5. Financial market models with local interactions

5.1 Clustering and herd behaviour

The non-Gaussian empirical results in Chapter 3 suggest that market participants do not behave entirely randomly. Chapter 4 examined the collective behaviour of agents in a market, assuming that these agents do not share any local information. The formation and action of crowds was seen to play a crucial role in determining the price dynamics, in particular the volatility. These crowds were *unintentional* in the sense that an agent became a member of a particular crowd via the strategy he was using, rather than because he decided to join that crowd *per se*. In short, this was a crowding in strategy space.

In this chapter we look at a complementary situation in which agents may interact and share opinions: they may hence imitate each other and thereby act as a crowd. Such crowding has been termed 'herd formation' in the Econophysics literature. As in Chapter 4, our goal is to build minimal yet justifiable models, which are consistent with the stylized facts observed in financial market price data. In the present case of addressing local interactions between agents, the models will not include as much detail as those of Chapter 4 concerning market microstructure. Although this could be added, we will focus instead on the statistical properties of various simple models. In particular, the herding mechanism in these models will be taken as fundamentally stochastic in nature. We will leave the interested reader to explore the intriguing possibility of combining the key elements of the models from this chapter and Chapter 4.

Cont and Bouchaud[1] proposed a simple agent-based model of such herd behaviour, with a demand generating mechanism, which depends on the collective action of the traders in a cluster (i.e. herd or crowd). Unlike the models of Chapter 4 where agents were independent, the agents in a cluster come to a collective decision and then all

[1] Cont, R. and Bouchaud, J. P. (2000) *Macroeconom. Dynam.* **4**, 170.

act in the same way. A cluster of agents may represent, for example, the investors in a mutual fund.

Consider N agents (traders) trading in a single asset. The cluster formation is modelled by establishing links between the agents. Let q_{ij} be the probability that the ith agent and the jth agent are connected. For simplicity q_{ij} is taken to be independent of i and j, that is, $q_{ij} = q$ where q is a constant. An agent therefore has an average of $q(N-1)$ links to the other agents. To ensure that the average number of links per agent is finite in the limit $N \to \infty$, we take $q = c/N$ where c is a constant, which is less than (but close to) unity, that is, $0 < 1-c \ll 1$. At each timestep of the market's evolution the N agents are partitioned stochastically into clusters of different sizes according to the parameter c, which characterizes inter-agent connectivity. The herding behaviour is introduced through the collective action of the agents in these clusters. Each agent i may take one of three possible actions, labelled by $a_i \in \{-1, 0, 1\}$. The action $a_i = +1$ denotes a buy order, $a_i = -1$ denotes a sell order, and $a_i = 0$ denotes not trading.[2] The variable a_i can thus be thought of as combining the roles of the 'investment suggestion' $a^{\mu[t]}$ and the 'confidence level' r from the agent models of Chapter 4. Agents in a given cluster have a common opinion and thus $a_i = a_j$ for i and j belonging to the same cluster. The probability of carrying out a transaction is characterized by a parameter ν, where $0 \le \nu \le 1$. The probabilities of a_i being $+1$ and -1 are assumed to be the same, that is, $p[a_i = +1] = p[a_i = -1] = \nu/2$ and hence $p[a_i = 0] = 1 - \nu$. The parameter ν characterizes how frequently a transaction occurs, and thus represents the *activity* in the market. The model is therefore completely characterized by the two parameters ν and c.

In this simple model the formation of herds (i.e. clusters) is purely geometrical and depends only on the parameter c. At each timestep of the market's evolution the herding process divides the N agents into clusters of various sizes. The probability distribution of cluster sizes $p[s]$ can be studied either by numerical simulation, or using probability and random graph theory. Most relevant to our discussion is the functional form of $p[s]$. For $c = 1$, the probability distribution function (PDF) has the form[3]

$$p[s] \sim \frac{A}{s^{5/2}}, \tag{5.1}$$

where A is a constant. For $0 < 1 - c \ll 1$,

$$p[s] \sim \frac{A}{s^{5/2}} \exp\left[\frac{-(1-c)s}{s_0}\right]. \tag{5.2}$$

[2] This is the same as the 'Grand Canonical' feature in Chapter 4, whereby agents can decide not to trade.

[3] We use the symbol $\sim$ to denote the functional dependence. For practical purposes, it can be thought of as meaning 'approximately proportional to'.

Equation (5.1) exhibits a pure power-law dependence of the cluster size, whereas Equation (5.2) exhibits a power-law dependence which becomes truncated at a cluster size of order s_0, after which an exponential dependence takes over. We will see that the power-law behaviour of $p[s]$ will carry over to the distribution of log-returns of the price when the collective herd behaviour is taken into account in driving price movements. We leave the discussion on determining the exponent of this power-law behaviour to Sections 5.2 and 5.3, where a related model with identical results will be studied. Interestingly, such power-law behaviour for cluster-size distributions also emerges from standard percolation problems (see Section 5.4). The connectivity in the present model is set-up via long-range interactions between the agents, in contrast to the nearest-neighbour interactions usually assumed when studying percolation problems on a lattice. The exponent $-\frac{5}{2}$ in the cluster-size distribution is consistent with that obtained in percolation theory for spatial dimensions higher than the upper critical dimension of the percolation problem. Percolation-type models of markets, which amount to putting the Cont–Bouchaud model onto a spatial lattice, will be discussed in Sections 5.4 and beyond.

Suppose the N participants are divided into a total of n_c clusters at a given point in the model's evolution. All the agents in the αth cluster of size s_α have the same demand a_α. Clearly then $\sum_{\alpha=1}^{n_c} s_\alpha = N$. Some of these agent-clusters decide to place buy orders ($a_\alpha = +1$), other agent-clusters may decide to place sell orders ($a_\alpha = -1$), and a fraction $(1 - \nu)$ of agent-clusters decide not to make any transaction at all ($a_\alpha = 0$). Price movements are small if the excess demand, which is (as before) the difference in the number of buy and sell orders, is small. Just before time t, the market-maker gathers all orders $a_i[t-1]$ and calculates an excess demand given by

$$D[t^-] = \sum_{i=1}^{N} a_i[t-1] = \sum_{\alpha=1}^{n_c} s_\alpha[t-1]\, a_\alpha[t-1]. \tag{5.3}$$

Using the argument from Chapter 4 relating excess demand to price change (Equation (4.1)), the single timestep log-return defined in Equation (1.4) is given by

$$z[t] = \ln x[t] - \ln x[t-1] = \frac{D[t^-]}{\lambda} \equiv \frac{1}{\lambda} \sum_{\alpha=1}^{n_c} s_\alpha[t-1]\, a_\alpha[t-1], \tag{5.4}$$

where we have used the shorthand $z[t] \equiv z[t, t-1]$. As in Chapter 4, the parameter λ is the market depth. The temporal ordering of this model is thus as follows. At time $t-1$ the N agents form n_c clusters. Within each cluster α the s_α agents each place an identical order $a_i[t-1] = a_\alpha[t-1]$. Just before time t the market maker collects all these orders, which results in an excess demand $D[t^-]$. The market maker then moves the price accordingly to a new level $x[t]$, the orders are executed and new clusters form.

