Essential bibliography

- **For an history of wavelets and some applications**

  [1] ***Wavelets, algorithms and applications***, *Yves Meyer*

- **Manuals for wavelet analysis**

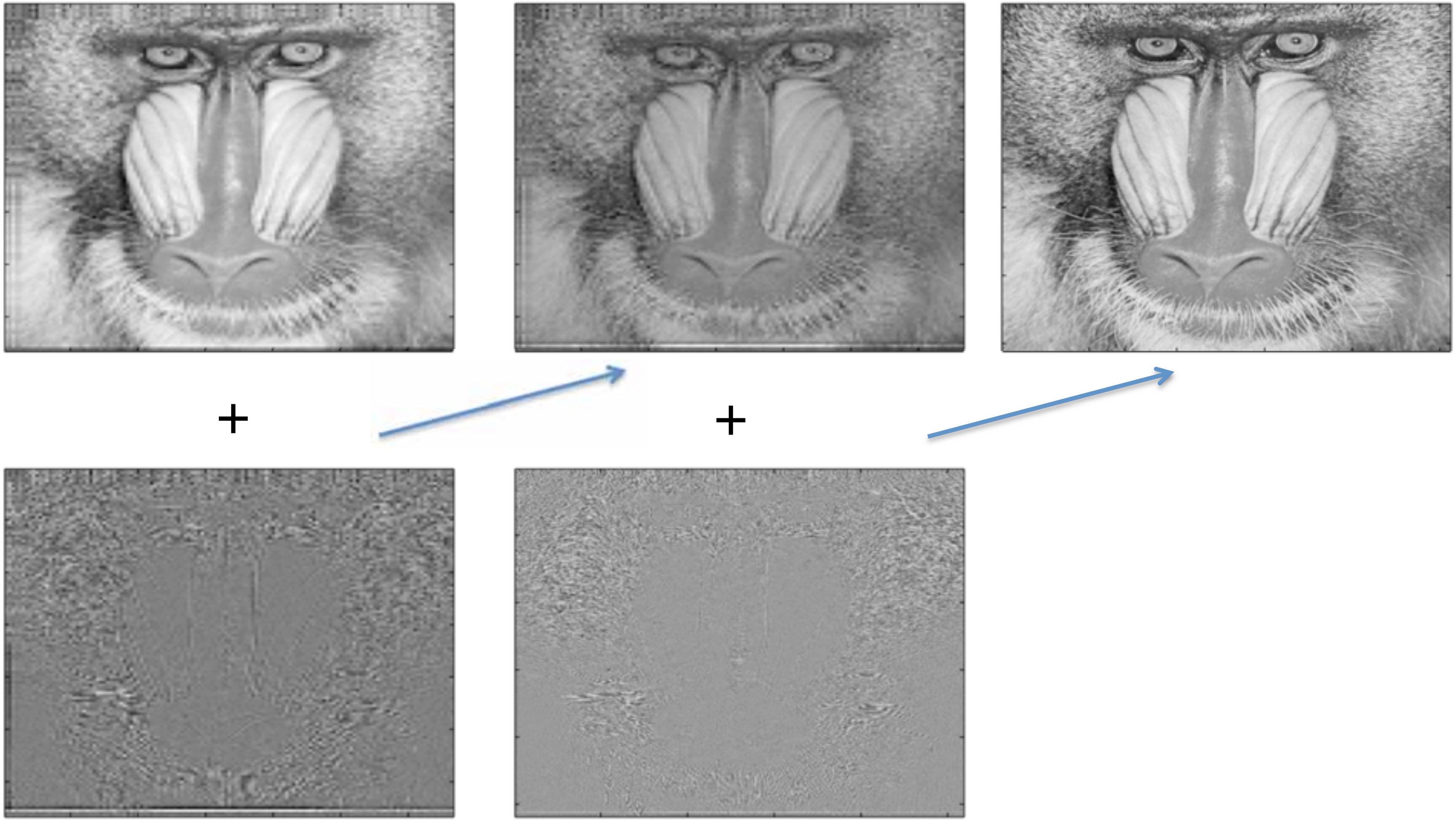
  [2] ***A primer on wavelets and their scientific applications***

  James S. Walker

  [3] ***Essential wavelets for statistical applications and data analysis,*** *R. Todd Ogden*

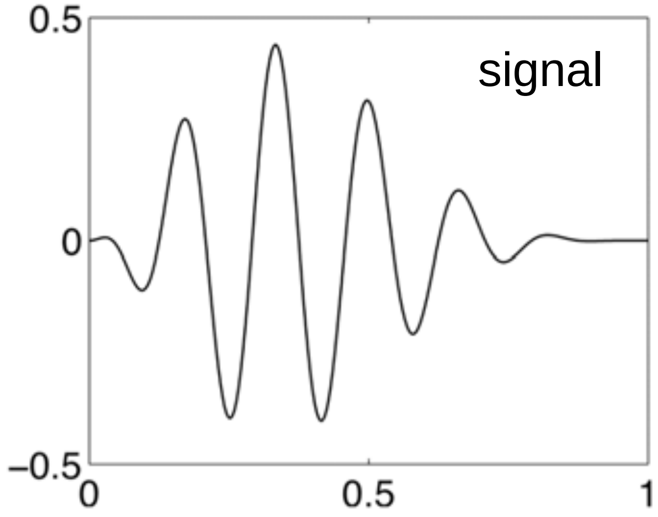
Examples presented in the following slides are taken from [2].
http://www.uwec.edu/walkerjs/Primer

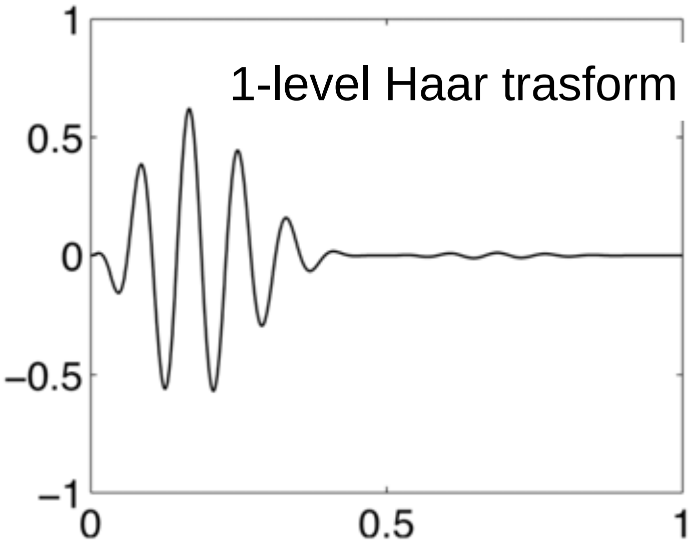
The wavelet representation of reality



Example: 1-level Haar transform

[2] chapter 2





$$g(x) = 20x^2(1-x)^4\cos(12\pi x)$$

N = 1024 => 10-level Haar transform

[C,L] = wavedec(g,1,'Haar');

C ($a^1|d^1$)

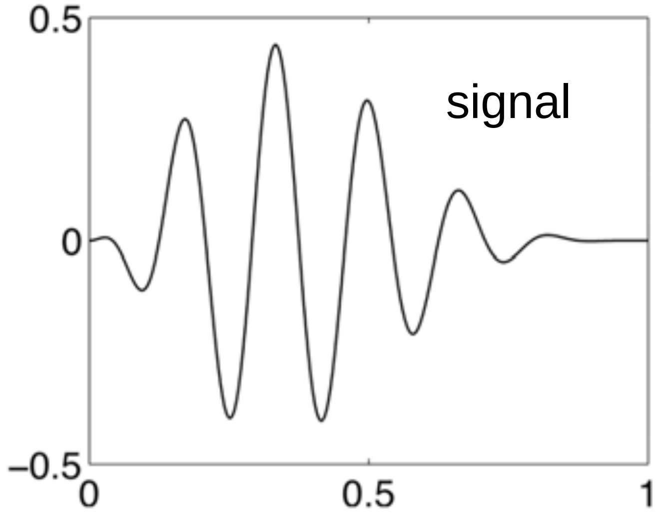
L = [512 512 1024]

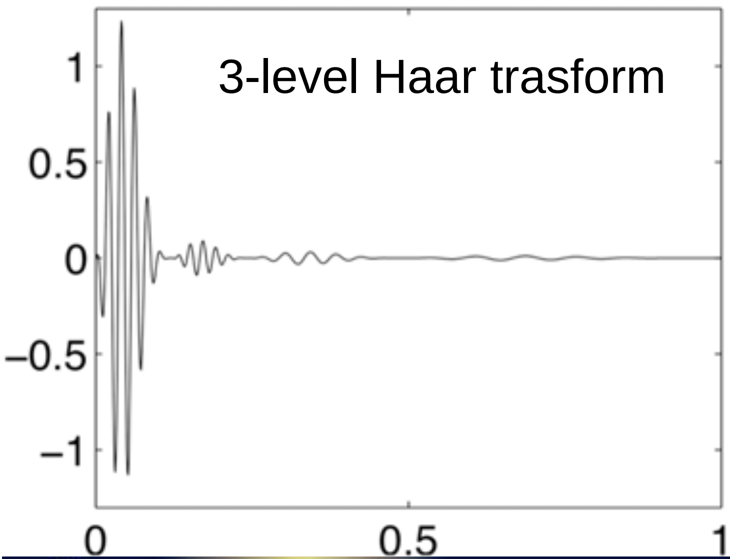
$$a_m \approx \sqrt{2}g(t_m)$$

$$d_m = \frac{g(t_{2m-1}) - g(t_{2m})}{\sqrt{2}} \approx 0$$

3-level Haar transform

[2] chapter 2



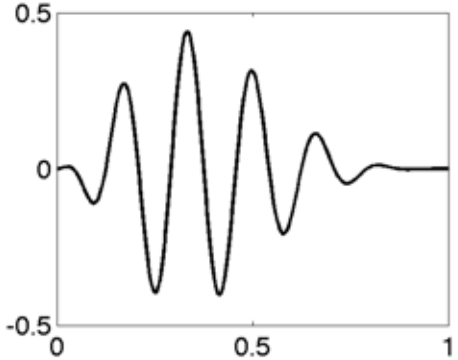


[C,L] = wavedec(g,3,'Haar');

C $(a^3|d^3|d^2|d^1)$
L = [128 128 256 512 1024]

