Viva topics for CO905 - 2013

general MC
- definition of a Markov process, transition probabilities,
- Chapman Kolmogorov equation, solution for discrete and cont time
- stationary distributions, interpretation as left eigenvector
- existence for finite state space, irreducibility and uniqueness
- time reversal of MC and reversible distributions, detailed balance

discrete time MC
- P-matrix, general solution of linear recursion in terms of eigenvalues
- branching processes, generating functions, survival probability
- SRW with boundary conditions as example for irreducibility

continuous time MC
- generator G, forward and backward equation, connection to generator
- master equation and detailed balance, general solution in terms of eigenvalues
- interpretation of G entries, exponential holding times, jump rates
- construction of a sample path, jump chain (the picture)
- Poisson process, characterization with stat independent increments
- birth death processes, queues
- Moran model, master equation, derivation of FP equation

ergodicity
- stationary distributions and recurrence times
- ergodicity, ergodic theorem, sufficient conditions for ergodicity
- aperiodicity for discrete time, Perron-Frobenius theorem

infinite state space
- transience, positive recurrence, null recurrence
- example: RW with/without drift, ~~Polya's theorem~~
- explosion, pure birth chain

stochastic particle systems
- local jump rates and graphical construction
- master equation and evolution of observables, generator
- properties of Poisson processes and random sequential update
- contact process: definition and ergodicity, expected number of infections on the complete graph, derivation of mean-field rate equation, simulations in 1D critical stationary behaviour, simulation results
- voter model: definition and ergodicity
- exclusion process: definition and ergodicity, conservation law and stationary current, fundamental diagram, generalization to traffic model, simulations
- the Ising model and MCMC, ergodicity, Metropolis and heat bath algorithm

Brownian motion
- scaling limit of sum of iidrv, CLT, LLN, Brownian motion
- definition of BM, characterization as Gaussian process with covariances
- Gaussian process and properties of Gaussians, characteristic functions, white noise
- Taylor expansion of generator and heat equation, solution to the heat equation
- scaling limit of continuous time RWs using expansion of the generator
- scaling limit of discrete time RWs (Donsker) using covariances

diffusion processes
- finite dimensional distributions and path properties
- transition kernels and densities, connection to fdds
- diffusion processes and the Fokker-Planck equation
- stationary solutions to Fokker-Planck with time-homogeneous coefficients
- Ornstein-Uhlenbeck process, evolution of observables, generator
- Kingman's coalescent, scaling to deterministic limit, ~~fluctuations~~

beyond diffusion
- jump process/compound Poisson process, generator as integral
- Levy processes, definition, generator, alpha-stable Levy process
- fractional BM, definition as Gaussian process, scaling properties, fractional noise

Stochastic calculus
- derivation of SDE/Langevin equ from diffusion process
- solution of SDE and stochastic integration
- stochastic integrals w.r.t. time (dt)
- Ito integral w.r.t. BM (dB_t), martingale property
- definition of martingales and martingale problem, applied to Poisson pr and BM
- computation of Ito integrals, Ito's formula (Taylor expansion up to dt)
- derivation of Fokker Planck equation from Ito's formula
- examples: expected values for OU process, Wright Fisher diffusion
- geometric Brownian motion and Black-Scholes formula

non-examinable
- Perron-Frobenius for periodic chains
- Polya's theorem
- solutions to PDEs with characteristics (hand-out)