The price is therefore driven by the effect of trading between different clusters of agents. Since a fraction $(1-\nu)N$ of agents have $a_i = 0$, then $\langle V[t]\rangle = \nu N$ is the average number of active agents and hence the average volume of orders received. Note that the cluster configuration at time $t+1$ is independent of that at time t in this present model, since the topology of agent–agent connections changes randomly at each timestep. Let us now consider how we might simulate this model on a computer: we start off with a randomly generated cluster configuration $\{s_\alpha[t]\}$ for a given parameter c. Using the consequent cluster sizes $s_\alpha[t]$ and the randomly generated orders $a_\alpha[t]$ for a given activity probability ν we form a demand $D[(t+1)^-]$. From this demand we can evaluate the log-return $z[t+1]$. In the next timestep, another random set of orders compatible with ν and c is generated and $z[t+2]$ is evaluated. Repeating the procedure gives a series of log-returns for which statistical properties can be analysed. Interesting generalizations of the model with a built-in dynamical process for the coagulation and fragmentation of clusters have been proposed. Some of these will be discussed in later sections.

This model readily gives power-law behaviour in the PDF of price returns, together with an exponential cutoff for large cluster sizes and hence large returns. For small values of ν, the net demand is dominated by one or just a few clusters. The PDF of log-returns then follows that of the cluster distribution. An analytic treatment of the excess kurtosis (recall Chapter 2) gives

$$\kappa - 3 = \frac{2c+1}{V(1-(c/2))(1-c)^3 A[c]} \tag{5.5}$$

for $0 < 1-c \ll 1$, where $A[c]$ is a normalization constant of order unity. The result that the excess kurtosis varies as $1/V$ implies that a small average volume of active agents leads to large price fluctuations. This is consistent with the fact that large price fluctuations occur more often in smaller or less active markets—it is also consistent with the numerical results of Chapter 4 for the Grand Canonical Minority Game (GCMG) in the ‘crowded’ regime corresponding to small memory m (see Fig. 4.4). Even though the average volume is small, its bursty structure creates large changes in demand and hence large changes in the price.

5.2 Transmission of information: the EZ model

The herd-formation model of Cont and Bouchaud is static in that the partition of the agents into clusters at one timestep *is independent* of the partition at an earlier

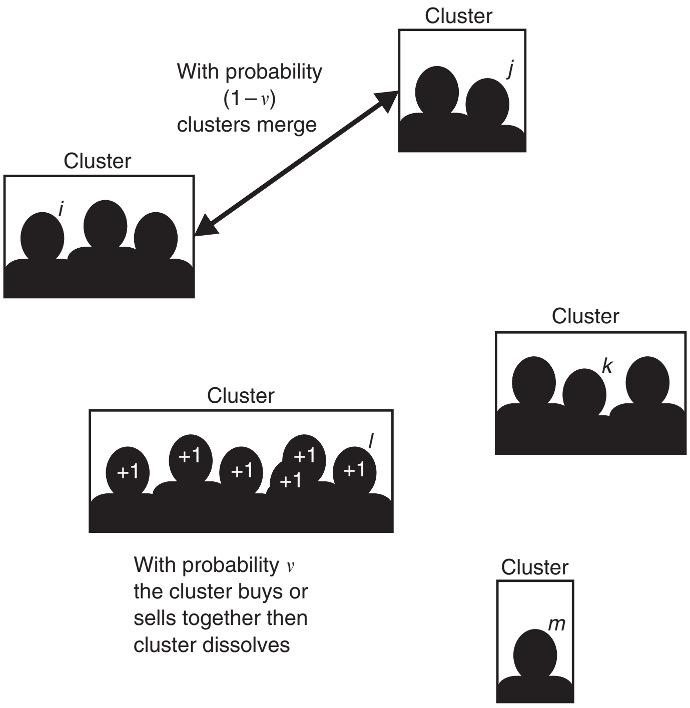

Fig. 5.1 The dynamical herd formation model, referred to as the EZ model.

timestep. However, we know that one of the reasons for herd formation is the spreading and sharing of opinion. Eguiluz and Zimmermann (EZ) generalized the Cont and Bouchaud model by incorporating a *dynamical* process of herd formation via the spread or transmission of opinion.[4] Consider again our market of N agents, in which agent i can be in one of three possible states represented by $a_i = +1$ for buying, $a_i = -1$ for selling, and $a_i = 0$ for the inactive state. Trading occurs with a probability ν, as in the previous model. At the beginning of the simulation, all the agents are isolated with their state set to $a_i = 0$. As time evolves, agents may spread their opinion and hence form clusters with a common opinion, as indicated in Fig. 5.1.

At any particular timestep, an agent belongs to a cluster of a certain size. An isolated agent represents a cluster of size one. At each timestep, an agent (e.g. the ith agent) is chosen at random. Let s_i be the size of the cluster to which this agent belongs. Since the agents within a given cluster have a common opinion, all agents within the same cluster will act together as a crowd. With probability ν, the agent and hence the whole cluster

[4] Eguiluz, V. M. and Zimmermann, M. G. (2000) *Phys. Rev. Lett.* **85**, 5659.

decides to make a transaction, for example, to buy or to sell with equal probability $\nu/2$ as in the model of Cont and Bouchaud. This collective action of a cluster of agents creates a price movement that depends on the size of the cluster. After the transaction, the cluster is then broken up into isolated agents all in the inactive ($a_i = 0$) state. With probability $(1 - \nu)$, the chosen agent decides not to make a transaction: instead he tries to accomplish further transmission of information rather than jumping into a decision to buy or sell. The other agents in the cluster follow. In this case, another agent j is chosen at random. The two clusters of sizes s_i and s_j then combine to form a bigger cluster. The EZ model is thus characterized by a single parameter ν. The set-up of the model is shown schematically in Fig. 5.1. The connectivity among the agents, characterized by the parameter c in the Cont–Bouchaud model, is now driven by the dynamics of the EZ model. The parameter ν can again be interpreted as the *activity* in the market. For small ν (i.e. $\nu \ll 1$), not many transactions take place. Instead, agents tend to spend their time in dispersing information. This leads to the build-up of internal connectivity and hence the formation of larger clusters. When a large cluster of agents eventually trades, the price movement (which is given in terms of the size of the cluster) is large. In the other limit of $\nu \to 1$, nearly every randomly chosen agent trades and the system consists of mostly isolated agents. The price movements will then be small and random. For intermediate values of ν, the price should consist of many small movements, with occasional larger ones arising when a cluster of agents submit their orders. For example, a value of $\nu = 0.01$ corresponds to about one cluster of buy or sell orders in every 100 iterations. The extreme case of forming one giant super-cluster containing all agents—which would be analogous to the phenomenon of Bose–Einstein condensation in physics—is unlikely unless ν is very small, that is, $\nu \ll 1/(N \ln N)$.

Demand in the EZ model is generated in a similar way to the Cont–Bouchaud model as given by Equation (5.3). However, here the total order size corresponds to the size of cluster to which the chosen agent i belongs (s_i), thus:

$$D[t^-] = s_i[t-1]\, a_i[t-1]. \tag{5.6}$$

The excess demand therefore has a magnitude given by $|s_i|$ and has equal probability $\nu/2$ of being positive or negative. Putting the excess demand in Equation (5.4) yields a log-return $z[t]$. We note that the EZ model can be easily implemented numerically.[5]

Figure 5.2 shows a time-series of price movements generated using the EZ model. Evidence of occasional large movements can be identified in the log returns. Since ν is small, we have re-scaled the time axis such that a time unit represents the average

[5] See the book-website www.occf.ox.ac.uk/books/fmc for a simple Fortran program, which runs on both UNIX and Windows platforms.