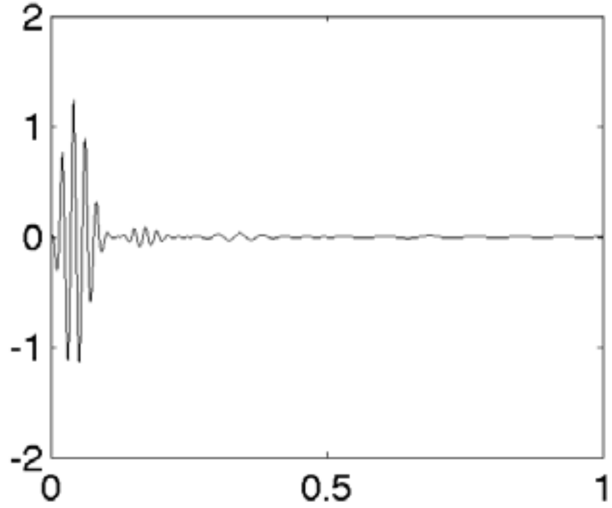
3-level Haar transform

[2] chapter 2

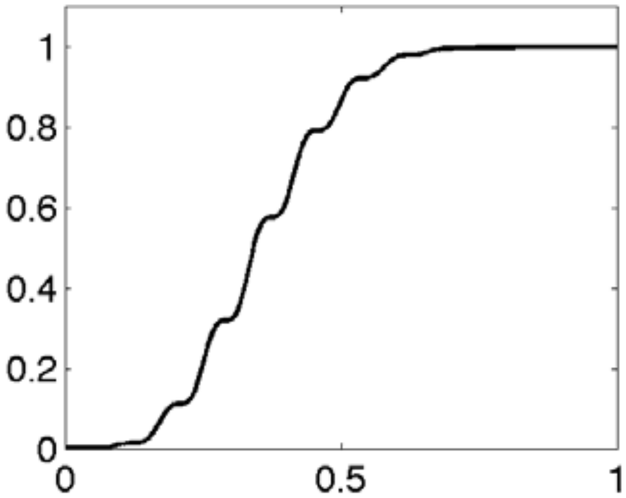


[C,L] = wavedec(g,3,'Haar');

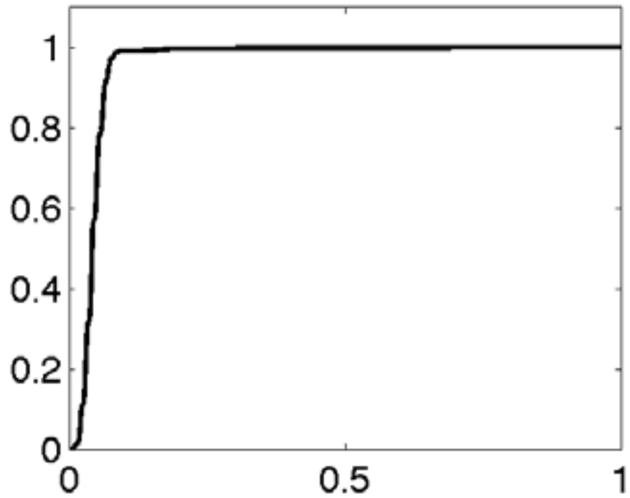
C ($a^3|d^3|d^2|d^1$)
L = [128 128 256 512 1024]



