

## The directed polymer with symmetric inverse gamma distribution.

The study of random polymer models constitutes one of the most active and interesting areas of modern probability theory with intimate connections to mathematical physics and material and biological sciences. While originating from modeling of physical and biological phenomena [2], they have far reaching and absolutely surprising connections to diverse areas of mathematics [3] (random matrices, particle systems, combinatorics, queuing theory etc.) as recently realised. Still, both the mathematics and the physics communities lie far from a complete understanding of its rich structure.

### Details.

Assume that on each site of the lattice  $\mathbb{N} \times \mathbb{N}$  there is a nonnegative random variable  $d_{ij}$  with a certain distribution (inverse gamma) and assume that all these variables are i.i.d. Consider, also, all possible directed paths starting from the origin and ending at  $(N, N)$ . Each path  $\pi$  is given a probability weight  $P(\pi) = Z_N^{-1} \prod_{(i,j) \in \pi} d_{ij}$ . The partition function  $Z_N = \sum_{\pi: (0,0) \rightarrow (N,N)} \prod_{(i,j) \in \pi} d_{ij}$  plays an important role in the study of the statistical properties of the paths. It is therefore important to obtain detailed statistical information on  $Z_N$ . In [1] the Laplace transform of  $Z_N$  was determined, opening the way to obtain exact asymptotics and limit theorems (that go far beyond the central limit theorem universality class) verifying and extending physical conjectures.

The goal of this project will be to study the case where the weights  $d_{ij}$  are symmetric, ie  $d_{ij} = d_{ji}$  and otherwise they are independent. We call this the symmetric polymer. This case turns out to be more subtle than the iid case. A resolution of this problem will open paths for the study of a wider class of random polymers, such as pinning models.

### Aims of the project.

The first part of the project will be to obtain a conjecture on the Laplace transform of the symmetric polymer, with the help of mathematical software/programming. At the same time the candidate will have the chance to get familiar with the mathematical techniques and the physics background. Time permitting, we can depart to an attempt of providing a mathematical proof of the conjectured formula.

### Scope for PhD.

This mini project will provide an excellent step towards a PhD thesis on random polymers or related disordered systems. In fact the expected outcome of the project will be of great interest on its own. The MSc candidate will be introduced in this way to an area, which lies in the center of the modern probability theory, and at the same time is of great significance for the corresponding statistical mechanics community.

### References.

- [1] Corwin, I.; O'Connell, N.; Seppäläinen, T.; Zygouras, N. *Tropical Combinatorics and Whittaker functions* ; <http://arxiv.org/abs/1110.3489>
- [2] Krug, J, *Origins of scale invariance in growth processes*. Adv. Phys. **46**(2), 139-282 (1996)
- [3] Corwin, I. *The Kardar-Parisi-Zhang equation and universality class* <http://arxiv.org/pdf/1106.1596.pdf>

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