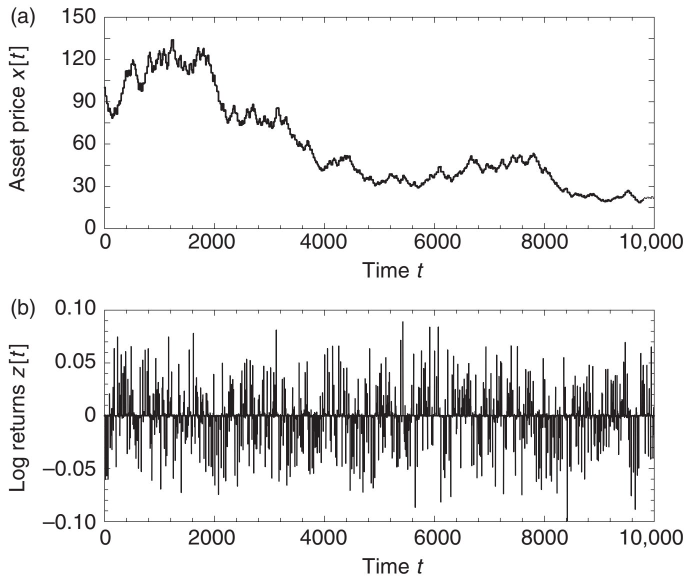

Fig. 5.2 (a) Price and (b) log-returns as a function of time, generated using the EZ model with $N =$ 10,000 agents, $\nu = 0.01$ and a market depth of $\lambda = 5 \times 10^4$.

time it takes for an order to arrive. The average number of connections per agent measures the connectivity within the EZ model. The connectivity is driven by the dynamics, hence it fluctuates with time. The simulation gives a mean connectivity $\langle c \rangle \approx 0.76$.

Let us now examine the statistics of the distribution of log-returns a little more closely. Figure 5.3 shows the PDF of positive returns $p[z]$ on a log–log plot. A power law is observed over a range of z values, with an exponent of -1.5. The width of the range over which the power law is observed, depends on the value of ν. If the volume of active agents is generally high, or equivalently the order flows are frequent, large returns are rare and hence an exponential tail is manifest in the PDF $p[z]$. For very small values of ν, large clusters are formed and rare but large returns appear. In between, there is a critical value of ν for which the power law is observed[6] over a large range of z. It follows from the definition of price returns in terms of excess demand that the distribution of price returns is determined by the distribution of cluster-sizes. This distribution is obtained by counting the number of occurrences n_{s} of a cluster of size s over time. The cluster size distribution is shown in Fig. 5.4 in terms of n_s/n_1, using the same set of parameters as before.

[6] There are also finite-size effects in numerical studies, caused by the finite number of agents.

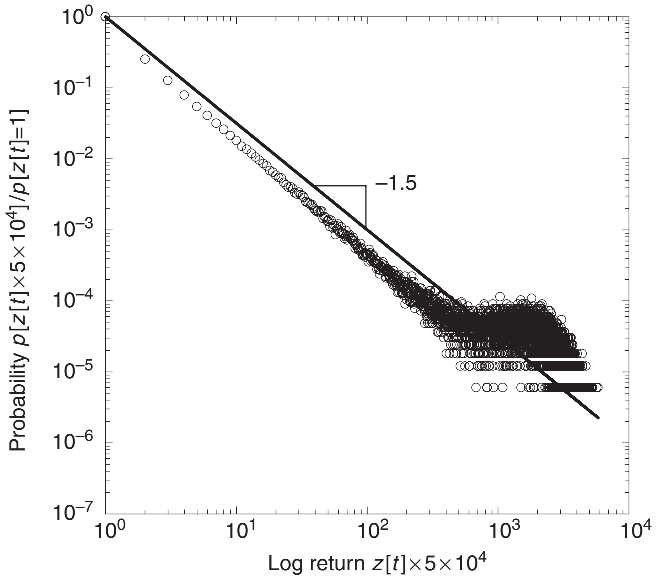

Fig. 5.3 The probability of positive price returns within the EZ model with $N = 10{,}000$ agents, $\nu = 0.01$ and a market depth of $\lambda = 5 \times 10^4$ as a function of the return on a log–log scale.

The distribution shows power law behaviour with an exponent $-5/2$, identical to the Cont–Bouchaud model and to percolation theory in high dimensions (i.e. >6 dimensions). The exponent $-5/2$ is thus independent of the details of the dynamics of herd formation and cluster fragmentation. The two exponents $-3/2$ (Fig. 5.3) and $-5/2$ (Fig. 5.4) are related in that $p[z]$ is determined by the probability of a randomly picked agent belonging to a cluster of size $s = \lambda z$. Hence $p[z]$ is proportional to the number of agents belonging to a cluster of size s, that is, $p[z] \sim n_s \times s \sim s^{-5/2} \times s \sim s^{-3/2} \sim z^{-3/2}$ as observed numerically. The EZ model can be generalized to include more elaborate mechanisms for herd formation and decision-making in a cluster of agents. It could also be extended to include feedback mechanisms between the agents' performance and their actions, as for the models of Chapter 4.

5.3 Analytic model: generating function approach

One can write a dynamical equation for the evolution of the EZ model at different levels of approximation. For example, one could start with a microscopic description of the system by noting that at any moment in time, the population can be described

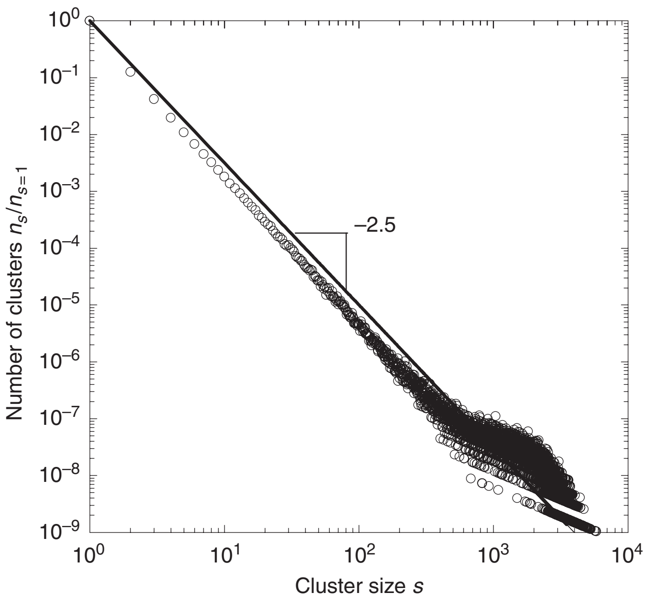

Fig. 5.4 Distribution of cluster sizes in the EZ model with $N = 10{,}000$ agents, $\nu = 0.01$.

by a partition $\{l_1, l_2, \ldots, l_N\}$ of the N agents into clusters. Here l_s is the number of clusters of size s. For example, $\{N, 0, \ldots, 0\}$ and $\{0, 0, \ldots, 1\}$ correspond to the extreme cases in which all agents are isolated and all agents belong to one big cluster, respectively. Clearly, the number of agents must be conserved $\sum_{i=1}^{N} il_i = N$. The dynamics could then be described by the time-evolution of the probability function $p[l_1, l_2, \ldots, l_N]$: in particular, taking the continuous-time limit would yield an equation for $\mathrm{d}p[l_1, l_2, \ldots, l_N]/\mathrm{d}t$ in terms of transitions between partitions. For example, the fragmentation of a cluster of s agents leads to a transition from the partition $\{l_1, \ldots, l_s, \ldots, l_N\}$ to the partition $\{l_1 + s, \ldots, l_s - 1, \ldots, l_N\}$. For our purposes, however, it is more convenient to work with the *average* number n_s of clusters of size s, which can be written as $n_s = \sum_{\{l_1,\ldots,l_N\}} p[l_1, \ldots, l_s, \ldots, l_N] \cdot l_s$. The sum is over all possible partitions of the system into clusters. Since $p[l_1, \ldots, l_N]$ evolves in time, so does $n_s[t]$. After the transients have died away, the system is expected to reach a steady-state in which $p[l_1, \ldots, l_N]$ and $n_s[t]$ become time-independent in the large-N limit. The time-evolution of $n_s[t]$ can be written down for the EZ model either by intuition, or by invoking a mean-field approximation to the equation for $\mathrm{d}p[l_1, l_2, \ldots, l_N]/\mathrm{d}t$. Taking the intuitive route, one can immediately write down the following dynamical equations in the continuous-time limit:

$$\frac{\partial n_s}{\partial t} = -\frac{\nu s n_s}{N} + \frac{(1-\nu)}{N^2}\sum_{s'=1}^{s-1} s' n_{s'}(s-s')n_{s-s'} - \frac{2(1-\nu)\, s\, n_s}{N^2}\sum_{s'=1}^{\infty} s' n_{s'}, \quad \text{for } s \geq 2 \tag{5.7}$$

$$\frac{\partial n_1}{\partial t} = \frac{\nu}{N}\sum_{s'=2}^{\infty}(s')^2 n_{s'} - \frac{2(1-\nu)n_1}{N^2}\sum_{s'=1}^{\infty} s' n_{s'}. \tag{5.8}$$