Cumulative energy - signal

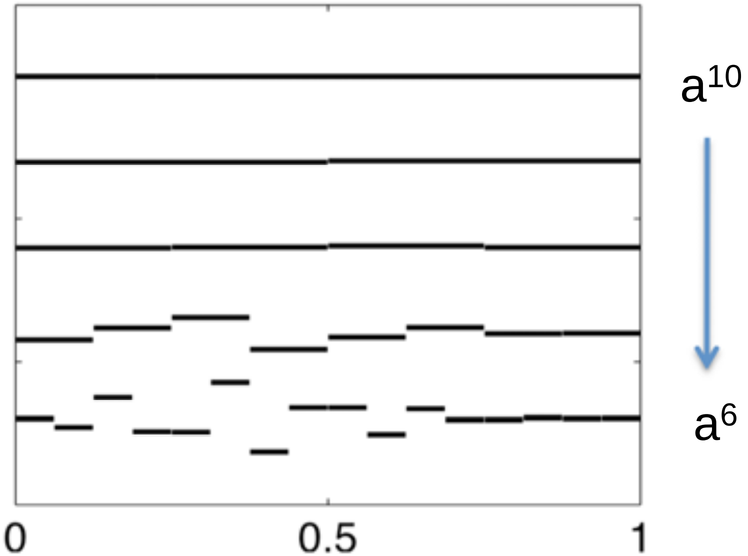


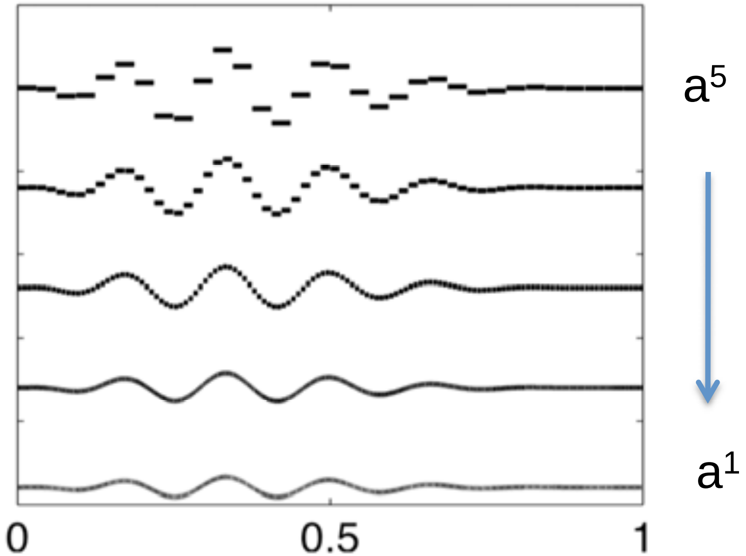
Cumulative energy - transform



10-level Haar transform [2] chapter 2

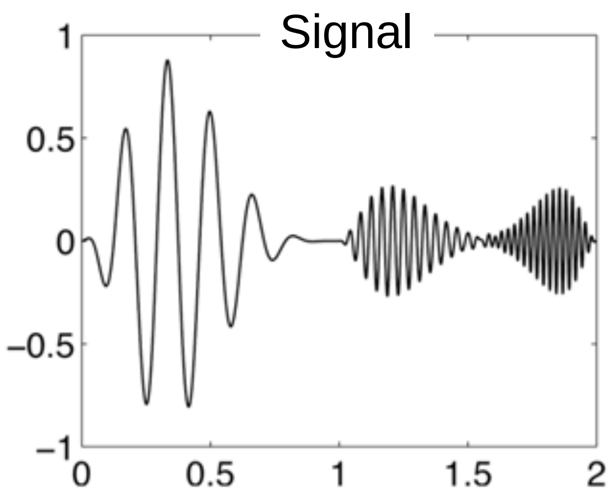
Trend values

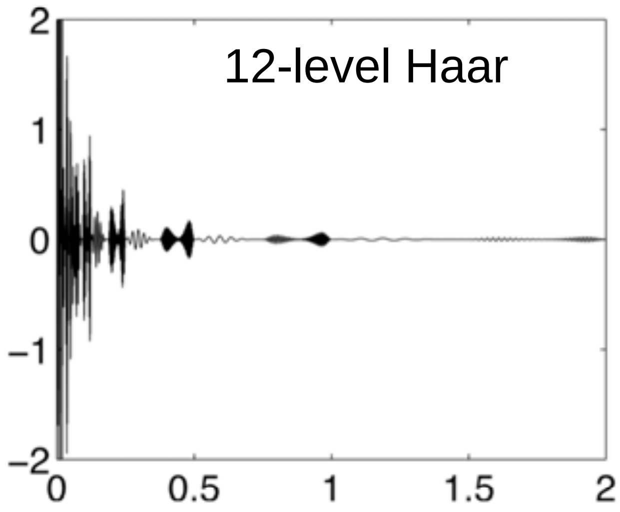




Compaction of energy

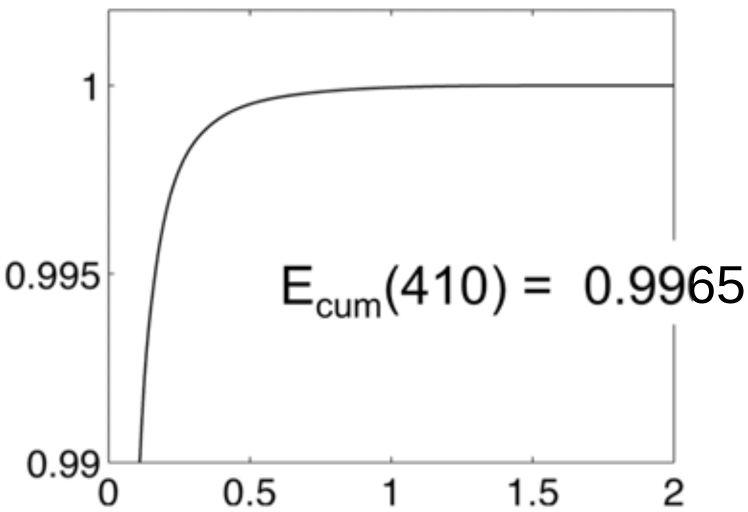
[2] chapter 2

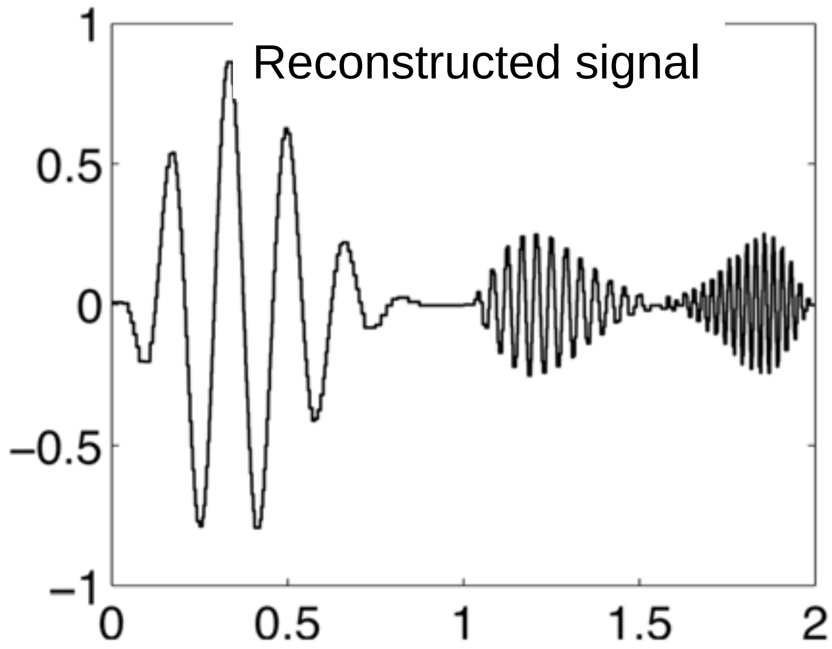




N = 4096
12-level Haar

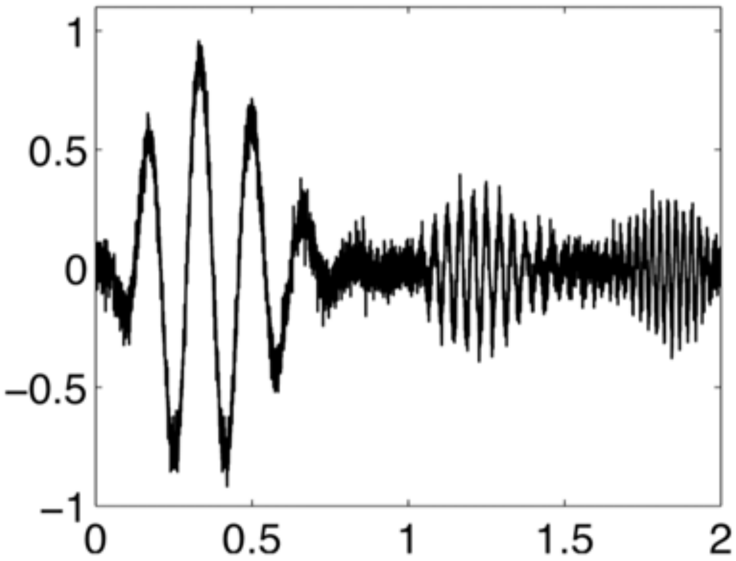
Try to get a
10:1 compression

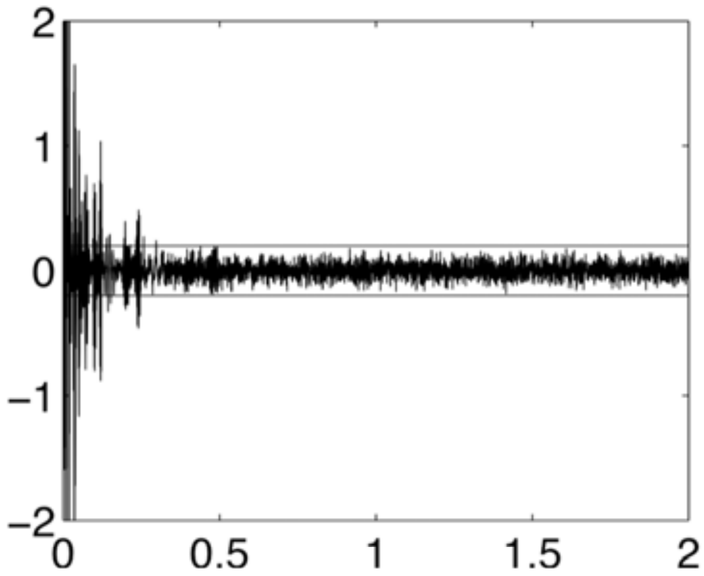


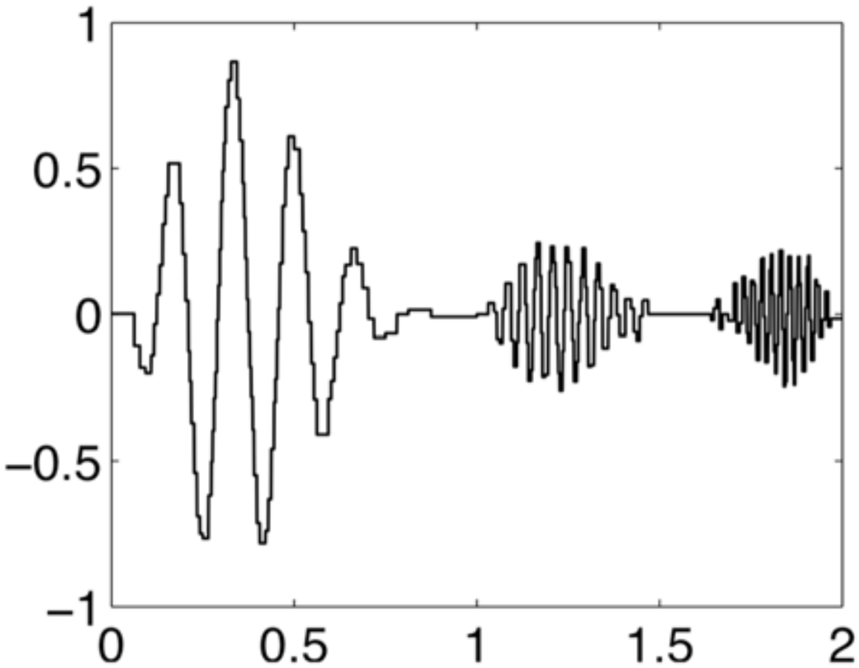


Removing the noise

[2] chapter 2







RMS error = 0.0339

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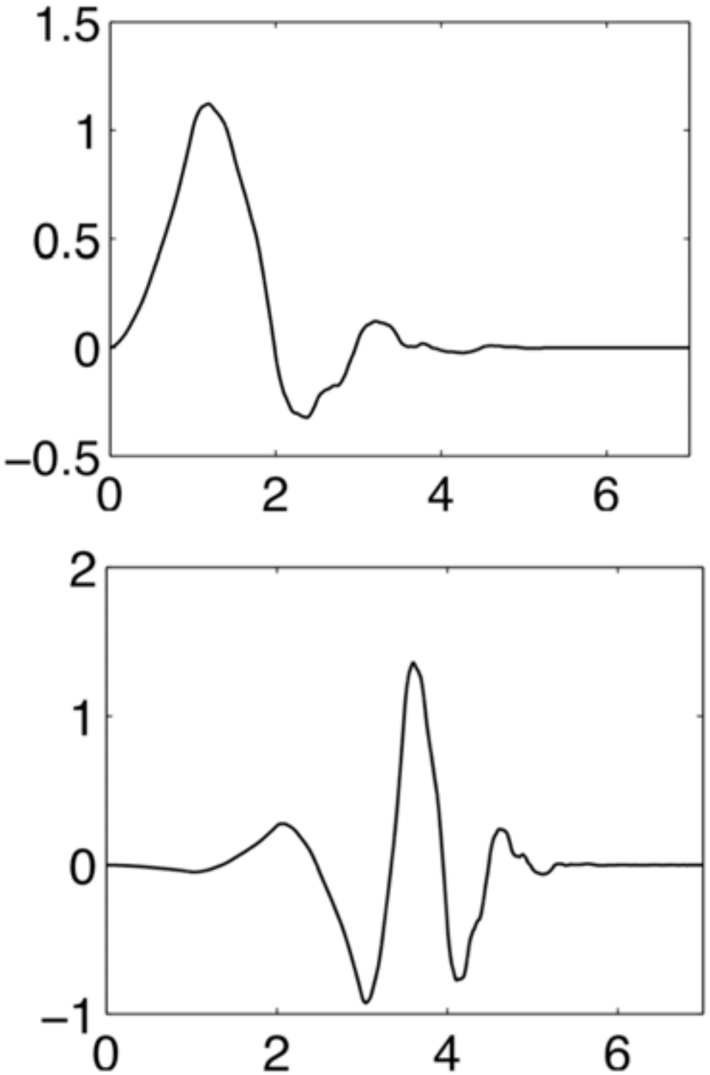
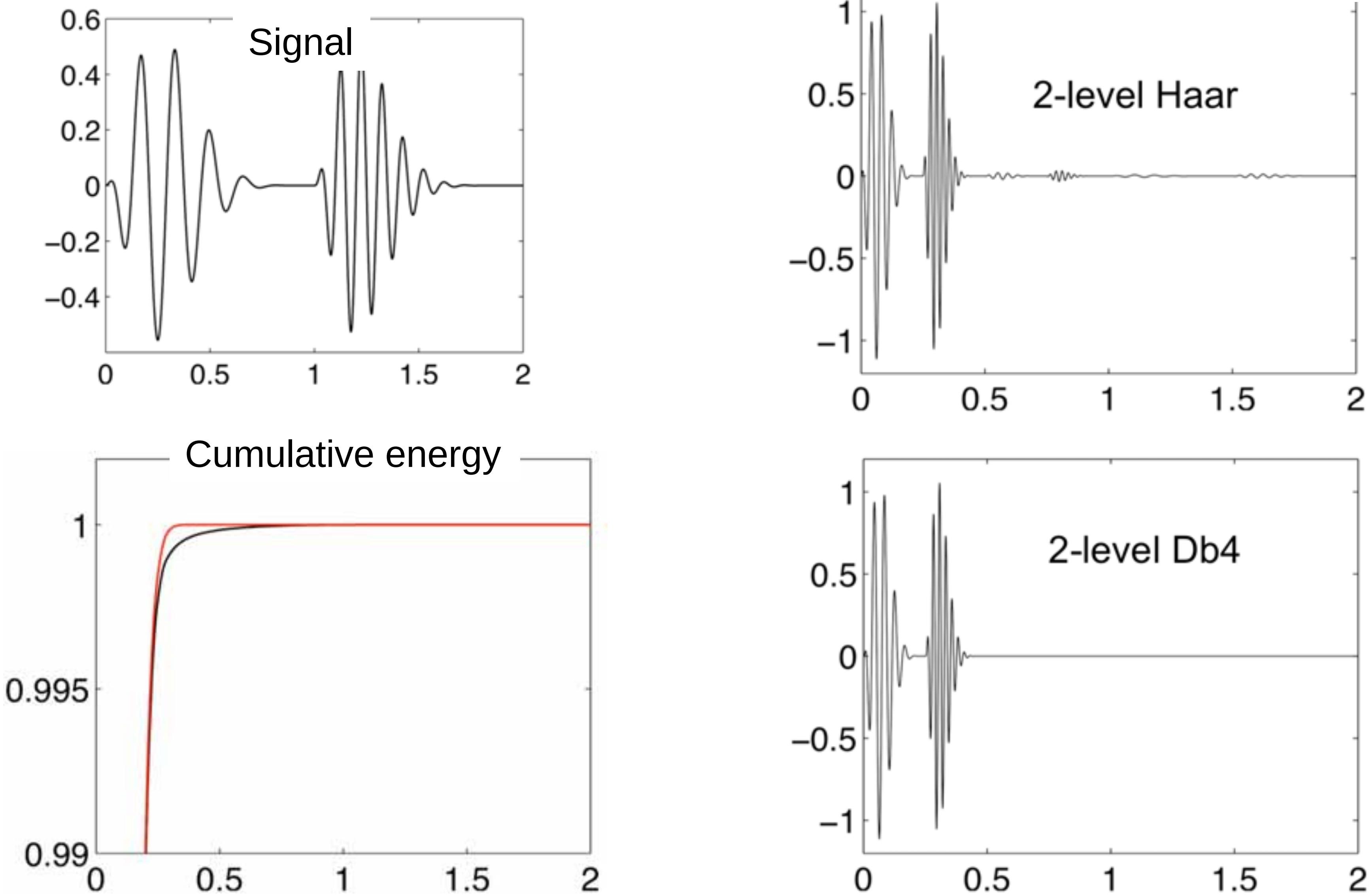
The Daub4 wavelet



