defined as the height of the highest block above position x. The roughness or width of the surface after $t\,L$ growth steps is defined as
$$w(t) = \sqrt{\left\langle |h_x - \bar{h}|^2 \right\rangle_{x,\mathrm{ens}}}$$
where the average is both over x and over the ensemble (independent realisations), and $\bar{h} = \langle h_x \rangle_x$. For sufficiently large L and ensemble, and for $1 \ll t \ll t_\times$, w scales with t as $w(t) \sim t^\beta$. Determine the value of β empirically.

   Do your computation by submitting a job to the COW. Restrict your run to at most 10MB memory and at most around 10 minutes CPU time.

   Hand in the following:

   - the size of your simulation: L, the maximum time t (the number of growth steps per L), and the number of independent realisations. This should fit within the limits above: mention how much memory you used for the arrays, and how much CPU time it took.
   - the plot used to determine β, the range of t you used for the fit, and the value of β obtained (preferably with some indication of uncertainty).
   - printout of your code and COW submission script.