The terms on the right-hand side of Equation (5.7) represent all the ways in which n_s can change. The first term represents a decrease in n_s due to the dissociation of a cluster of size s: this happens only if an agent belonging to a cluster of size s is chosen and that agent decides to make a transaction. The former occurs with probability $s\, n_s/N$ and the latter with probability ν. The second term represents an increase in n_s as a result of the merging of a cluster of size s' with a cluster of size $(s-s')$. The third term describes the decrease in n_s due to the merging of a cluster of size s with any other cluster. For the $s = 1$ case described by Equation (5.8), the chosen agent remains isolated after making a transaction; thus Equation (5.8) does not have a contribution like the first term of Equation (5.7). The first term that appears in Equation (5.8) reflects the increase in the number of single agents due to fragmentation of a cluster after a collective transaction is taken. Similarly to Equation (5.7), the last term of Equation (5.8) describes the merging of a single agent cluster with a cluster of any other size. Equations (5.7) and (5.8) are so-called ‘master equations’ describing the dynamics within the EZ model.

In the steady-state, Equations (5.7) and (5.8) yield:

$$s n_s = \frac{(1-\nu)}{(2-\nu)N}\sum_{s'=1}^{s-1} s' n_{s'}(s-s')n_{s-s'}, \quad \text{for } s \geq 2 \tag{5.9}$$

$$n_1 = \frac{\nu}{2(1-\nu)}\sum_{s'=2}^{\infty}(s')^2 n_{s'}. \tag{5.10}$$

Equations of this type are most conveniently treated using the general technique of ‘generating functions’. As the name suggests, these are functions which can be used to generate a range of useful quantities. Consider

$$G[y] = \sum_{s'=0}^{\infty} s' n_{s'} y^{s'}, \tag{5.11}$$

where $y = \mathrm{e}^{-\omega}$ is a parameter. Note that $s\,n_s/N$ is the probability of finding an agent belonging to a cluster of size s. If $G[y]$ is known, $s\,n_s$ is then formally given by

$$sn_s = \frac{1}{s!}G^{(s)}[0], \tag{5.12}$$

where $G^{(s)}[y]$ is the sth derivative of $G[y]$ with respect to y. $G^{(s)}[y]$ can be decomposed as

$$G[y] = n_1 y + \sum_{s'=2}^{\infty} s' n_{s'} y^{s'} \equiv n_1 y + g[y], \tag{5.13}$$

where the function $g[y]$ governs the cluster size distribution n_s for $s \geq 2$. The next task is to obtain an equation for $g[y]$. This can be done in two ways. One could either write down the terms in $(g[y])^2$ explicitly and then make use of Equation (5.9), or one could construct $g[y]$ by multiplying Equation (5.9) by $\mathrm{e}^{-\omega s}$ and then summing over s. The resulting equation is:

$$(g[y])^2 - \left(\frac{2-\nu}{1-\nu}N - 2n_1 y\right) g[y] + n_1^2 y^2 = 0. \tag{5.14}$$

First we solve for n_1. From Equation (5.13), $g[1] = G[1] - n_1 = N - n_1$. Substituting $n_1 = N - g[1]$ into Equation (5.14) and setting $y = 1$, yields

$$g[1] = \frac{1-\nu}{2-\nu}N. \tag{5.15}$$

Hence

$$n_1 = N - g[1] = \frac{1}{2-\nu}N. \tag{5.16}$$

To obtain n_s with $s \geq 2$, we need to solve for $g[y]$. Substituting Equation (5.16) for n_1, Equation (5.14) becomes

$$(g[y])^2 - \left(\frac{2-\nu}{1-\nu}N - \frac{2N}{2-\nu}y\right) g[y] + \frac{N^2}{(2-\nu)^2}y^2 = 0. \tag{5.17}$$

Equation (5.17) is a quadratic equation for $g[y]$ which can be solved to obtain

$$\begin{aligned} g[y] &= \frac{(2-\nu)N}{4(1-\nu)}\left(1 - \sqrt{1 - \frac{4(1-\nu)}{(2-\nu)^2}y}\right)^2 \\ &= \frac{(2-\nu)N}{4(1-\nu)}\left(2 - \frac{4(1-\nu)}{(2-\nu)^2}y - 2\sqrt{1 - \frac{4(1-\nu)}{(2-\nu)^2}y}\right). \end{aligned} \tag{5.18}$$

Some undergraduate mathematics now becomes useful: using the expansion[7]

$$(1-x)^{1/2} = 1 - \frac{1}{2}x - \sum_{k=2}^{\infty} \frac{(2k-3)!!}{(2k)!!} x^k, \tag{5.19}$$

we have

$$g[y] = \frac{(2-\nu)N}{2(1-\nu)} \sum_{k=2}^{\infty} \frac{(2k-3)!!}{(2k)!!} \left(\frac{4(1-\nu)}{(2-\nu)^2} y \right)^k . \tag{5.20}$$

Comparing the coefficients in Equation (5.20) with the definition of $g[y]$ in Equation (5.13), the probability of finding an agent belonging to a cluster of size s is given by:

$$\frac{s n_s}{N} = \frac{(2-\nu)}{2(1-\nu)} \frac{(2s-3)!!}{(2s)!!} \left(\frac{4(1-\nu)}{(2-\nu)^2} \right)^s . \tag{5.21}$$

It hence follows that the average number of clusters of size s is

$$n_s = \frac{(2-\nu)}{2(1-\nu)} \frac{(2s-3)!!}{s(2s)!!} \left(\frac{4(1-\nu)}{(2-\nu)^2} \right)^s N = \frac{(1-\nu)^{s-1}(2s-2)!}{(2-\nu)^{2s-1}(s!)^2} N. \tag{5.22}$$

The s-dependence of n_s is implicit in Equation (5.22), with the dominant dependence arising from the factorials. Recall Stirling's series for $\ln[s!]$:

$$\ln[s!] = \frac{1}{2}\ln[2\pi] + \left(s + \frac{1}{2} \right) \ln[s] - s + \frac{1}{12s} - \cdots . \tag{5.23}$$

Retaining the few terms shown in Equation (5.23) is in fact a very good approximation, giving an error of <0.05 per cent for $s \geq 2$. It hence follows from Equation (5.22) that

$$n_s \approx \left(\frac{(2-\nu)e^2}{2^{3/2}\sqrt{2\pi}(1-\nu)} \right) \left(\frac{4(1-\nu)}{(2-\nu)^2} \right)^s \frac{(s-1)^{2s-3/2}}{s^{2s+1}} N. \tag{5.24}$$