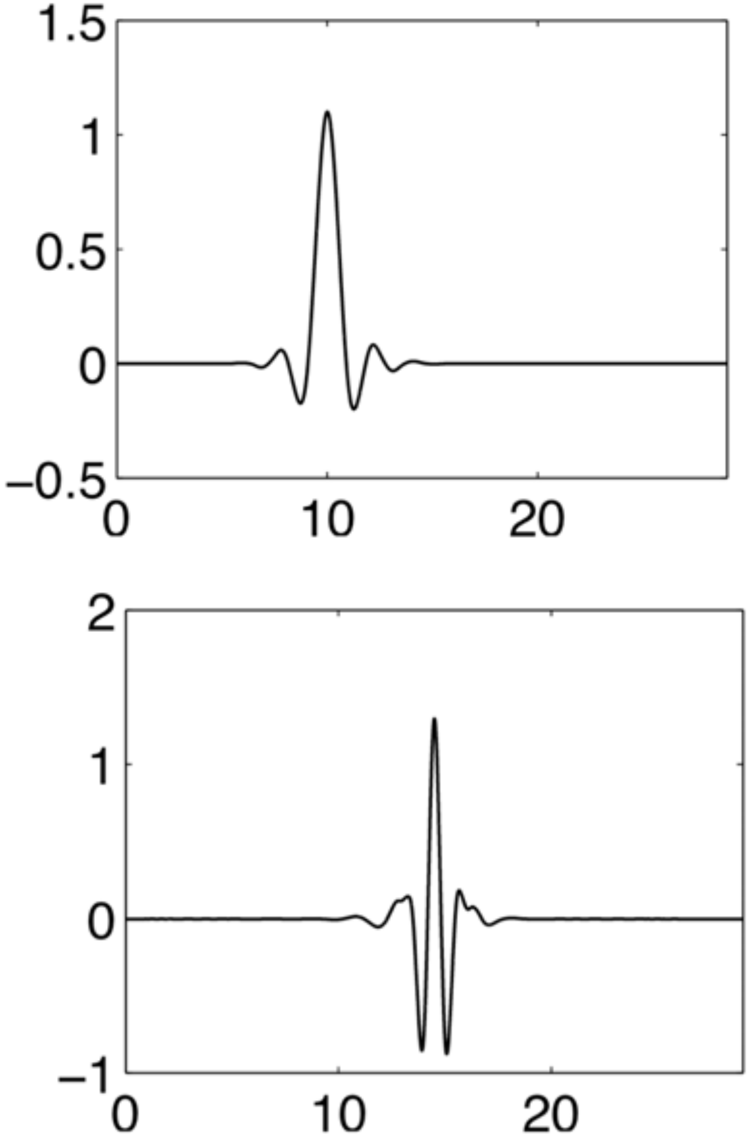
$[\phi,\psi]$ = wavefun('db4',10)

Daubechies wavelet transforms

[2] chapter 3



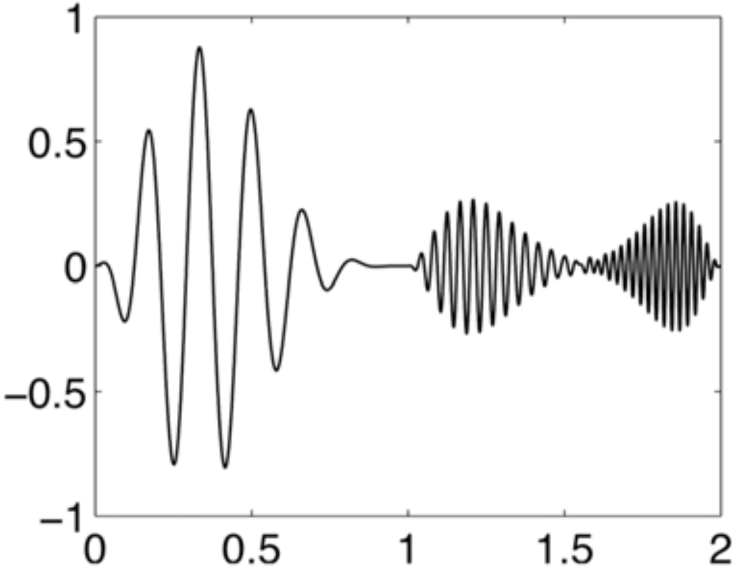
The Coiflets

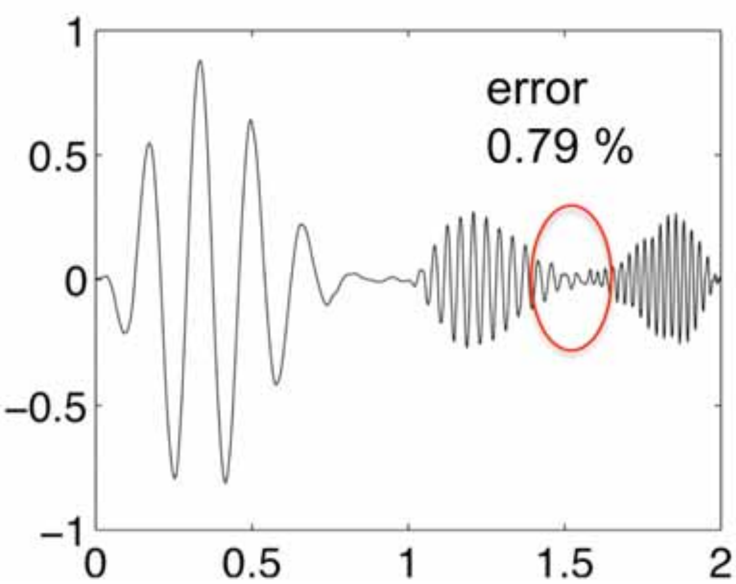


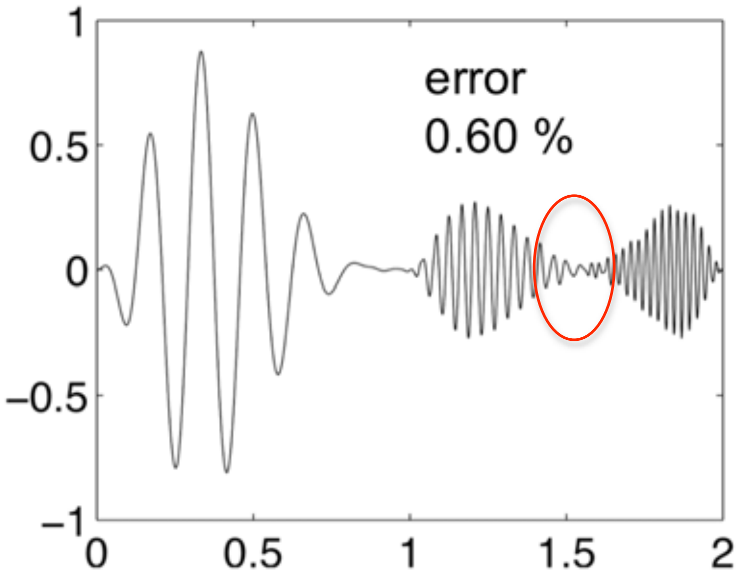
$[\phi,\psi]$ = wavefun('coif5',ITER)

