The transition from chromatic number 3 to 4 in random graphs – supervisor Keith Briggs

The chromatic number of a graph is defined as the smallest number of colours with which it is possible to colour the nodes of a graph so that adjacent (i.e. linked by edges) nodes do not have the same colour. To compute the chromatic number is one of the best-known examples of a computationally hard problem. In practice this means that the computation takes time exponential in the number of nodes, and typically this limits us to about 50 to 100 nodes (depending on the number of edges). The chromatic number of several families of random graphs has been much studied from a theoretical viewpoint.

This problem becomes a practical one when assigning channels to radio networks, with the aim of attaining interference-free operation. An edge represents a potentially interfering pair of nodes, and such nodes must use different frequencies (i.e. channels or "colours"). In wifi networks, there are often only 3 channels, and if the network is too dense, there will be interference.

This project will look in details at this transition from 3-colourable to not-3-colourable as graphs get more dense (i.e. get more edges). What is the shape of the curve of probability of 3-colourability as edges are added? This could be done for both Erdős-Rényi graphs ($G(n,p)$, n nodes, each pair linked with probability p), which has some theory available but does not represent a radio network; and with geometric random graphs, which more realistically represents a radio network. The work will involve using existing software for computing the chromatic number.