The s-dependence can be deduced as

$$n_s \sim N \left(\frac{4(1-\nu)}{(2-\nu)^2} \right)^s s^{-5/2}. \tag{5.25}$$

For small values of ν, the dominant dependence on s is found to be

$$n_s \sim s^{-5/2}, \tag{5.26}$$

[7] The 'double factorial' operator !! denotes the product: $n!! = n(n-2)(n-4)\cdots$.

which is exactly the behaviour observed numerically for the EZ model (see Fig. 5.4) and is also the behaviour of n_s in the Cont–Bouchaud model. For large s, the power law behaviour is masked by the function in parentheses in Equation (5.25). The present results are valid when there is a spread in the size of clusters: for very small values of ν, transactions are so infrequent that agents tend to crowd into one big cluster. The behaviour will be quite different when such a big cluster dominates. Note that the general technique described here can be applied to other problems. Related to the generating function are other similar functions such as the characteristic function, and the moment-generating function. As a general rule, one should try to work with the function that is most convenient for evaluating the quantity concerned: the general procedure for getting at the solution is then more or less the same.[8]

5.4 The percolation problem

The specific value of the exponent characterizing the distribution of cluster sizes (i.e. 5/2) suggests a connection to the percolation problem, which we will now explore. The percolation problem[9] is a geometrical problem related to connectivity. Take, for example, a two-dimensional (2D) square lattice on which a fraction q of the sites are randomly occupied by black dots, as shown in Fig. 5.5. Dots on neighbouring sites are regarded as belonging to the same cluster. For small values of q, the dots are mostly isolated and hence form clusters of only one agent, together with a few clusters of larger sizes. As q increases, larger clusters are formed. The 'percolation transition' refers to the occurrence of a connected path of dots from one side of the lattice to another, say from top to bottom. In the limit of large lattices, this occurs at a critical fraction q_c called the 'percolation threshold', the precise value of which depends on the lattice type and geometric dimension. For one-dimensional (1D) systems, $q_c = 1$ since any missing dot in a 1D array interrupts the path from one end to another. For a 2D square lattice, $q_c \simeq 0.593$. At q_c, dots in the connected path form an infinitely connected cluster, with the remaining dots forming clusters of various sizes. Right at q_c, it has been found that the distribution of cluster sizes n_s follows a power law of the form $n_s \sim s^{-\tau}$, where τ is the exponent conventionally used in percolation theory to characterize the cluster-size distribution. The value of τ depends only on the geometric dimension of the system and is independent of other details, for example,

[8] See also D'Hulst, R. and Rodgers, G. J. (2000) *Int. J. Theor. Appl. Finance* **3**, 609; Xie, Y., Wang, B. H., Quan, H. J., Yang, W. S., and Hui, P. M. (2002) *Phys. Rev. E* **65**, 046130.

[9] See, for example, Stauffer, D. and Aharony, A. (1994) *Introduction to Percolation Theory*. Taylor and Francis, London.

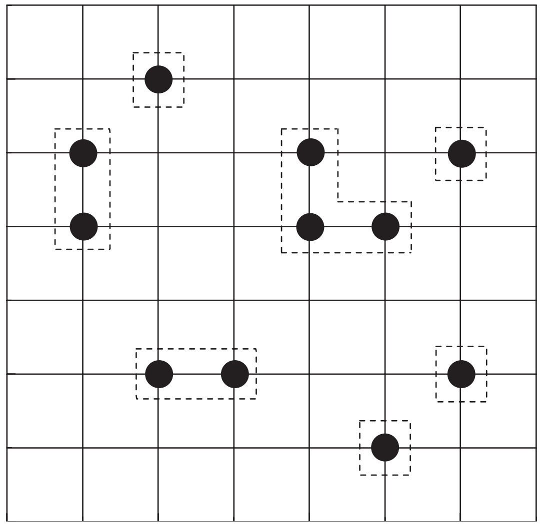

Fig. 5.5 Schematic diagram showing cluster formation on a 2D lattice. Dots represent agents and the dashed lines grouping these agents represent the extent of clusters.

lattice types. For 2D $\tau \approx 2$, while for dimensions higher than six (which is the 'upper critical dimension' of the percolation problem) we have $\tau = 5/2$.

5.5 Cont–Bouchaud model on a lattice

The connection between the Cont–Bouchaud model, the EZ model, and percolation problem now becomes clear. Since there is no underlying lattice in the Cont–Bouchaud and EZ models, each agent has the chance to be connected to any one of the other agents in the population. Therefore, the number of 'neighbouring sites' is large—hence the Cont–Bouchaud and EZ models can be regarded as lattice models in very high spatial dimensions, for which the exponent $\tau = 5/2$ follows immediately. It would be interesting, therefore, to investigate the effects of putting the Cont–Bouchaud and EZ models onto a lower dimensional lattice. Although such a model might seem abstract, the lattice could be thought of as an ordered version of the population of traders standing on a trading floor: traders then can 'connect' to their nearest neighbours by talking, facial gestures, or eye contact in order to pass on an opinion. Stauffer and coworkers studied a series of models based on the percolation problem.[10] The simplest one goes as follows. In a 2D square lattice, a fraction q of the sites is occupied. The occupied sites represent the agents. Agents occupying neighbouring sites form

[10] See, for example, Stauffer, D., de Oliveira, P. M. C., and Bernardes, A. T. (1999) *Int. J. Theor. Appl. Finance* **2**, 83; Stauffer, D. and Sornette, D. (1999) *Physica A* **271**, 496.

a cluster with a common perception of the market. All agents within a cluster act collectively. They buy with probability $\nu_{\text{buy}} = \nu/2$, sell with probability $\nu_{\text{sell}} = \nu/2$, and remain inactive with probability $(1 - \nu)$. The limit $\nu \ll 1$ is typically considered. The price movements are again determined by the excess demand (as in Equations (5.3) and (5.4)), with each agent contributing the same amount.

In the simplest form of the model one simply generates a geometric configuration of agents on the lattice (traders on the trading floor) and then lets them trade randomly in accordance with the probabilities ν_{buy} and ν_{sell}. Since $\nu \ll 1$, only one or a few clusters actually trade at any given timestep. The distribution of price movements is therefore expected to follow that of the cluster sizes. To analyse the statistics of this simple market model one should in practice average over many timesteps for a given geometric configuration, as well as over many independent configurations for given parameters ν and q. The power law behaviour in the cluster size distribution for the percolation problem near the percolation threshold, naturally leads to a power law in the price-returns with an exponent τ' equal to the exponent τ characterizing the cluster size distribution. We have then $p[z] \sim z^{-\tau'=\tau}$, which was also the case in the basic Cont–Bouchaud model. This model has been studied using hypercubic lattices in spatial dimensions from 2 to 7; the corresponding exponent τ varies from $\tau \approx 2$ in 2D to $\tau = 5/2$ for dimensions higher than or equal to 6. The resulting asset-return distributions are consistent with the fat-tailed behaviour of real financial asset returns as demonstrated in Chapter 3. Besides the fat tails in the price-returns, the percolation model can also be made geometrically dynamical (just as the EZ model is a dynamical version of the Cont–Bouchaud model) in order to reproduce the additional feature of volatility clustering. This can be achieved by allowing for a dynamical correlation between the connectivity of agents from one timestep to the next. The simplest way to do this is to allow a certain percentage of agents, say 1 per cent, to try and move to an empty neighbouring site after each timestep. This models the behaviour of information transport across, for example, a trading floor through the direct movement of traders. With this modification, the distribution of returns shows the same features as in the basic percolation models, with the same value for the exponent. In addition the resulting volatility shows, via its autocorrelation function, the kind of clustering effect seen in real markets (recall Chapter 3).