Beyond Daubechies wavelets

[2] chapter 3

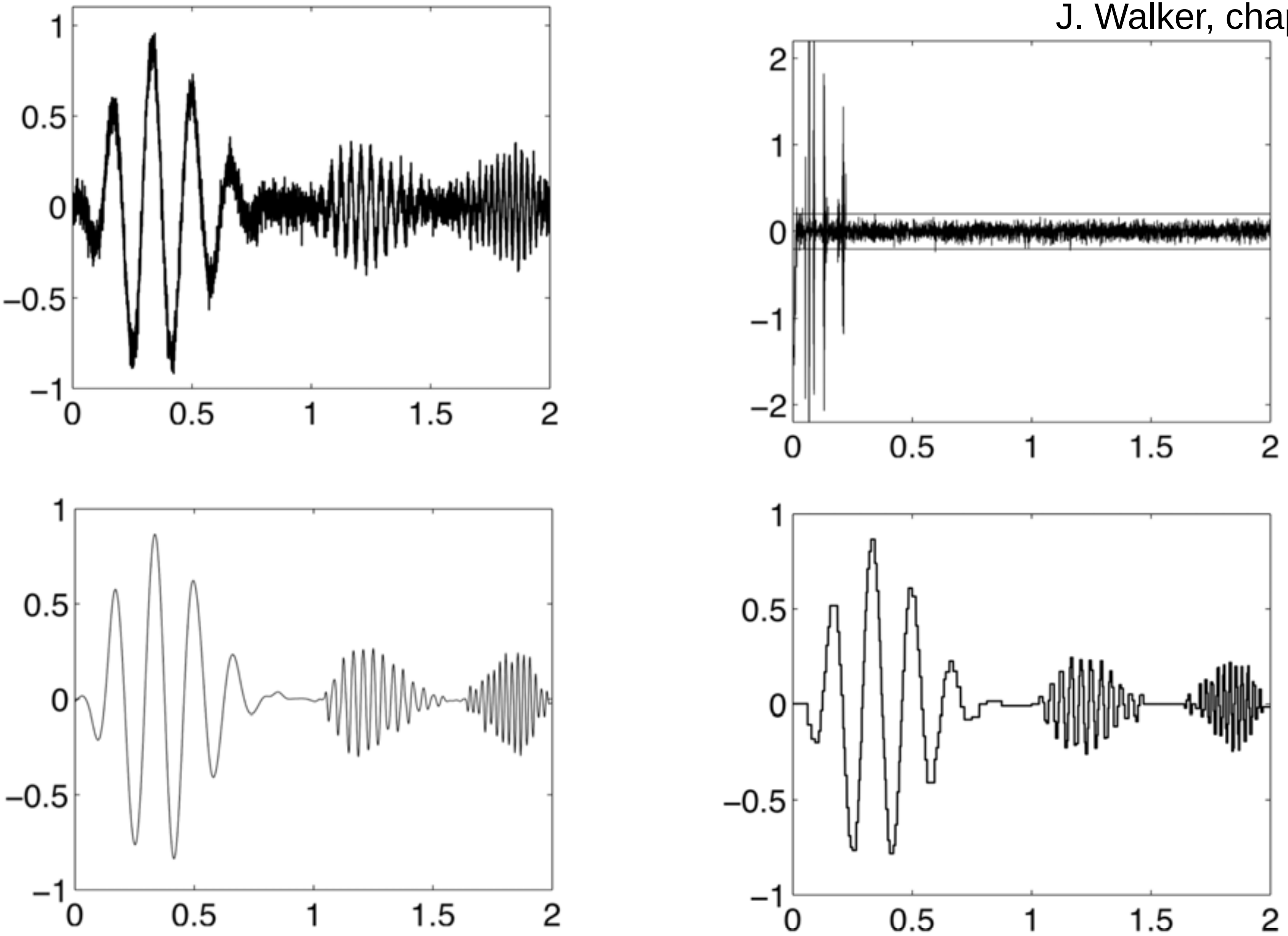






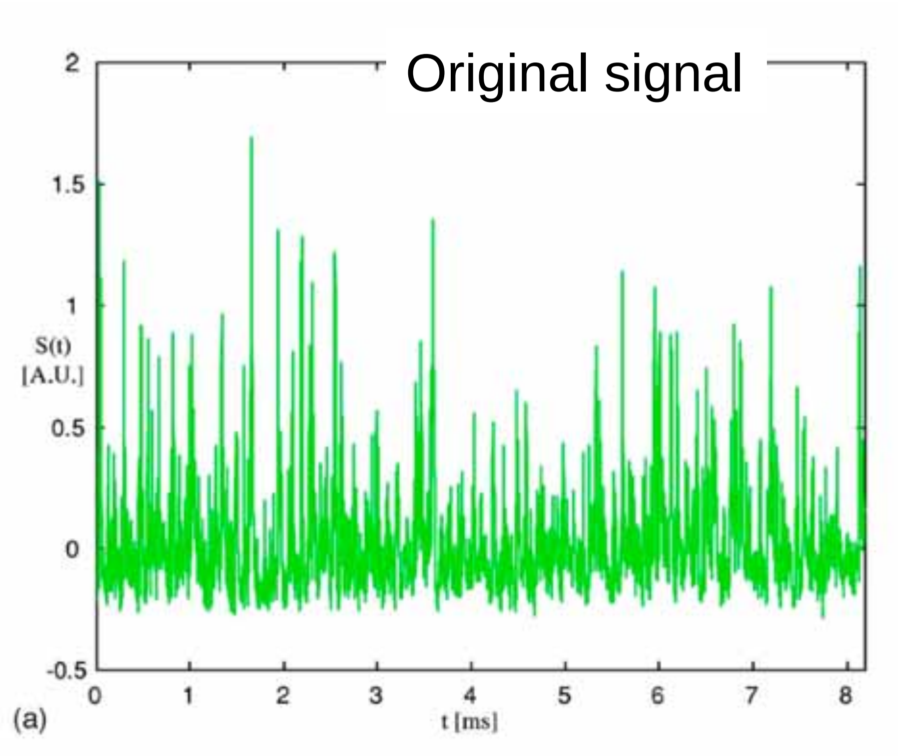
Noise removal

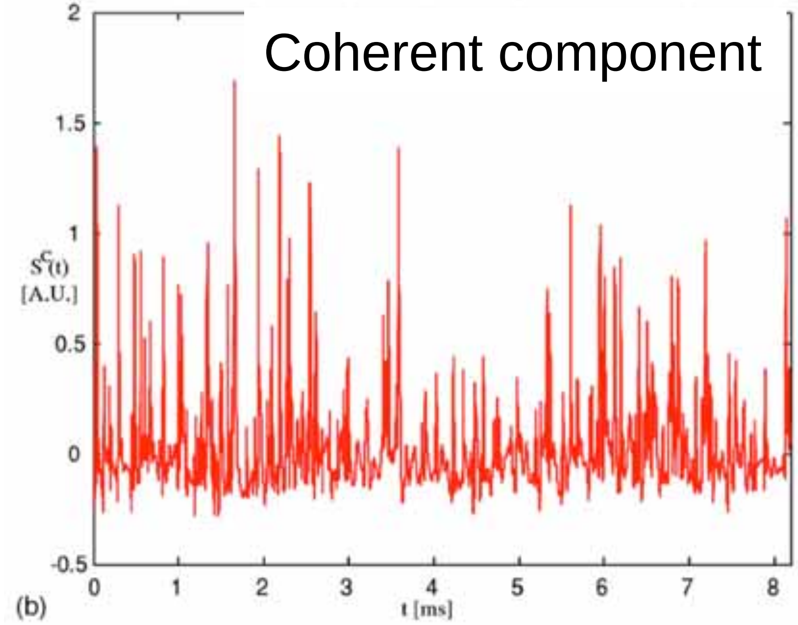
J. Walker, chapter 3

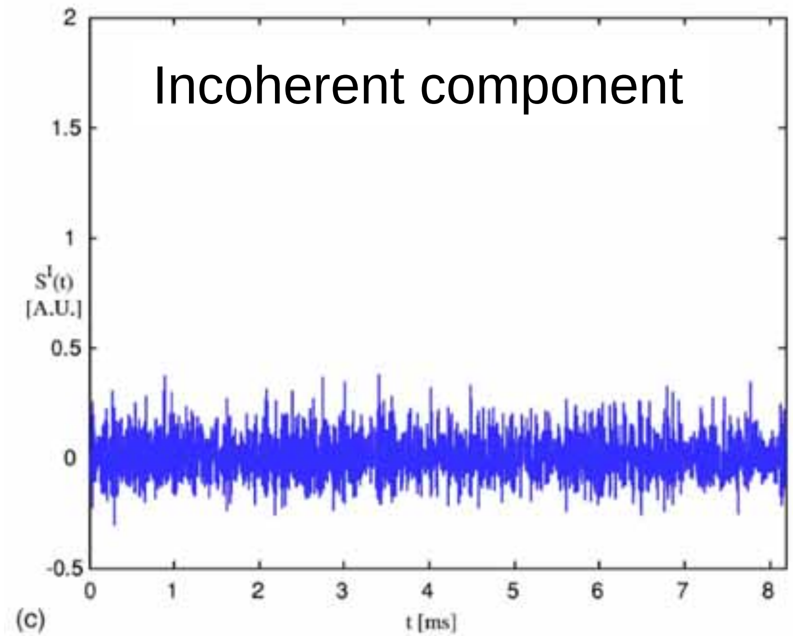


How to isolate coherent bursts

M. Farge et al, Phys. Plasmas 13 042304 (2006)



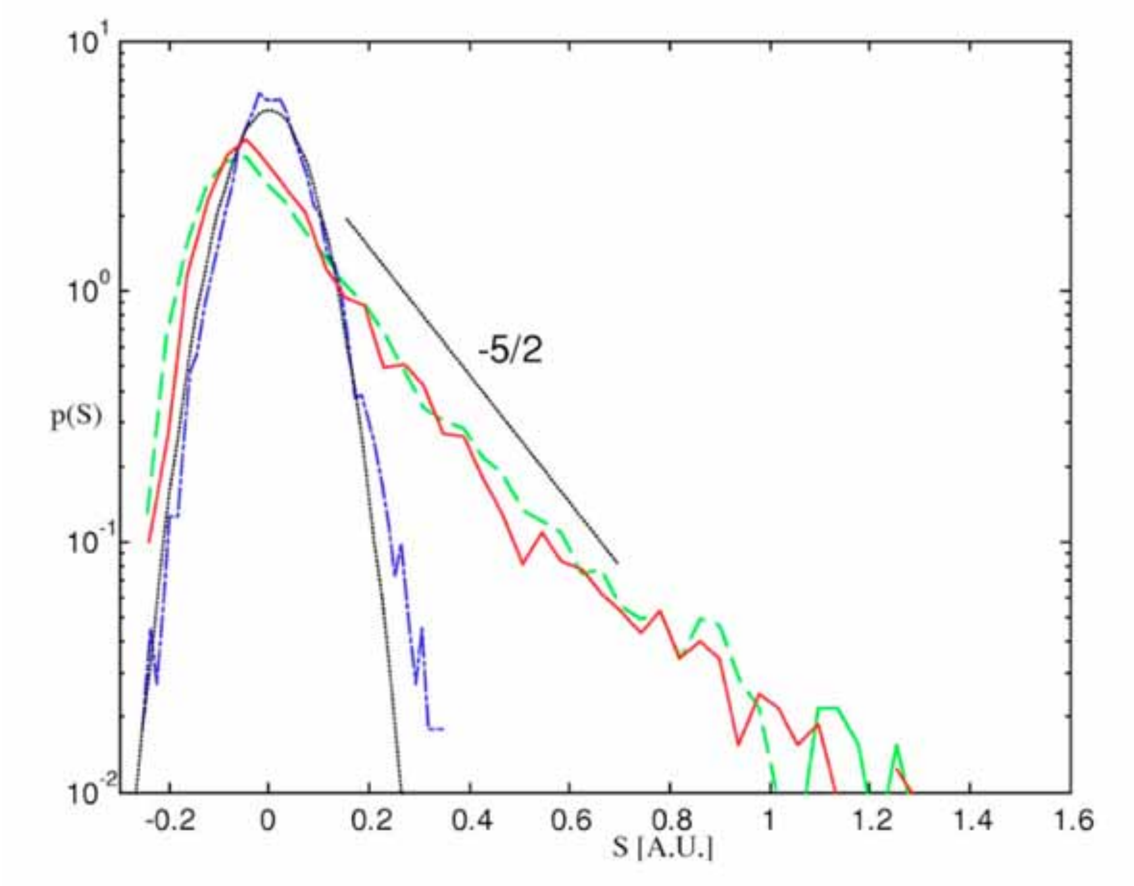




Use Coif12 to decompose
Iterative choice of threshold

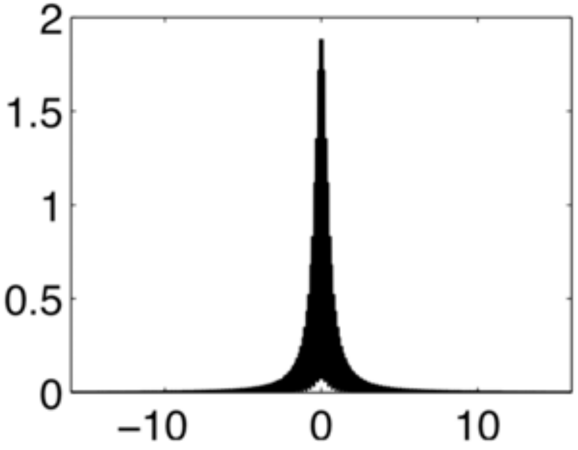
… to separate statistics

M. Farge et al, Phys. Plasmas 13 042304 (2006)

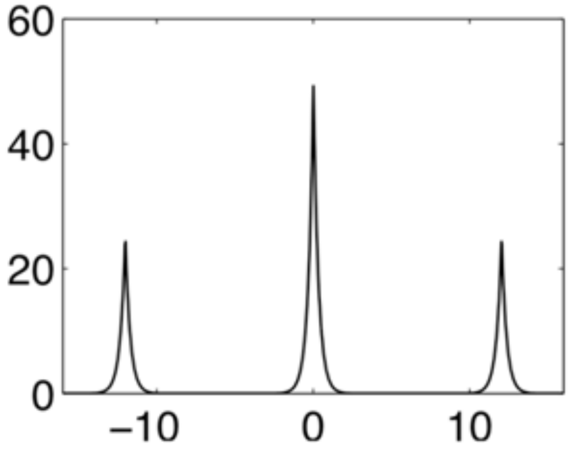


Discrete Fourier Transform

[2] Chapter 5

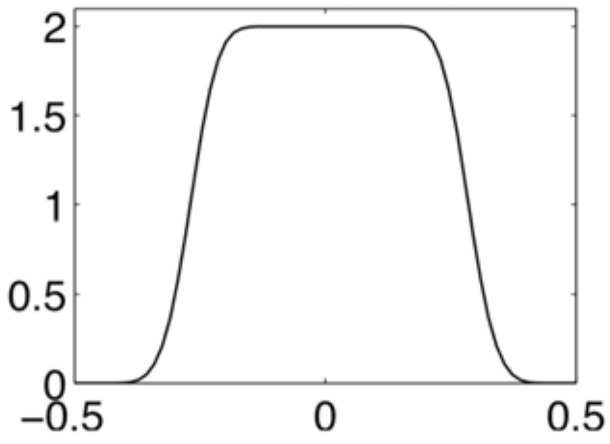


