CO905	Term 2
Stochastic models of complex systems	12 CATS

Commitment: 20 lectures, 10 classes

Prerequisites: basic knowledge on Stochastic Processes and basic programming skills (e.g. in C, Matlab or any equivalent language)

Content: This module provides an introduction to basic stochastic models of collective phenomena arising from the interactions of a large number of identical components. We will discuss various examples from physics, biology and social sciences, such as spread of epidemics/opinions, traffic flow, crystal surface growth or population dynamics, and get a qualitative understanding of possible phase transitions. The second main aspect of the course is a mathematical description of these models as stochastic processes, and an introduction to basic probabilistic and simulation tools for their analysis.

Outline:

- Short introduction to basic theory
  Markov processes, graphical construction, semigroups and generators, stationary distributions and reversibility, conservation laws, symmetries, absorbing states

- Population models
  branching processes, Moran model, Wright-Fisher diffusion and duality with Kingman's coalescent, fixation times, diffusion limits

- Epidemic models
  Contact process, survival and extinction, mean field rate equations, critical values, general remarks on the DP universality class

- Random walks with exclusion interaction
  exclusion processes, stationary currents and conservation laws, hydrodynamic limits, dynamic phase transition; maybe also connection to surface growth and the KPZ universality class

- Theoretical techniques (introduced along in lectures)
  scaling limits and Fokker-Planck approximations, mean-field rate equations, generating functions, duality; maybe also monotonicity and coupling

- Computational techniques (covered in classes)
  - how to simulate discrete and continuous-time models: random sequential update and other update rules, sampling rates and jump chains, construction with Poisson processes and rejection
  - how to measure: stationary averages and ergodic theorem, equilibration times
  - maybe also: classical Monte Carlo with heat-bath and Metropolis algorithm
  - implementation of 2 simulations with measurements and plots (homework, basic codes will be provided in C and have to be adapted)

Objectives: At the end of the module students should be acquainted with simple mathematical models of collective phenomena and apply basic probabilistic and compuational techniques for their analysis.

Organisation: 2h lectures per week; 1h class, where homework topics are covered
3 problem sheets with homework, roughly half the marks are computational the other half theoretical

Assessment: 50% homework and 50% viva

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