Viva topics for CO905

general MC
- definition of a Markov process, transition probabilities,
- Chapman Kolmogorov equation, solution for discrete and cont time
- stationary distributions, interpretation as left eigenvector
- existence for finite state space, irreducibility and uniqueness
- time reversal and of MC and reversible distributions, detailed balance

discrete time MC
- P-matrix, general solution of linear recursion in terms of eigenvalues
- branching processes, generating functions, survival probability
- SRW with boundary conditions as example for irreducibility

continuous time MC
- generator G, forward and backward equation
- master equation and detailed balance, general solution
- interpretation of G entries, exponential holding times, jump rates
- construction of a sample path, jump chain
- Poisson process, characterization with stat indep increments
- birth death processes, queues

ergodicity
- stationary distributions and recurrence times
- ergodicity, ergodic theorem, sufficient conditions for ergodicity
- aperiodicity for discrete time, Perron-Frobenius theorem

infinite state space
- transience, positive recurrence, null recurrence
- example: RW in dimensions 1,2, 3 and higher
- explosion, birth chain

stochastic particle systems
- local jump rates and graphical construction
- master equation and evolution of observables, generator
- properties of Poisson processes and random sequential update
- contact process: definition and ergodicity, expected number of infections
  on the complete graph, derivation of mean-field rate equation,
  critical stationary behaviour, simulation results
- voter model: definition and ergodicity, number of opinion 1 persons
  (average and distribution), scaling limit to Wright-Fisher diffusion
- exclusion process: definition and ergodicity, conservation law and stationary
  current, fundamental diagram, generalization to traffic model, simulations
- the Ising model and MCMC, ergodicity, Metropolis and heat bath algorithm

Brownian motion
- scaling limit of sum of iidrv, CLT, LLN, Brownian motion
- definition of BM, characterization by covariances
- Gaussian process and properties of Gaussians, characteristic functions
- scaling limit of a SRW, Taylor expansion and heat equation
- solution to the heat equation, heat kernel
- scaling properties of Gaussians and BM

diffusion processes
- finite dimensional distributions and path properties
- transition kernels and densities, connection to fdds
- diffusion processes and the Fokker-Planck equation
- stationary solutions to Fokker-Planck with time-homogeneous coefficients
- Ornstein-Uhlenbeck process, evolution of observables, generator

Stochastic calculus
- derivation of SDE/Langevin equ from diffusion process
- solution of SDE and stochastic integration
- stochastic integrals w.r.t. time (dt)
- Ito integral w.r.t. BM (dB_t), martingale property
- computation of Ito integrals, chain rule
- Ito's formula (Taylor expansion up to dt)
- derivation of Fokker Planck equation from Ito's formula
- examples: expected values for OU process, geometric BM

non-examinable
- Perron-Frobenius for periodic chains
- proof of Polya's theorem
- Kolmogorov consistency of fdds
- Section 3.5: noise perturbed gradient flows and connection to stat mech in $d>=1$
- Levy processes, existence & uniqueness of solutions of SDEs