$$x(t) = \frac{1 + \cos(24\pi t)}{1 + 4t^2}$$

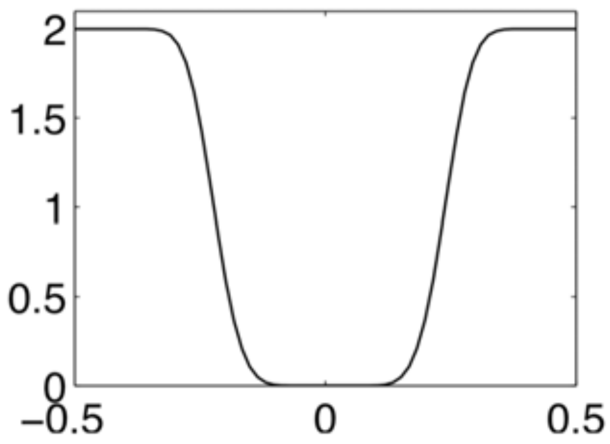


Frequency content of wavelets

[2] Chapter 5



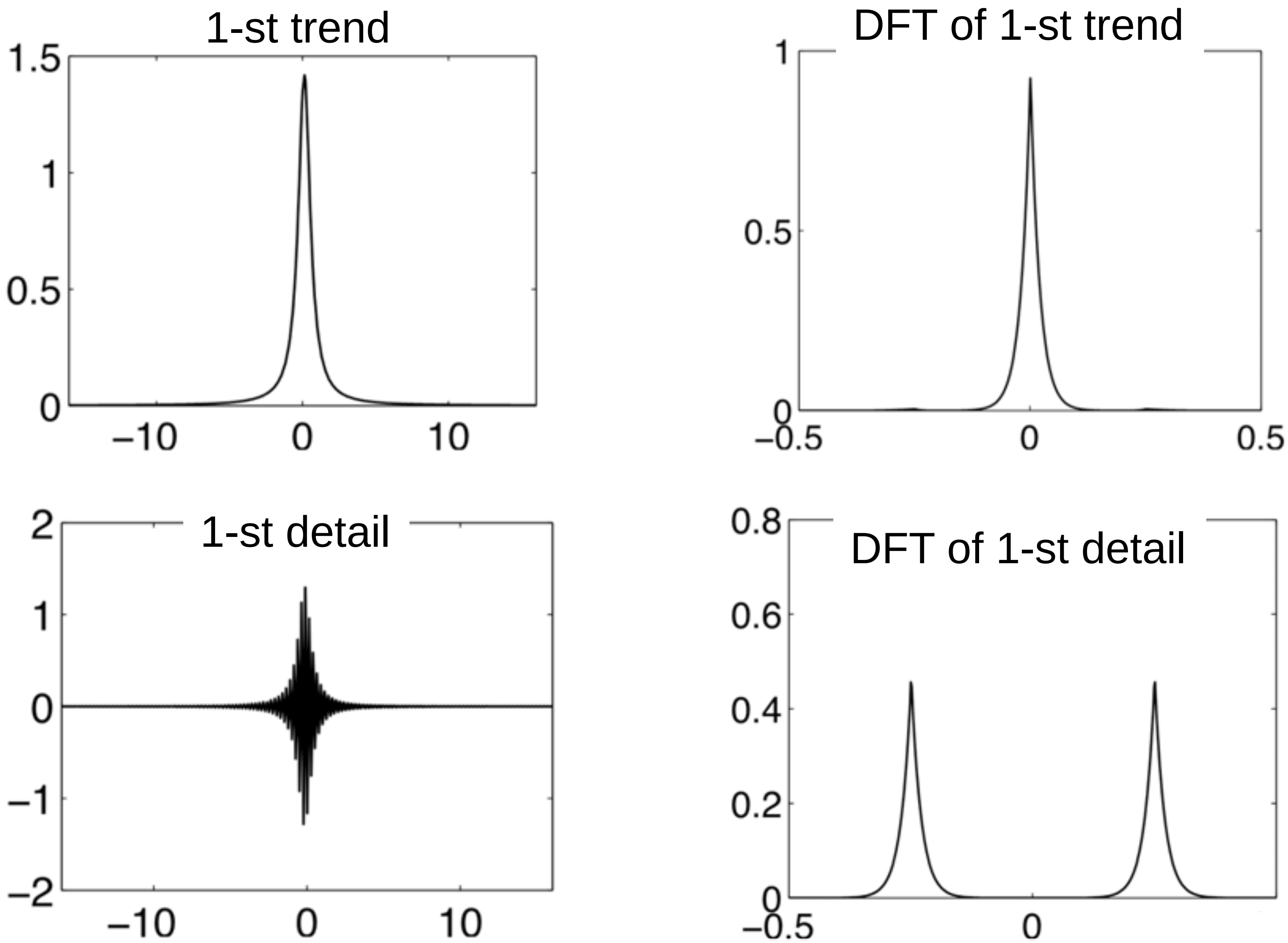
DFT of Coif5 scaling signal



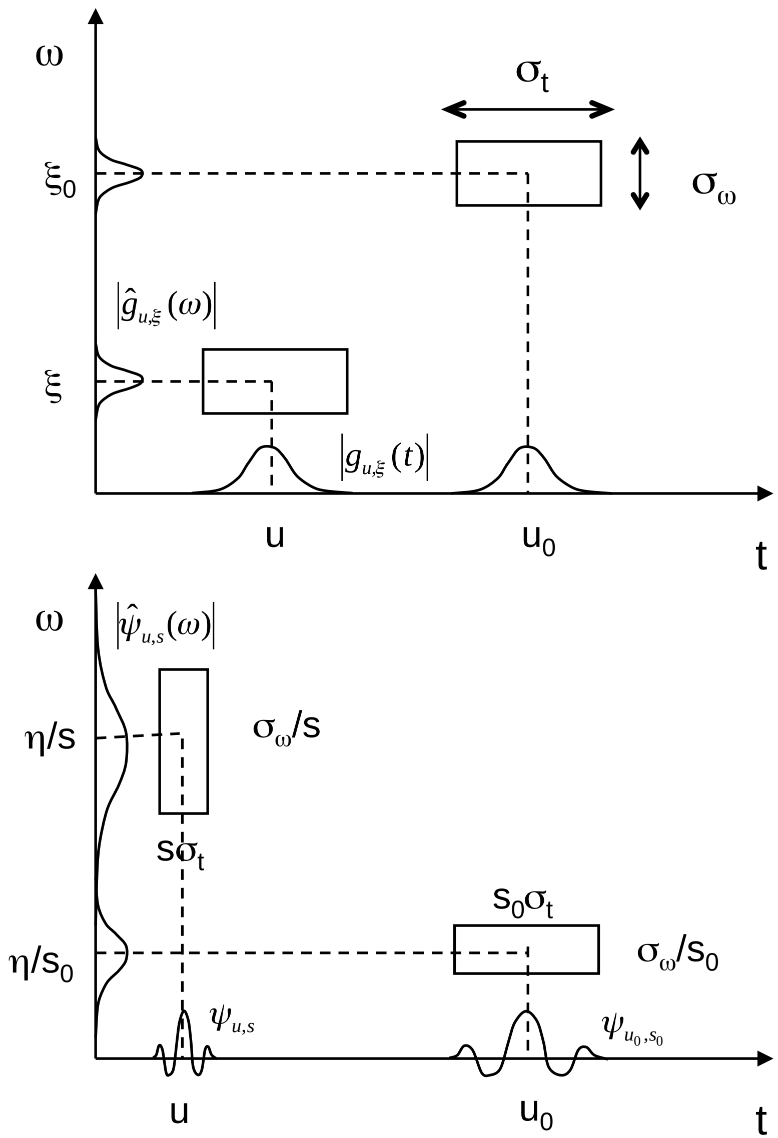
DFT of Coif5 wavelet

Frequency description

[2] Chapter 5



Fourier vs wavelets



Short Time Fourier Transform

$$S_{m,k} = \sum_{l=0}^{N-1} x_l g[l-m] e^{-\frac{2\pi ikl}{N}}$$

$$\omega_k = \frac{2\pi k}{N\delta t} \qquad k = 1,\ldots,N$$

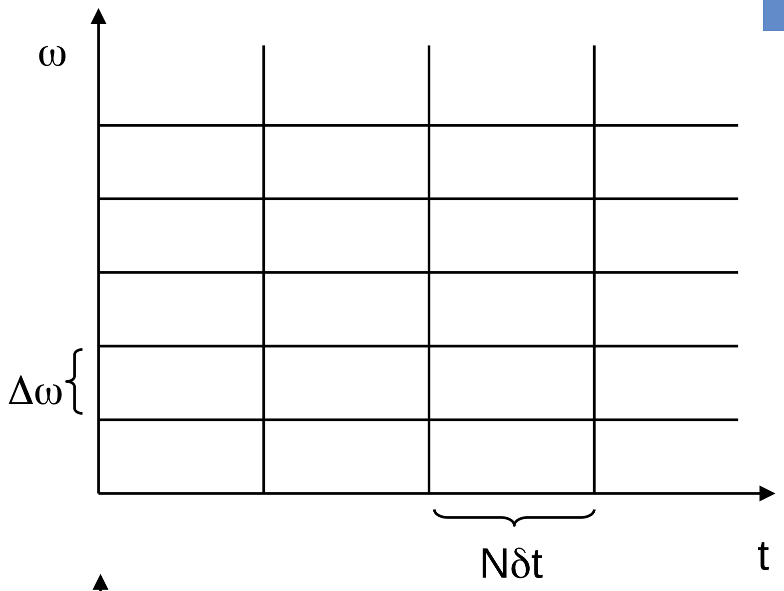
Continuous Wavelet Transform

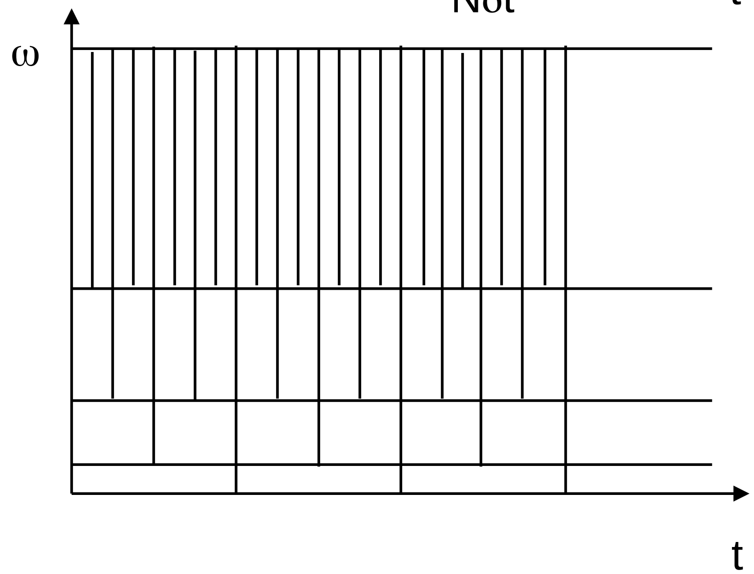
$$W_{m,s} = \frac{1}{\sqrt{s}} \sum_{l=0}^{N-1} x_l \psi^* \left(\frac{l-m}{s} \delta t \right)$$

