Properties of Extremely Heterogeneous Networks

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A network is defined by a set of nodes $\mathcal{N}$ and a set of links between them $\mathcal{L} \subseteq \mathcal{N} \times \mathcal{N}$. The *degree* of a node is the number of links that it is part of. For many real networks, the degrees of nodes are observed to be extremely heterogeneous [6], and it is often claimed that the proportion of nodes with degree k,

$$p_k \sim k^{-\alpha} \text{ for large } k \ . \tag{1}$$

Such graphs can be generated and analysed using a stochastic process known as preferential attachment [1], and are often called 'scale free' because the power-law behaviour of (1) is invariant under a rescaling $k \to \beta k$.

Scale free networks have many unusual properties, for example they are very resilient to random attack but susceptible to targeted attack [4], they are all sparse [3], and asymptotically they do not have an epidemic threshold [7, 4] (although this need not hold for finite networks [5] or if high degree nodes are less transmissible on average [2]).

While much is known about scale-free networks, many open questions remain to be answered and new techniques to be developed. For example, how does the existence of a maximum degree interact with the epidemic threshold, and the value of α in (1)? How do two-point correlations (e.g. degree assortativity) and three-point correlations (e.g. clustering) interact with each other in the presence of extreme heterogeneity? And is it possible to find solid evidence for asymptotic behaviour like (1) in messy, real-world data?

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