5.6 Variations on a theme

The cluster-size exponent τ leads to a power law in the distribution of returns $p[z] \sim z^{-\tau'}$ with $\tau' = \tau = 5/2$ in high dimensions (within the Cont-Bouchaud model, with or without an underlying lattice). However several modifications have been made in an effort to make the value of τ' larger, and hence closer to the values observed in real markets. In addition, the stochastic nature of these models implies that trading is noisy and prices may go up and up (or down and down) without any 'restoring force' to bring the price back to some kind of mean or 'fundamental' level. It is debatable whether such a fundamental price-level really exists in practice, but it could easily be incorporated into the model as follows. Instead of having the probabilities of selling and buying be equal, we may impose a higher (lower) buying probability ν_{buy} when the current price $x[t]$ is below (above) this imposed fundamental level x_0. Stauffer and coworkers introduced the form

$$\nu_{\text{buy}} = (1 - \varepsilon \log[x/x_0])\frac{\nu}{2} \quad \text{and} \quad \nu_{\text{sell}} = (1 + \varepsilon \log[x/x_0])\frac{\nu}{2},$$

where ε is a parameter characterizing the strength of this fundamental price restoring-force effect. Note that $\nu_{\text{buy}} + \nu_{\text{sell}} = \nu$, and in the case of $\varepsilon = 0$ the basic model is recovered. Detailed numerical studies show that while the price time-series looks very different for $\varepsilon \neq 0$, the resulting price-return distribution and the autocorrelation of volatility are nearly identical to those with $\varepsilon = 0$ for lattices from 2D to 7D. Thus the stylized facts of this model are unaffected by the addition of this restoring force, even though the price produced is prevented from moving too far away from the imposed fundamental value.

For percolation models operating exactly at $q = q_c$, the exponent τ' follows the exponent τ. However, there is no a priori reason why a market should know, and hence adjust itself, to behave as a system exactly at q_c. For $q \ll q_c$, the price changes will be small since only small clusters exist. As q increases, larger price changes will occasionally arise. Hence sweeping from $q \ll q_c$ to $q = q_c$ can mimic the build-up of a speculative bubble wherein agents start forming larger and larger clusters of the same opinion. Numerical results obtained by sweeping a system from $q = 0.1$ to $q = q_c$ have shown that $p[z] \sim z^{-\tau'}$ over a range of z, with τ' in 2D enhanced by about 0.5 to reach $\tau' \simeq 2.5$, as opposed to $\tau' \simeq 2.0$. Another possible way to change the seemingly robust τ' value, is to consider a size-dependent activity $\nu[s]$. This is not unreasonable since, for example, large pension funds would avoid high risk investments, and would trade less often than small professional investors. Size-dependent activity has also been observed in the analysis of other economical data, for example, in the growth dynamics of firms and economies of countries. Stauffer and Sornette have proposed the form $\nu[s] = 1/\sqrt{s}$. This has the effect of removing the parameter ν from the model. With this size-dependent activity incorporated into the percolation model, the effect of sweeping towards q_c gives an exponent of $\tau' \simeq 3$ in 2D. In all the models discussed so far, the change in the logarithm of price is taken to be a linear function of the excess demand as in Equation (5.4). Obviously the relationship need not be linear. In fact a non-linear dependence would also shift the value of τ'.

5.7 Modified EZ models

The EZ model is characterized by the activity ν together with the mechanism for cluster coagulation and fragmentation. A cluster of agents dissolves with 100 per cent certainty after a transaction, and combines with another cluster with 100 per cent certainty if the cluster remains inactive. Under these conditions, the distribution of cluster sizes is then characterized by the same exponent $\tau = 5/2$ as in the percolation problem in high spatial dimensions. We will now demonstrate how the EZ model can be extended to give rise to model-dependent exponents for the cluster-size distribution, and how the generating function approach can be applied to describe the modified model analytically.

We will keep the essential structure of the basic EZ model here but with the modification that a cluster will dissolve after a transaction with a probability depending on cluster size, and it will combine with another cluster with a probability depending on the size of the two clusters involved. At each timestep an agent, say the ith one, is chosen at random. Thus, the agent belongs to a cluster of size s_i. With probability ν the agent—and hence the whole cluster since agents in a cluster share the same opinion—decides to make a transaction, that is, to buy or sell with equal probability $\nu/2$. After the transaction, the cluster is then broken up into isolated agents with a probability $f[s_i]$, which depends on s_i. With probability $(1 - \nu)$ the agent—and hence the whole cluster—decides not to trade. In this case, another agent j is chosen at random. The jth agent belongs to a cluster of size s_j. The two clusters of size s_i and s_j then combine to form a bigger cluster with probability $f[s_i]f[s_j]$, but remain separated otherwise. With the choice $f[s] = 1$ the original EZ model is recovered. Analytically, this particular formulation of the fragmentation and coagulation process can be readily treated by the generating function approach discussed earlier.

This probabilistic cluster-formation process may even mimic certain aspects of behaviour in a real financial market. One such aspect is the effect of news arrival. Imagine that one of the agents is in a cluster of size s_i, which receives some external news at a given timestep with probability ν. This external news suggests to the members of the cluster that they should immediately trade. Since the news is external, the cluster acts together in this one moment, subsequently the cluster has a finite probability of dissociation. The agents may sense that they are members of a large crowd (e.g. through their effect on the asset price); their probability of dissociation is therefore likely to be a decreasing function of the crowd size. The principle of this is that agents may like to feel the assurance of being part of a large crowd, as opposed to acting alone. By contrast, with probability $(1 - \nu)$ there is no news arrival from outside. The agent in the chosen cluster, uncertain about whether to buy or to sell, makes contact with an agent in another cluster of size s_j. The agents share information and

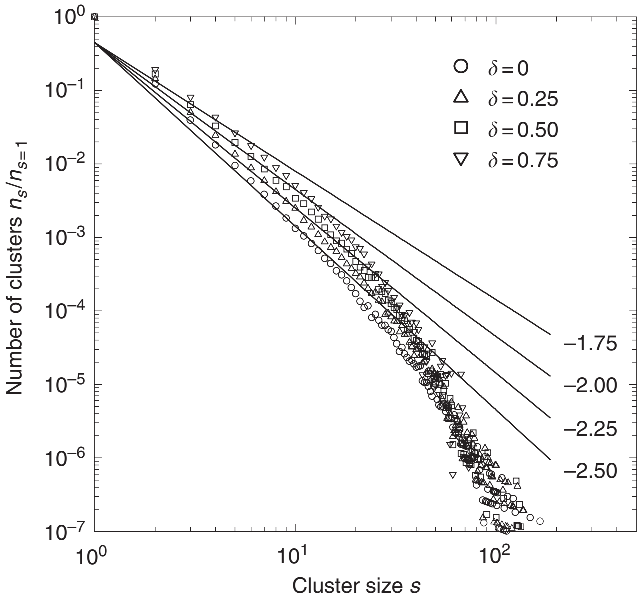

Fig. 5.6 Cluster-size distribution for the modified EZ model with $N = 10{,}000$ agents, $\nu = 0.01$ and with $f[s] = s^{-\delta}$, for different values of δ.

come up with a new opinion. Each of them then separately tries to persuade the other members of their cluster to accept the new opinion. With probability $f[s_i](f[s_j])$ the opinion of cluster $i(j)$ changes to the new opinion. Thus, the two clusters combine with probability $f[s_i]f[s_j]$.

