Attractive transport, urn models and condensation

Stochastic particle systems are simple discrete models of transport phenomena of interacting entities, called particles. Such systems can exhibit interesting phase transitions such as condensation, where a finite fraction of all particles in the system concentrate in a small volume or a single lattice site. An interesting question under current theoretical investigation is whether condensation is compatible with a property called 'attractivity'.

Configurations η can be partially ordered, i.e. $\eta \leq \zeta$ if $\eta_x \leq \zeta_x$ for all lattice sites x. A system is 'attractive', if this partial order of configurations is conserved under the dynamics. A paradigmatic model in this class is the zero-range process (ZRP) [1] where particles on site x jump to a neighbouring site with rate $g(\eta_x)$ which depends only on the occupation number at the departure site (zero-range interaction). It is well known that if g is an incresing function of η_x then the ZRP is attractive, and does NOT exhibit condensation. But it is not clear if this holds for other systems as well or even in general. This project is a step towards understanding this question.

Details.

Attractivity implies that also the stationary distributions $\mu_{L,N}$ on a fixed lattice of size L are stochastically ordered in the total number of particles N. This has interesting practical consequences: For a given sample η from the stationary distribution $\mu_{L,N}$, one can generate a sample from $\mu_{L,N+1}$ by adding a single particle to η. The site x where this particle should be added is random, and its distribution depends on η. This provides a potentially much faster way of sampling with computation times scaling linear with N, without having to use MCMC techniques where equilibration times are typically of order L^2 or even L^3.

For two simple ZRP with $g(k) = k$ (independent particles) and $g(k) = 1$ ($M/M/1$ server queues) one can derive an explicit rule for adding the next particle. In a first step, this should be confirmed by simulations, and it should be checked if this holds also for other ZRP with general increasing rates g. This has interesting connections to Polya Urns, which can be interpreted as models of size-bias advantage in population dynamic models.

In a second step, the rule can be implemented for ZRP with decreasing rates that are known to exhibit condensation [2]. In this case, the distributions $\mu_{L,N}$ are known to be NOT ordered in N, and it should be understood why exactly the algorithm fails to give a stationary sample. Then more general particle systems can be studied by simulations and theoretically, with a goal to establish a general algorithm for growing a stationary sample for a stochastic particle system. This would provide a great computational benefit over usual MCMC methods.

Collaboration, prospect for PhD project.

Paul Chleboun did his PhD in Complexity in this area and is now an IAS Postdoctoral fellow in Warwick Mathematics for the next 3.5 years. Condensation, and related metastability effects are a topic with considerable recent research interest in applied probability, and there are various opportunities for PhD projects in that area in computational as well as mathematical directions.

References.

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[2] S. Grosskinsky, *Phase transitions in nonequilibrium stochastic particle systems with local conservation laws*. PhD thesis, Technical University of Munich (2004), available online at: `http://www.warwick.ac.uk/~masgav/` under publications

[3] P. Chleboun, *Large Deviations and Metastability in Condensing Stochastic Particle Systems*. PhD thesis, University of Warwick (2011), available online at: `http://www2.warwick.ac.uk/fac/sci/maths/people/staff/paul_chleboun/`

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