Cluster-Cluster Aggregation with Levy Diffusion

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Project Outline and Objectives

Consider a collection of particles subject to a transport mechanism, such as diffusion, which stick irreversibly to each other upon contact. Cluster-cluster aggregation refers to the process whereby such a system of particles will evolve in time to produce heavier and heavier clusters of particles. Cluster-cluster aggregation is interesting for many reasons. It has applications in physics and chemistry relating to the science of colloidal suspensions, aerosols and gels. It also provides an interesting example of a nonequilibrium phase transition between weakly and strongly fluctuating regimes as the spatial dimension of the system is changed. The critical dimension is two. Above two dimensions clusters are essentially uncorrelated from each other and a mean field treatment describes the system well. Below two dimensions, clusters are correlated in space, an effect which can be traced back to the recurrence property of random walks in low dimensions. Thus the scaling and statistical properties of the clusters differs greatly between these two regimes.

In recent work [1] we found that for certain classes of aggregation problems, there is a special correlation function, known as the mass flux correlation function, which, counter to most intuitions in the subject, should be insensitive to the transition from weak to strong fluctuations. The objective of this project is to verify this theoretical prediction via analysis and simulation of some concrete stochastic lattice models of cluster aggregation.

It is not convenient to study a transition which requires varying the spatial dimension so we propose, instead, to study a one dimensional model in which the diffusive transport is replaced by a levy flight or some other non-local transport process. The idea, which we must test as part of the project, is that by tuning the probability of the large jumps one can mimic the breaking of correlations which occurs due to strong mixing in higher dimensions. The objective will be to characterise the spatial correlations as the strength of jumps is varied and confirm that the scaling of the flux correlation function is independent of the strength of the fluctuations.

Required Background and Methodology

The student will be required to implement Monte Carlo simulations of some lattice models of cluster cluster aggregation in one dimension and analyse the data. Basic knowledge of stochastic processes will be required and a willingness to explore a little bit the theory of cluster-cluster aggregation and nonequilibrium phase transitions.

Research Outcomes and Outlook

The principal research outcome will be a better understanding of the role of spatial fluctuations in determining the statistics of cluster cluster aggregation. Lattice models of cluster-cluster aggregation can be mapped fully or partially to a surprising variety of models in other disciplines such as drainage problems, dynamics of growing random networks and the directed sandpile model of self-organised criticality. Hence this research feeds into the general complexity effort at Warwick and beyond.

References

[1] C. Connaughton, R. Rajesh, and O. Zaboronski. Constant Flux Relation for diffusion limited cluster–cluster aggregation. *Phys. Rev. E*, 78(041403), October 2008.

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