To illustrate how particular forms of $f[s]$ may alter the exponent τ in the cluster size distribution, we consider the case of $f[s] = s^{-\delta}$. Figure 5.6 shows n_s for several values of δ. The solid lines are guides to the eye indicating that the exponent τ is described by $\tau = 5/2 - \delta$, and hence is now model-dependent. It follows that the exponent τ' of the price-returns also becomes model-dependent in this modified EZ model. Analytically, the master equations for the case of $f[s] = s^{-\delta}$ can readily be written as

$$\frac{\partial n_s}{\partial t} = -\frac{\nu s^{1-\delta} n_s}{N} + \frac{(1-\nu)}{N^2} \sum_{s'=1}^{s-1} (s')^{1-\delta} n_{s'} (s-s')^{1-\delta} n_{s-s'} - \frac{2(1-\nu) s^{1-\delta} n_s}{N^2} \sum_{s'=1}^{\infty} (s')^{1-\delta} n_{s'}, \quad \text{for } s \geq 2 \tag{5.27}$$

$$\frac{\partial n_1}{\partial t} = \frac{\nu}{N} \sum_{s'=2}^{\infty} (s')^{2-\delta} n_{s'} - \frac{2(1-\nu) n_1}{N^2} \sum_{s'=1}^{\infty} (s')^{1-\delta} n_{s'}, \tag{5.28}$$

with the physical meaning of each term being similar to that for Equations (5.7) and (5.8). The steady-state equations become

$$s^{1-\delta} n_s = A \sum_{s'=1}^{s-1} (s')^{1-\delta} n_{s'} (s-s')^{1-\delta} n_{s-s'} \tag{5.29}$$

$$n_1 = B \sum_{s'=2}^{\infty} (s')^{2-\delta} n_{s'}. \tag{5.30}$$

The constant coefficients A and B are given by

$$A = \frac{1-\nu}{N\nu + 2(1-\nu)\sum_{s'=1}^{\infty}(s')^{1-\delta} n_{s'}} \quad \text{and} \quad B = \frac{N\nu}{2(1-\nu)\sum_{s'=1}^{\infty}(s')^{1-\delta} n_{s'}}.$$

Setting $\delta = 0$ in Equations (5.29) and (5.30) recovers Equations (5.9) and (5.10) for the basic EZ model. A generating function

$$G[y] = \sum_{s'=0}^{\infty} (s')^{1-\delta} n_{s'} y^{s'} = n_1\, y + g[y] \tag{5.31}$$

can be introduced where $g[y] = \sum_{s'=2}^{\infty} (s')^{1-\delta} n_{s'}\, y^{s'}$ and $y = \mathrm{e}^{-\omega}$. The function $g[y]$ satisfies a quadratic equation of the form

$$(g[y])^2 - \left(\frac{1}{A} - 2n_1\, y\right) g[y] + n_1^2\, y^2 = 0, \tag{5.32}$$

which is a generalization of Equation (5.14). With $n_1 + g[1] = \sum_{s'=1}^{\infty}(s')^{1-\delta} n_{s'}$ and Equation (5.32), n_1 can be obtained as

$$n_1 = \frac{(1-\nu)^2 - \nu^2 A^2 N^2}{4(1-\nu)^2 A}. \tag{5.33}$$

Solving Equation (5.32) for $g[y]$ gives

$$g[y] = \frac{1}{4A}\left(1 - \sqrt{1 - 4n_1\, A\, y}\right)^2. \tag{5.34}$$

Following the steps leading to Equation (5.25), we obtain n_s in the modified EZ model:

$$n_s \sim N \left(\frac{4(1-\nu)\left((1-\nu) + \left(N\nu / \sum_{s'=1}^{\infty}(s')^{1-\delta} n_{s'}\right)\right)}{\left(\left(N\nu / \sum_{s'=1}^{\infty}(s')^{1-\delta} n_{s'}\right) + 2(1-\nu)\right)^2} \right)^s s^{-(5/2-\delta)}. \tag{5.35}$$

For $\delta = 0$, $\sum_{s'=1}^{\infty}(s')^{1-\delta}n_{s'} = N$ and hence Equation (5.35) reduces to the result in Equation (5.25) for the EZ model. For $\delta \neq 0$, it is difficult to solve explicitly for n_s. However, the summation simply gives a constant, and thus for small ν the dominant dependence on the cluster size s is $n_s \sim s^{-(5/2-\delta)}$. This dependence on s therefore agrees with the numerical results (see the discussion for Fig. 5.6).

It is interesting to extend the treatment to arbitrary, but properly normalized, forms of $f[s]$. In this case, we start with the master equations for $s \geq 2$ and $s = 1$ respectively:

$$\frac{\partial n_s}{\partial t} = -\frac{\nu\, s\, n_s}{N} f[s] + \frac{(1-\nu)}{N^2}\sum_{s'=1}^{s-1} s' n_{s'}(s-s')n_{s-s'} f[s'] f[s]$$
$$- \frac{2(1-\nu)s\, n_s}{N^2}\sum_{s'=1}^{\infty}(s' n_{s'} f[s'] f[s] - s(f[s])^2)$$
$$\frac{\partial n_1}{\partial t} = \frac{\nu}{N}\sum_{s'=2}^{\infty}(s')^2 n_{s'} f[s'] - \frac{2(1-\nu)n_1}{N^2}\sum_{s'=1}^{\infty}(s' n_{s'} f[s'] f[1] - (f[1])^2).$$

The derivations can be carried out in a similar way as before, although the algebra is slightly more complicated. The dominant s-dependence turns out to be

$$n_s \sim \frac{s^{-5/2}}{f[s]}. \tag{5.36}$$

This result confirms that the exponent τ, and hence τ', can be tuned by introducing a size-dependent fragmentation probability $f[s]$. Besides being proposed as a model for trading behaviour in financial markets, such modifications of the EZ model have also been proposed for the study of, for example, different methods for decision-making in a population with herd formation, and for the size distribution of customer groups and businesses.[11]

5.8 Other microscopic market models

We close with a brief discussion of two other interesting models which have been proposed in order to understand financial market behaviour.[12]

[11] See, for example, Zheng, D., Hui, P. M., Yip, K. F., and Johnson, N. F. (2002) *Eur. Phys. J. B* **27**, 213; Zheng, D., Rodgers, G. J., Hui, P. M., and D'Hulst, R. (2002) *Physica A* **303**, 176; Zheng, D., Rodgers, G. J., and Hui, P. M. (2002) *Physica A* **310**, 480.

[12] In addition, we encourage the reader to investigate the interesting model of Lux, T. and Marchesi, M. (1999) *Nature* **397**, 498, and the article by Farmer, J. D., 'Market force, ecology and evolution', see xxx.lanl.gov, adap–org/9812005.

5.8.1 Diffusion-reaction model

Bak and coworkers[13] proposed a model based on a population of $N/2$ sellers, each of which is holding one share of a stock, and $N/2$ buyers, each of which wants to buy a share. The population is a mixture of noise traders and rational traders, thus forming an inhomogeneous population. Each of the rational agents knows his own 'fair' price for selling or buying an asset (y_i in the discussion of limit orders in Chapter 1), the value of which is based on his own perception of the value of the asset and the level of risk acceptable to him. The rational agents only trade when their own fair price is met. A seller sells when a buyer is willing to pay at a price that is equal to (or higher than) the seller's fair price. After a transaction, a buyer becomes a seller of the same character and vice versa. A population with pure rational agents will eventually evolve to a situation in which trading stops, when the range of buying prices becomes separated from the range of selling prices. The noise traders, by contrast, decide their selling price or buying price randomly within some preset range. Of course, they expect to buy within a lower range of prices and to sell within a higher range of prices. When their price is not met, they randomly change their expected price by one unit up or down. This random movement of the expected prices stimulates more transactions. After a transaction, a new noise buyer picks a price randomly within the range $0 \to x[t]$, where $x[t]$ is the current price, that is, the price of the most recent transaction. A new noise seller picks a price randomly within the range $x[t] \to x_{\max}$, where $x_{\max}$ is a preset upper bound on the stock price. The behaviour of a population consisting entirely of noise traders yields a non-trivial time-correlation in the price changes. The problem can be mapped onto a diffusion-reaction problem: the agents' buying and selling prices diffuse, and when the prices coincide a reaction (i.e. a transaction) occurs. The correlations, and hence the tail exponent for the distribution of price returns, can be modified by changing the behaviour of the noise traders in the mixed population. Note that these models describe trading in a financial asset through submission of *limit orders* to the market-maker. These models are thus in contrast to the microscopic market models described so far in this book, which consider trading via *market orders* only. Ideally, a realistic microscopic market model would incorporate the effects of both types of trading.

