Contents

1 Ordinary Differential Equations (ODE's) 3
- 1.1 Ordinary Differential Equations . . . 3
- 1.2 Dynamical Systems . . . 5
- 1.3 Approximation of Derivatives by Finite Differences . . . 6
- 1.4 Time-stepping Algorithms 1 : Euler Methods . . . 7
  - 1.4.1 Forward Euler Method . . . 7
  - 1.4.2 Backward Euler Method . . . 8
  - 1.4.3 Stepwise vs Global Error . . . 9
  - 1.4.4 Explicit vs Implicit Methods . . . 9
  - 1.4.5 Improving on the Euler Method . . . 9
- 1.5 Time-stepping Algorithms 2 : Predictor–Corrector Methods . . . 10
  - 1.5.1 Implicit Trapezoidal Method . . . 10
  - 1.5.2 Improved Euler Method . . . 11
  - 1.5.3 Multistep Methods . . . 12
  - 1.5.4 Adams–Bashforth Methods . . . 12
  - 1.5.5 Adams–Moulton Methods: . . . 13
- 1.6 Time-stepping Algorithms 3 : Runge–Kutta Methods . . . 13
- 1.7 Adaptive Timestepping . . . 15
- 1.8 Snakes in the Grass . . . 17
  - 1.8.1 Singularities . . . 17
  - 1.8.2 Stiffness . . . 17

2 Partial Differential Equations (PDE's) 19
- 2.1 Introduction to PDE's and their Mathematical Classification . . . 19
  - 2.1.1 Classification of Second Order PDEs . . . 19
  - 2.1.2 Types of Boundary Conditions . . . 20
- 2.2 Non-Dimensionalisation . . . 21
- 2.3 First Order PDEs: Method of Characteristics . . . 23
- 2.4 Similarity Solutions and Travelling Waves . . . 28
  - 2.4.1 Travelling waves . . . 28
  - 2.4.2 Similarity Solutions . . . 30

3 The Diffusion Equation 33
- 3.1 Forward Time Centred Space (FTCS) Scheme . . . 33
- 3.2 Source Terms and Boundary Conditions in the FTCS Scheme . . . 34
  - 3.2.1 Including a source term in the FTCS scheme . . . 34
  - 3.2.2 Imposing Dirichlet Boundary Conditions in the FTCS scheme . . . 35
  - 3.2.3 Imposing Neumann Boundary Conditions in the FTCS scheme . . . 36
- 3.3 Consistency and Stability of the FTCS Scheme . . . 37
  - 3.3.1 Consistency of the FTCS Scheme . . . 38
  - 3.3.2 Stability of the FTCS Scheme . . . 39
  - 3.3.3 Von Neumann Stability Analysis . . . 40
- 3.4 The Crank–Nicholson Method . . . 40

3.5 Stability of the Crank–Nicholson Scheme 42
3.6 Similarity solutions as attractors: rescaling 42

4 Hyperbolic PDE's **44**
4.1 The Advection Equation Revisited 44
4.2 Lax Method and CFL Criterion 47
4.3 Conservation Laws and Lax–Wendroff Method 49
4.4 The Wave Equation 51
4.5 Boundary Conditions for the Wave Equation 52

Bibliography **55**