Automata over Infinite Alphabets

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Lecture 4: Fresh-Register Automata

Freshness

We saw automata for recognising languages like:

$$\mathcal{L} = \{\, d_1 d_2 \cdots d_n \in \mathcal{D}^* \mid n \geq 0 \wedge \forall i.\, d_i \neq d_{i+1} \,\}$$

$$\mathcal{L} = \{\, d_0 d_1 d_2 \cdots d_n \in \mathcal{D}^* \mid n \geq 0 \wedge \forall i > 0.\, d_i \neq d_0 \,\}$$

Such languages are based on being able to capture *local freshness*: being able to distinguish a name from a *bounded* number of names in memory.

However, consider this language that describes e.g. a memory allocator in Java or ML:

$$\mathcal{L}_{\text{fresh}} = \{\, d_1 d_2 \cdots d_n \in \mathcal{D}^* \mid n \geq 0 \wedge \forall_{i \neq j}\, d_i \neq d_j) \,\}$$

Such examples require *global freshness*, which we examine in this lecture.

Fresh-Register Automata

An r-**Fresh-Register Automaton (r-FRA)** is a tuple $\mathcal{A} = \langle Q, q_I, \tau_I, \delta, F \rangle$, where:

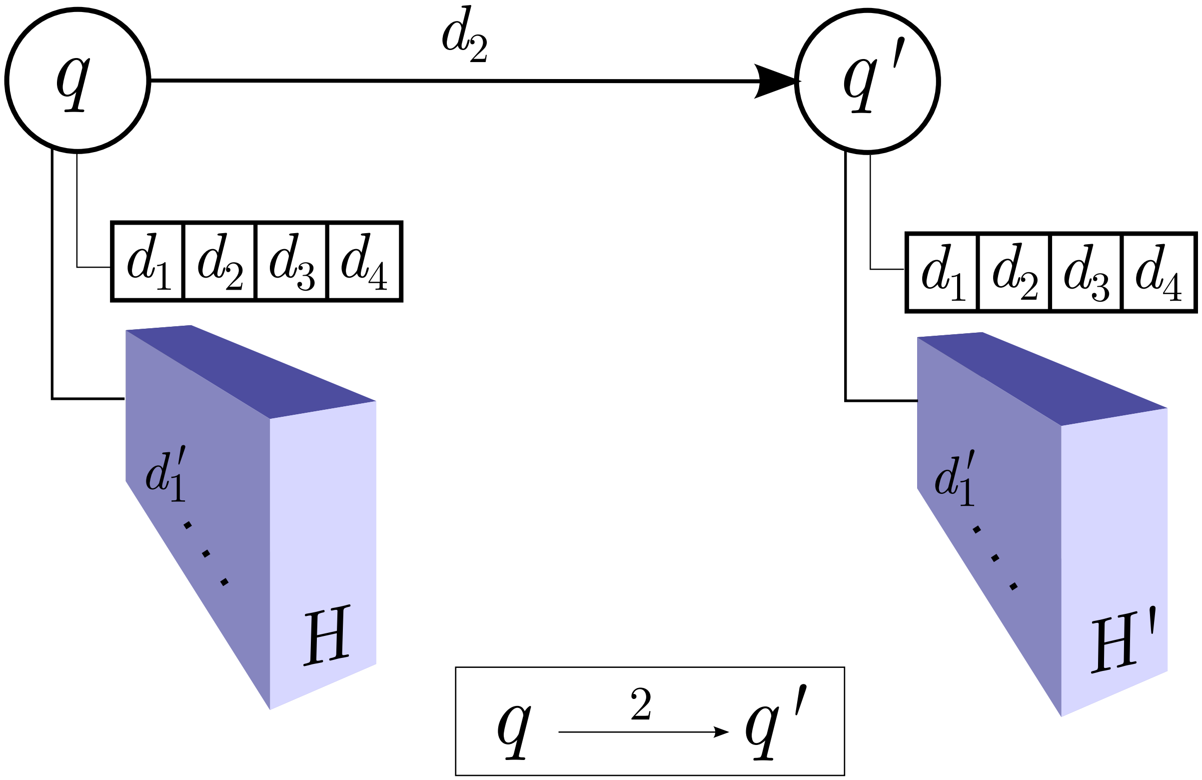
- Q is a finite set of states,
- $q_I \in Q$ is the initial state,
- $F \subseteq Q$ is the set of final states,
- $\tau_I \in Reg_r^{\mathtt{i}}$ is the initial r-register assignment,
- and $\delta \subseteq Q \times Op_r \times Q$ is the transition relation,

where $Op_r = \{\, i, i^{\bullet}, i^{\circledast} \mid 1 \leq i \leq r \,\}$.