5.8.2 Small-world networks

The potential importance of herd behaviour in a range of human settings, has recently led to intensive investigations concerning the structure of connectivity

[13] Bak, P., Paczuski, M., and Shubik, M. (1997) *Physica A* **246**, 430.

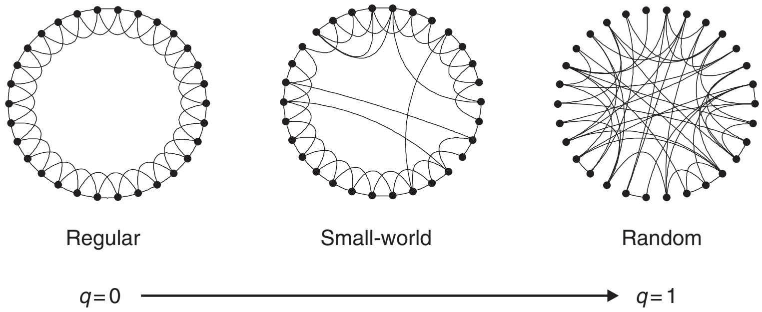

Fig. 5.7 A small-world network with $N = 30$ agents and $k = 4$, is obtained by randomly rewiring the connections in a regular world with probability q.

between members of a population (agents). One of the most interesting models is the *small-world network*.[14]

The construction of a small world network can be achieved as follows. For a population of N agents, a 'regular world' of k neighbours is first constructed by arranging the agents in a circle, that is, in a 1D chain with periodic boundary conditions. Each agent is then connected to his k neighbours having the shortest possible separations. Figure 5.7 shows a regular world of $N = 30$ agents and $k = 4$. A parameter q is then introduced which characterizes the geometrical features in the small world: in particular, q is the probability that an existing link in the regular world is *replaced* by a link to a randomly picked agent in the population. To construct a small world in practice, we start with the regular world and scan through each of the links, replacing them by a randomly established link with probability q. For $q \approx 1$, nearly all the links are replaced and the resulting network is referred to as a random network or random graph (see Fig. 5.7). The parameters are chosen so that $N \gg k \gg \ln N$ hence there is typically no isolated site. For smaller values of q, we obtain what is known as a 'small world network'. It is important to note that 'small world' does not mean that the number of agents in the population is small, but instead refers to the particular geometrical connectivity established for $q \ll 1$. A small world network shows several interesting features. We discuss two aspects here. Consider an agent j in the network having k_j neighbours. Among these k_j neighbouring agents, there are at most $k_j(k_j - 1)/2$ links. This is the limiting case where all the agents linked to agent j are also linked among themselves. In a small world network, if we count the actual number of links in the cluster and divide this number by $k_j(k_j - 1)/2$, this will provide us with the fraction of all possible links actually established in the cluster originating

[14] Watts, D. J. and Strogatz, S. H. (1998) *Nature* **393**, 440; Watts, D. J. (1999) *Small Worlds: The Dynamics of Networks between Order and Randomness*, Princeton University Press.

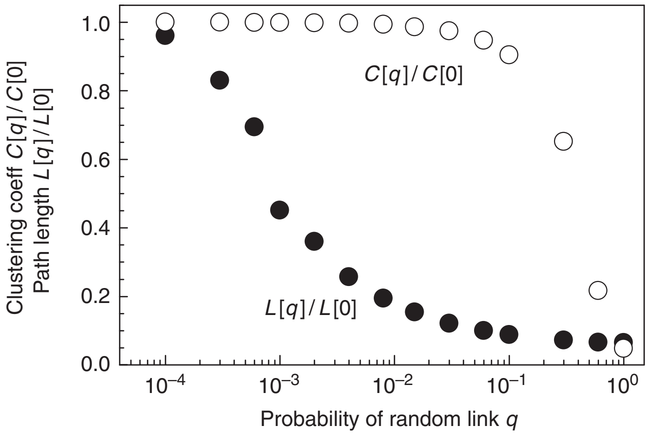

Fig. 5.8 The clustering coefficient $C[q]$ and the path length $L[q]$ in a network with $N = 1000$ agents and $k = 4$.

from agent j. Taking linked neighours as signifying trusted friends or 'confidantes', this link-fraction then reflects the extent to which friends of agent j are also friends of each other. The clustering coefficient $C[q]$ is the average of this link-fraction over all the agents in a network. For $q \to 0$, $C \sim 3/4$ while for $q \to 1$, $C \sim k/N$ which is *small* in a population with large N. The clustering coefficient is a *local* property of the network that measures the 'cliquiness' of a circle of friends.

Figure 5.8 shows $C[q]/C[0]$ for the entire range of q in a network with $N = 1000$ agents and $k = 4$. It is found that for a broad range of q, $C[q]$ is substantially higher than $C[q \to 1]$, and that $C[q]$ remains practically unchanged for small q if the network is rewired. By contrast the path length $L[q]$, which is the number of edges in the shortest possible path between two vertices when all possible pairs of vertices are averaged over, shows very different behaviour. For $q \to 0$ we have $L \sim N/2k$, which is large for a big population. For $q \to 1$ we have $L \sim \ln N$. The path length is a *global* property that reflects the average number of friendships in the shortest path connecting two agents. For example, think of someone that you would like to know. It turns out that he or she will be a friend of your friend's friend, and so on. One might expect that the behaviour of $L[q]$ would follow that of $C[q]$. However it turns out that $L[q]$ behaves quite differently (see Fig. 5.8). In the range of small but increasing q for which $C[q]$ remains unchanged, $L[q]$ drops quite rapidly. The network, through probabilistic rewiring, is effective in establishing links between different agents: a few connections could take you to any one you may want to know. It is a small world after all! The difference in the behaviour of $C[q]$ and $L[q]$ indicates that the transition to a small world cannot be detected just by looking at a local property of the network. A small world network is, therefore, characterized by its high clustering coefficient and small characteristic path length. It turns out that real world networks such as the networks of film actors, power grids, and even networks of terrorists, show features that are consistent with a small world, and cannot be explained using either a regular network ($q = 0$) or a random graph ($q = 1$).

The role of such small-world networks in the financial world is yet to be fully explored. For example, one could look at connections between agents, or common shareholders in different companies, or different market sectors and geographical regions, or different financial instruments, or even different financial news sources. Most importantly for the present discussion, the geometrical connection between agents in a small world would be different from that based on percolation models. Since financial markets involve the transmission of information and opinion sharing over a large network of agents, the rich and complicated connectivity among the agents in a small world network could have profound effects on herding behaviour and hence on the resulting financial market dynamics.

As suggested at the beginning of this chapter, the ultimate prize probably lies with joining together the key elements of Chapters 4 and 5, such that agents have access to both global information *and* in addition have the possibility to form local, possibly temporary, networks. Not only would this be of great potential interest for understanding the financial system as a whole, but also for a wide range of other complex system applications. This fascinating prospect awaits future research.