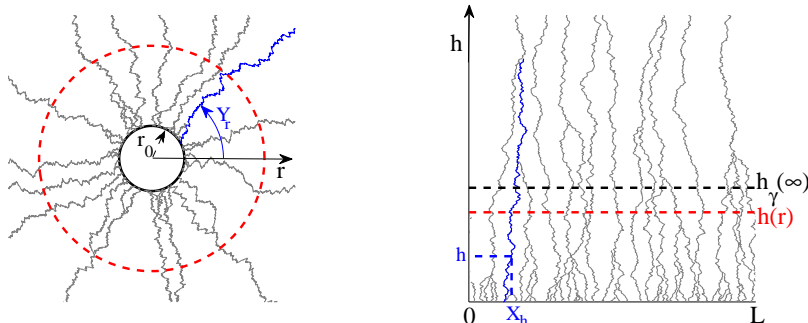# Expansion as a source of diversity

Consider a system of coalescing (picture) or annihilating particle trajectories in a radially expanding domain vs. a fixed, linear domain with periodic boundary conditions. In his PhD in Complexity Adnan Ali derived a mapping [1]

$$h(r) = r_0 \frac{\gamma}{1-\gamma} \left(1 - (r/r_0)^{\frac{\gamma-1}{\gamma}} \right)$$

to explain the behaviour in the expanding system at radius r in terms of the simpler linear one at height $h(r)$. Here $\gamma \in (0,1)$ is the Hurst exponenet describing local fluctuations of the trajectories $dX_h \sim (dh)^\gamma$ and $dY_r \sim (dr)^\gamma$ which are the same in both systems. In the limit $r \to \infty$ we have

$$h(r) \to h_\gamma(\infty) = r_0 \gamma/(1-\gamma) < \infty \quad \text{as} \quad r \to \infty \,,$$

which leads to the interesting effect that while in the finite linear system eventually all trajectories coalesce or annihilate, in the expanding system a random number of arms can survive. The distribution of this number and all other spatial statistics is exaclty given by the mapping.

In **biological applications** to microbial growth or range expansions of certain species on land the trajectories can be thought of as boundaries between different strains [2]. While in a fixed size system one strain will eventually dominate, expansion of the habitat of the population can act as a source of biodiversity.

- A possible project would be to extend this theory to include selective pressure (different fitness/reproduction rate for different types), mutations or to include spatial inhomogeneities. In the simplest case for domain boundaries with constant drift analytic results should be possible, for more complicated scenarios simulations will be necessary which are rather straightforward.

Another well-known system undergoing expansion is the **universe**. In the early universe gravity was not important and one could assume that elementary matter and antimatter particles performed undirected random motions due to huge short-range electro-weak and strong forces and high temperatures. Upon collision, matter and antimatter annihilate, and this leads to a spatial separation of both. Resulting radiation is believed to lead to the cosmic microwave background (CMB), which shows interesting spatial correlations. To our knowledge, there is no widely accepted explanation for the apparent matter/antimatter imbalance and those correlations.

- There is scope for a **crazy** project here: Model the early expanding universe by a 3D-sphere with annihilation between matter and antimatter using the above mapping, and extract characteristic scales of matter-antimatter variation as a function of γ and the initial density. Can this be brought in agreement with the CMB correlations?
  Various extensions are possible, e.g. including the radiation field, falling temperatures with expansion...

There are more areas where expansion can lead to interesting effects, such as financial markets or possible tumor growth, and there are various opportunities to extend this resaerch to PhD in a particular direction.

References.

[1] A. Ali, R.C. Ball, S. Grosskinsky, E. Somfai, *Scale-invariant growth processes in expanding space*, Phys. Rev. E **87**, 020102(R) (2013). arXiv:1210.4990

[2] O. Hallatschek, P. Hersen, S. Ramanathan, D.R. Nelson, *Genetic drift at expanding frontiers promotes gene segregation*. PNAS **104**(50):19926.9930 (2007)

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