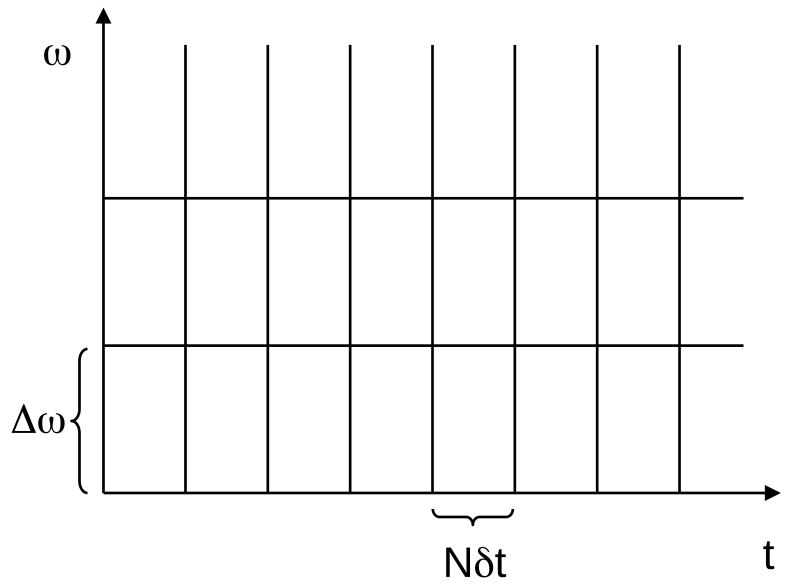
$$s_j = a^j \qquad a = 2^{1/v}$$

$$f_j = \frac{s_0}{s} \frac{1}{\delta t}$$

Time-frequency resolution

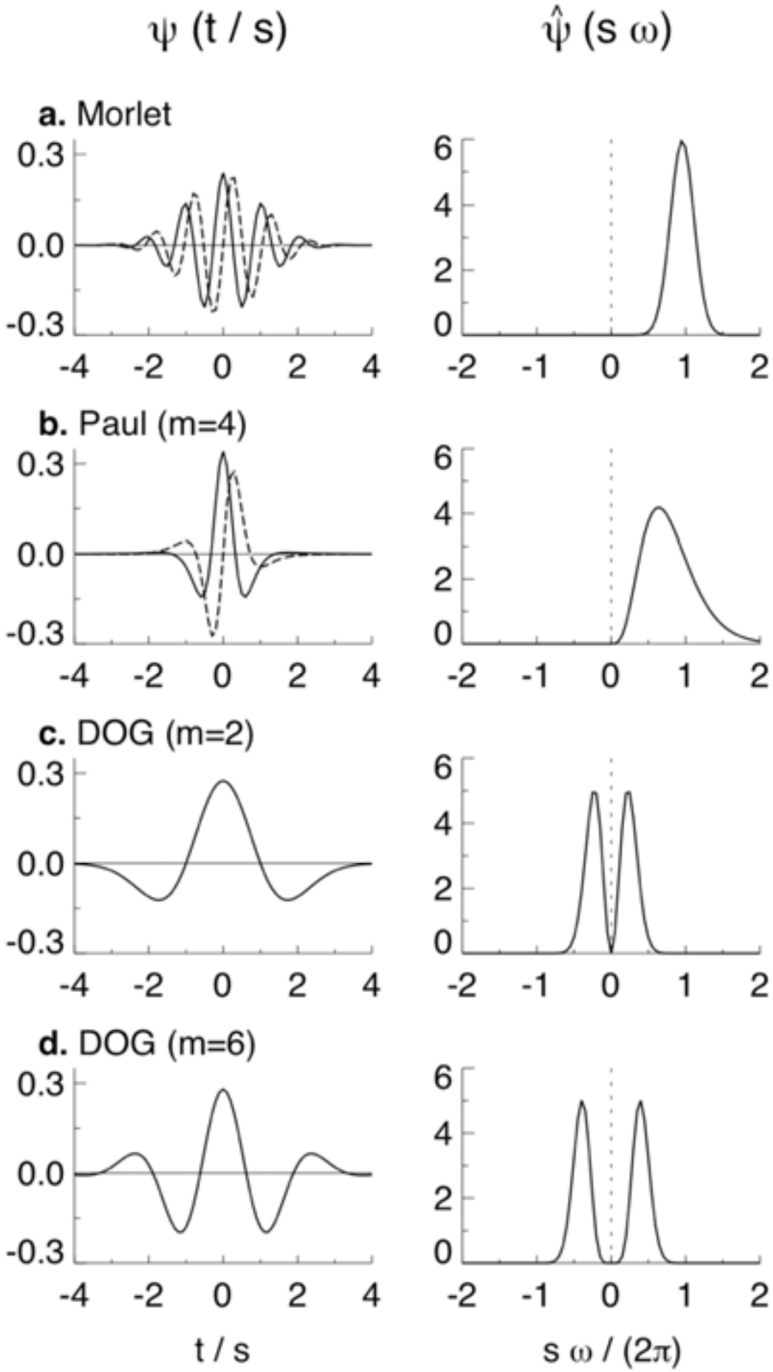






The time-frequency plan is divided into **octants** with constant area.

Good resolution in time at the expenses of resolution in frequency



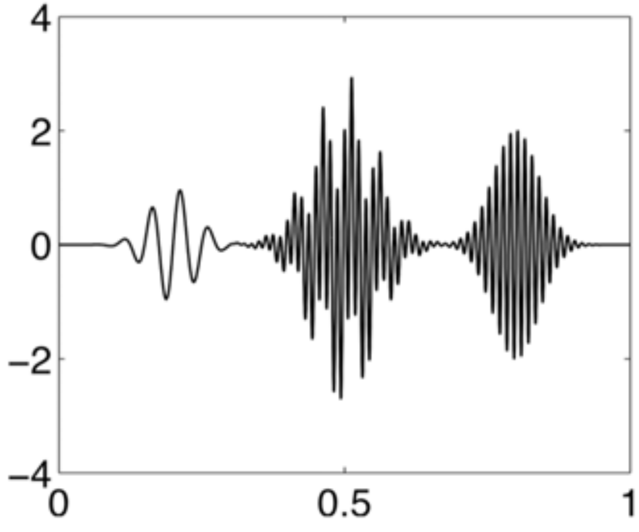
$$\psi(t) = \frac{1}{\pi^{1/4}} e^{i\omega_0 t} e^{-t^2/2\sigma^2}$$

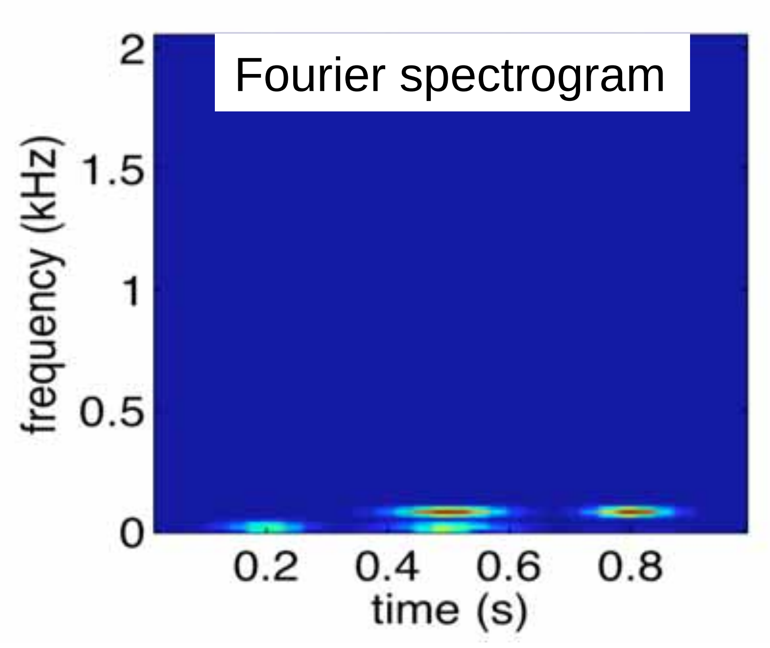
$$\frac{2^m i^m m!}{\sqrt{\pi(2m)!}}(1 - i\eta)^{-(m+1)}$$

$$\frac{(-1)^{m+1}}{\sqrt{\Gamma\left(m + \frac{1}{2}\right)}} \frac{d^m}{d\eta^m}\left(e^{-\eta^2/2}\right)$$

