Pattern formation in traffic flow

Since the pioneering works for highway traffic in the 90's, traffic in one-dimensional geometries continues to be an area of major research, with applications also in other areas of science [1]. Traffic flow can be modelled for example by cellular automata (Markov chains) on a microscopic level following the motion of individuals, and by PDEs for their density on a macroscopic level. To fully understand the connection between two such descriptions on different scales, is one of the fundamental questions of complexity science. In terms of highway traffic, simple Markov chain models are capable of reproducing all basic space-time patterns of real traffic, such as free flow, congestion and stop&go waves. On the other hand, the corresponding macroscopic models are not able to correctly predict all patterns, and this discrepancy poses a fundamental question which has not been answered satisfactory so far [2].

Details.

A simple model for highway traffic is given by a particle system (as considered in CO905, sheet 3) with rates

$$0100 \xrightarrow{1} 0010 \;, \quad 1101 \xrightarrow{\alpha} 1011 \;, \quad 0101 \xrightarrow{\beta} 0011 \;, \quad 1100 \xrightarrow{\gamma} 1010 \;. \tag{1}$$

It turns out that this simple model is already rich enough to reproduce the three basic space-time patterns. The corresponding macroscopic model is given by the conservation law

$$\frac{\partial}{\partial t}\rho(x,t) + \frac{\partial}{\partial x}j\big(\rho(x,t)\big) = 0 \tag{2}$$

for the car density $\rho(x,t)$, also called LWR model (Lighthill, Whitham and Richards) [2]. It can be derived as a scaling limit of (1), and the flux of cars j as a function of the density is called the fundamental diagram. It is of great importance since it captures the basic physics of the problem on a macroscopic level and is directly accessible to measurements via induction loops on highways. The solutions of (2) should reflect the large scale behaviour of the microscopic particle system (1), but they fail to predict more complicated patterns correctly, such as the downstream interface of stop&go waves. At present there are several approaches to fix this problem, none of which is entirely satisfactory (see section 4 of [2] for a nice discussion).

Aim of the project.

The project is a step towards a (hopefully) more satisfactory solution of this problem along the lines of [3]. The idea is to remember that solutions to the above PDE given by characteristics of the equation are a-priori not unique. There are several established admissability criteria [3] to single out the physically relevant solution, such as viscosity or entropy methods. The upshot is that the PDE alone cannot provide a full description of the system and additional information is required. This depends of course on the 'physics' of the problem which should be very different for cars than it is for gases, for which the classical criteria have been derived.

The practical part of the project will consist of Monte-Carlo simulations of the model (1) to determine the exact shape of the downstream density profile of stop&go waves. These should be compared to a suitably adjusted admissible solution of (2). To be on the safe side, the choice of parameters in (1) can focus on the so-called KLS model (see section 2.2.3 of [4] for a short introduction), for which the exact fundamental diagram can be computed. Furthermore, we can compute the thermodynamic entropy of the KLS model and check wether it provides us with a reasonable admissability criterion.

The project will also provide a good introduction to fundamental relations between macroscopic and microscopic modelling of critical phenomena. The understanding of these relations in complex systems outside the realm of classical thermodynamics is far from complete, and traffic flow is just one example. So there is ample room for continuing this project for PhD in theoretical directions, but the focus can of course also be on applications. Highway traffic is only one example here, there are also many interesting transport phenomena in molecular biology or social sciences (for an overview see e.g. a talk by A. Schadschneider, available at `math1.unice.fr/~bertheli/Page_Web/WorkshopTrafficJam/schadschneider.ppt`).

References.

[1] D. Helbing, B.A. Huberman, *Coherent moving states in highway traffic*. Nature **396**, 738-740 (1998)
Dussutour et al, *Optimal traffic organization in ants under crowded conditions*. Nature **428**, 70-73 (2004)

[2] R.E. Wilson, *Mechanisms for spatio-temporal pattern formation in highway traffic models*. Philos Transact A Math Phys Eng Sci. **366**(1872), 2017-2032 (2008)

[3] J.K. Knowles, *On entropy conditions and traffic flow models*. Z. Angew. Math. Mech. **88**(1), 6473 (2008)

[4] S. Grosskinsky, *Phase transitions in nonequilibrium stochastic particle systems with local conservation laws*. PhD thesis (2004), see `www.warwick.ac.uk/~masgav/` under publications

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