

Large deviations and current fluctuations in driven diffusive systems

Phase transitions are a common collective phenomenon observed in complex interacting systems, and there is a well developed mathematical theory for systems in thermal equilibrium. This theory can be formulated in terms of a minimization problem for the free energy, which incorporates energetic as well as entropic contributions from the distribution of the system. However, many phenomena of interest in applications are not in equilibrium. Large fluctuations and atypical behaviour in such systems can be studied using the formalism of large deviations; of particular interest are typical trajectories that lead to rare events, which are important also for the control of complex systems. The large-deviation function hence plays a central role in non-equilibrium systems, which is essentially the one of the free energy. When available techniques do not allow for an exact evaluation of this function, one turns to simulations but direct numerical simulation of large deviations is hard, since, by definition, they are rare. Recently, an adapted form of biased (importance) sampling has been introduced in [1], which involves parallel simulations of many systems with particular cloning rules. There has also been theoretical progress in particular on large deviations for the current in driven diffusive systems [2,3]. Motivated by some concrete research questions outlined below, the main **aim of the project** is to offer an accessible introduction to the basic theory and methods for the study of large deviations in complex systems, and state-of-the-art simulation techniques to study atypical behaviour of such systems.

Details.

The asymmetric simple exclusion process (ASEP) is a one-dimensional particle system where random walkers jump preferably in one direction on a lattice and interact via hard-core exclusion. The large deviations for generic observables such as the particle current or density profiles in the ASEP have been a subject of major recent research interest with many theoretical results [2]. The recent work [3] contains theoretical predictions on typical density profiles for the ASEP conditioning on a large particle current, which would be interesting to confirm and study by simulations. Results conditioning on low currents are contained in [1,2], and lead to relatively simple phase separated profiles with high and low density regions. These could be understood on a more coarse-grained hydrodynamic level, and an interesting applied question would be to study their dynamic stability, and understand possible connections to stop-and-go waves observed in traffic flow.

Both low and high flux cases are also related to fluctuation theorems, the applicability of which continues to be a subject of great interest. These theorems can be characterized by a particular symmetry property of the large deviation function, and seem to be quite universal even for non-Markovian systems [4]. A second direction of the project could be to use the algorithm in [1] to study fluctuation symmetries for more advanced systems.

Collaboration, prospect for PhD project.

The project offers various routes for an extension to PhD. The algorithm in [1] is similar to so-called Fleming-Viot processes, on which there is a rich mathematical literature (see e.g. [5]). These are often used to simulate quasi-stationary distributions with a very long lifetime and related metastability phenomena for systems with absorbing states (e.g. infection models). A proper mathematical understanding of the approach in [1] and of its applicability for dynamic metastability phenomena would be an interesting theoretical question. There are also various interesting applications in population genetic models, traffic flow or fluctuation theorems.

The project is jointly supervised by Rosemary Harris, Queen Mary, University of London (QMUL), an expert in non-equilibrium statistical mechanics and fluctuation theorems, and Stefan Grosskinsky, Warwick Complexity. In general, Mathematics in QMUL has a very active research group in statistical mechanics and dynamical systems, with many experts in the area of Complexity Science. Our centre can profit from that expertise, in particular through joint supervision of MSc and PhD students, which we seek to set up in the future.

References.

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