Example: superposition of modes

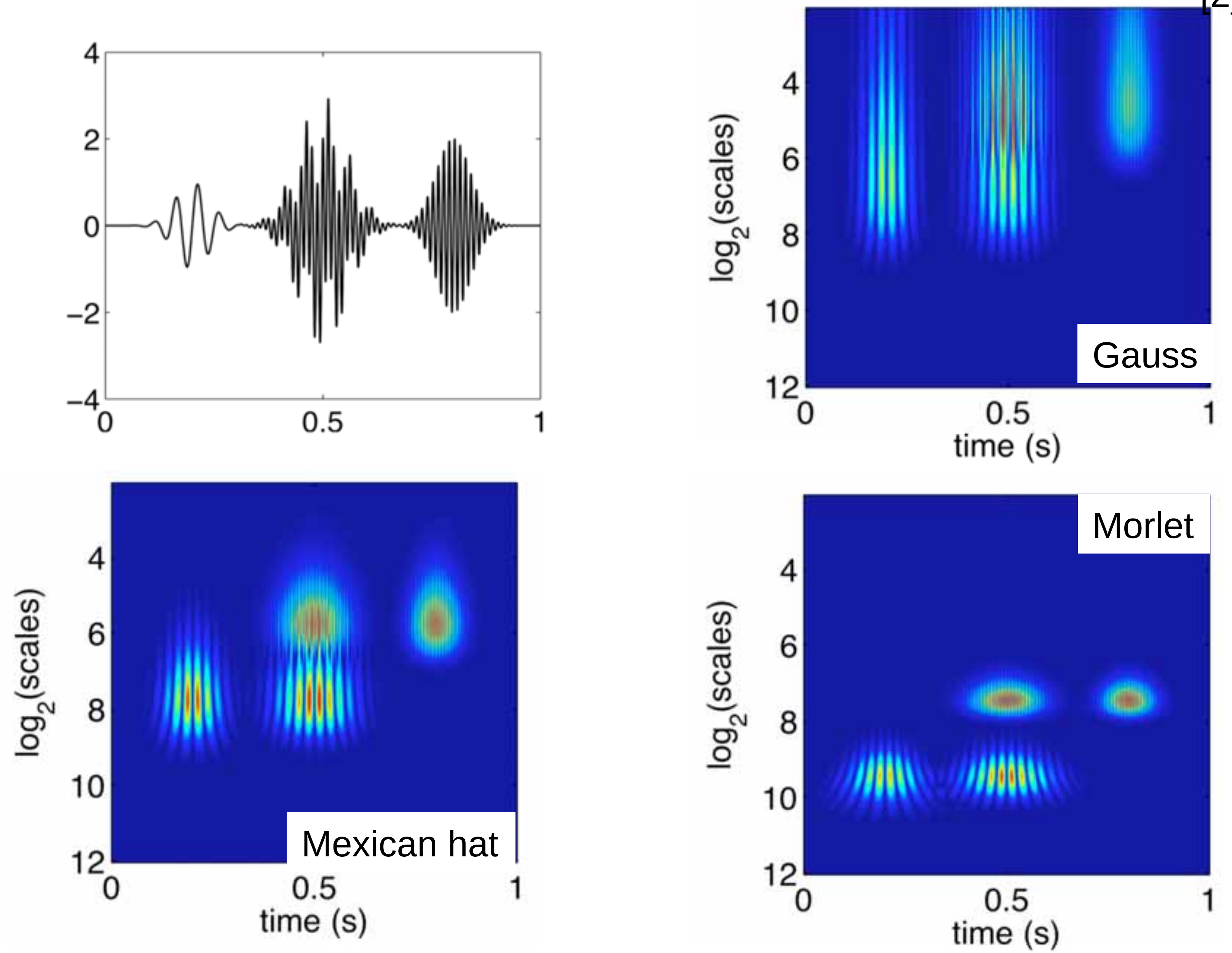
[2] Chapter 6





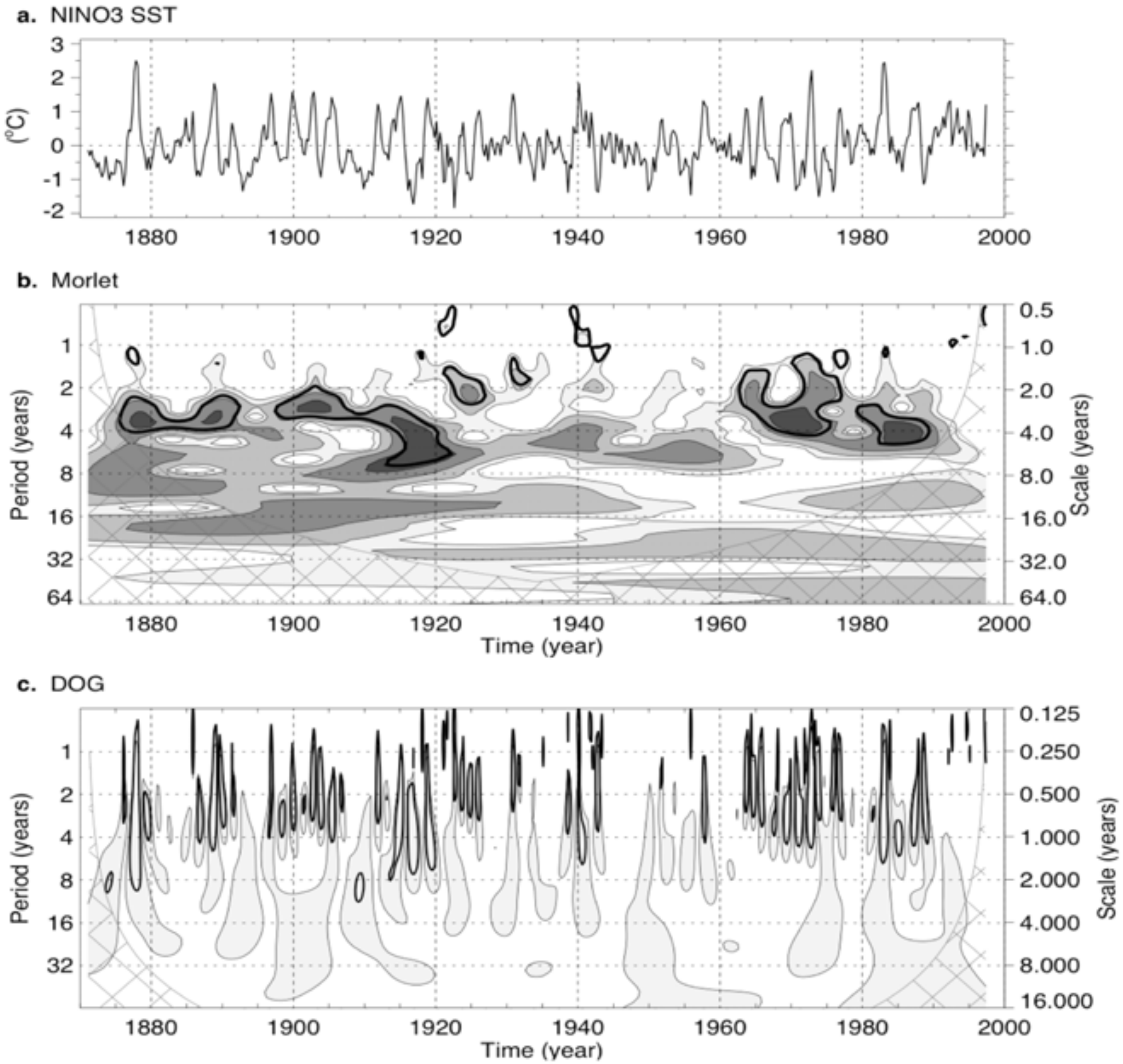
Example: superposition of modes

[2] Chapter 6

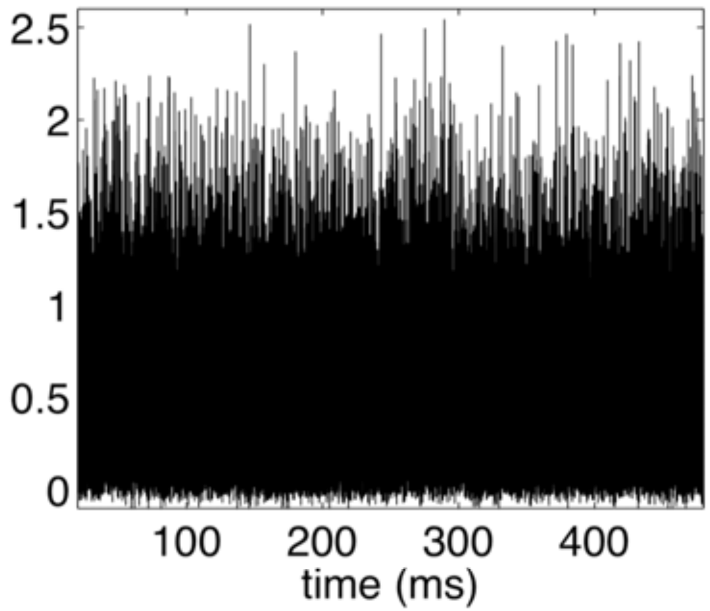


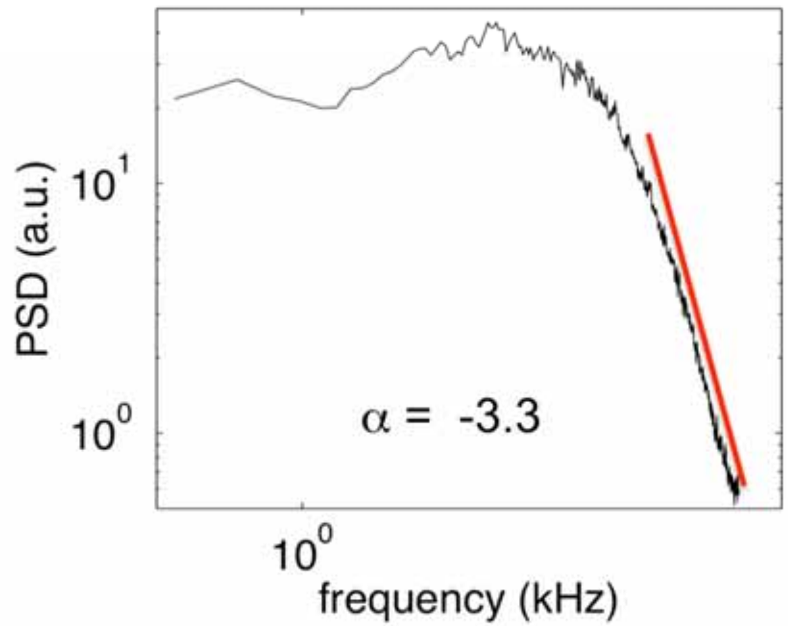
Application: CWT of EL Niño

Torrence and Compo, Bull. Am. Meteo. Soc. **79**, 61 (1998)

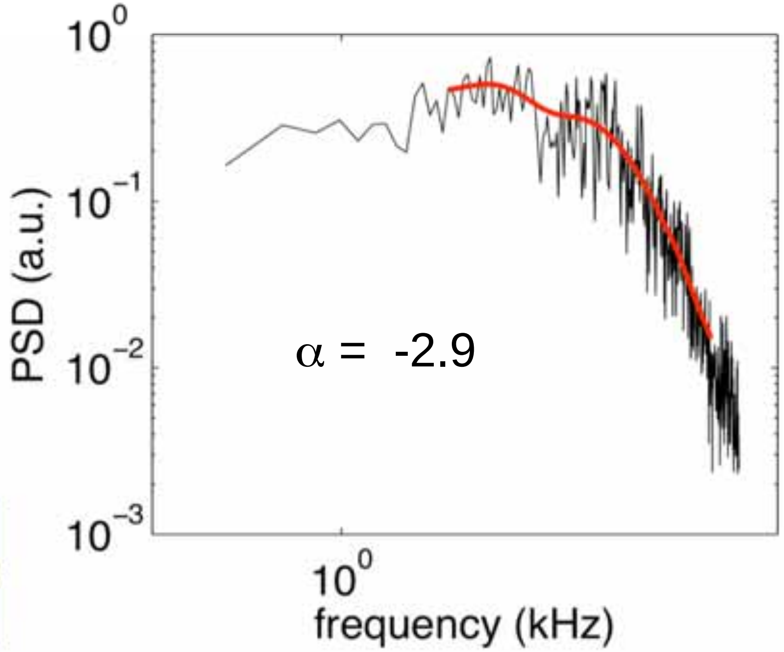


Application: spectral index





C = cwt(data,[2:1:48],'db4')



Fully turbulent spectrum