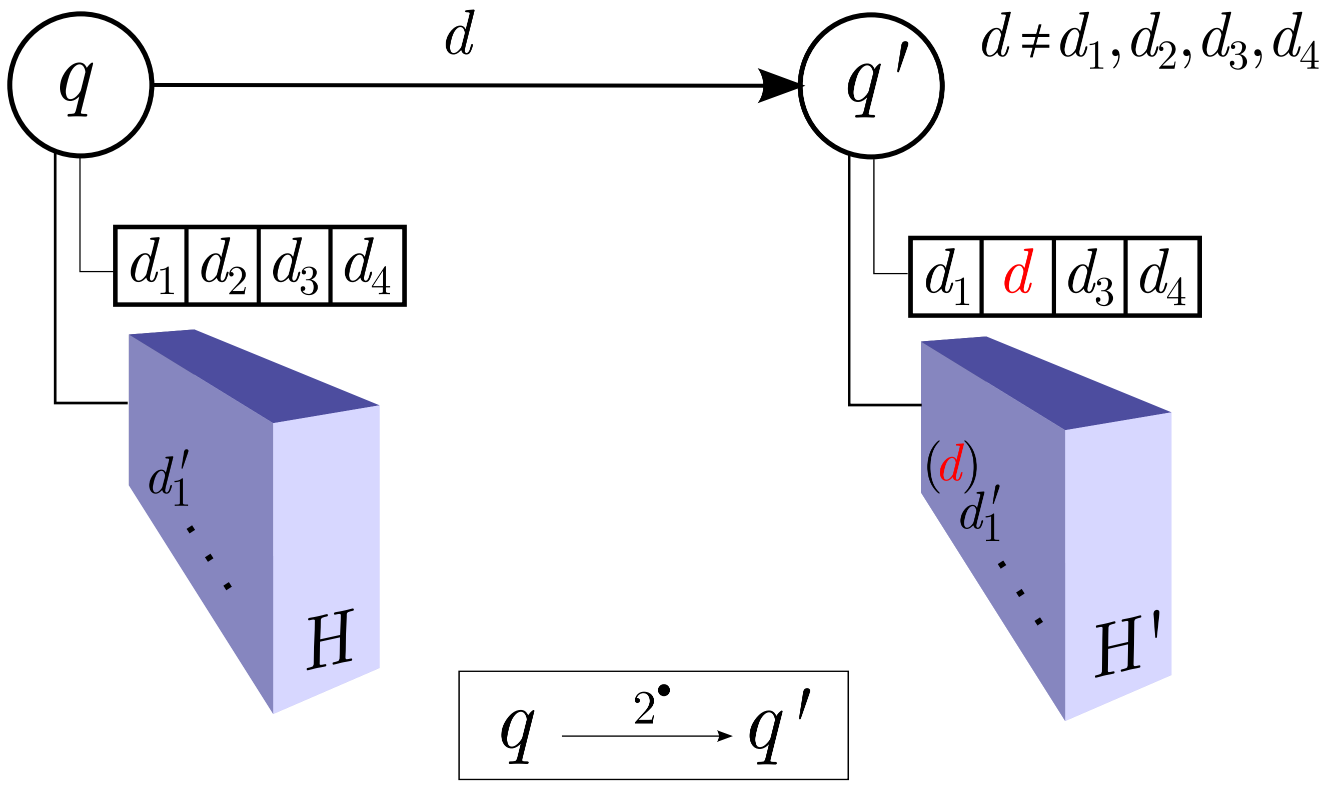
Thus, the new operation is: $q \xrightarrow{i^{\circledast}} q'$

It means: *accept a globally fresh name and store it in register* i

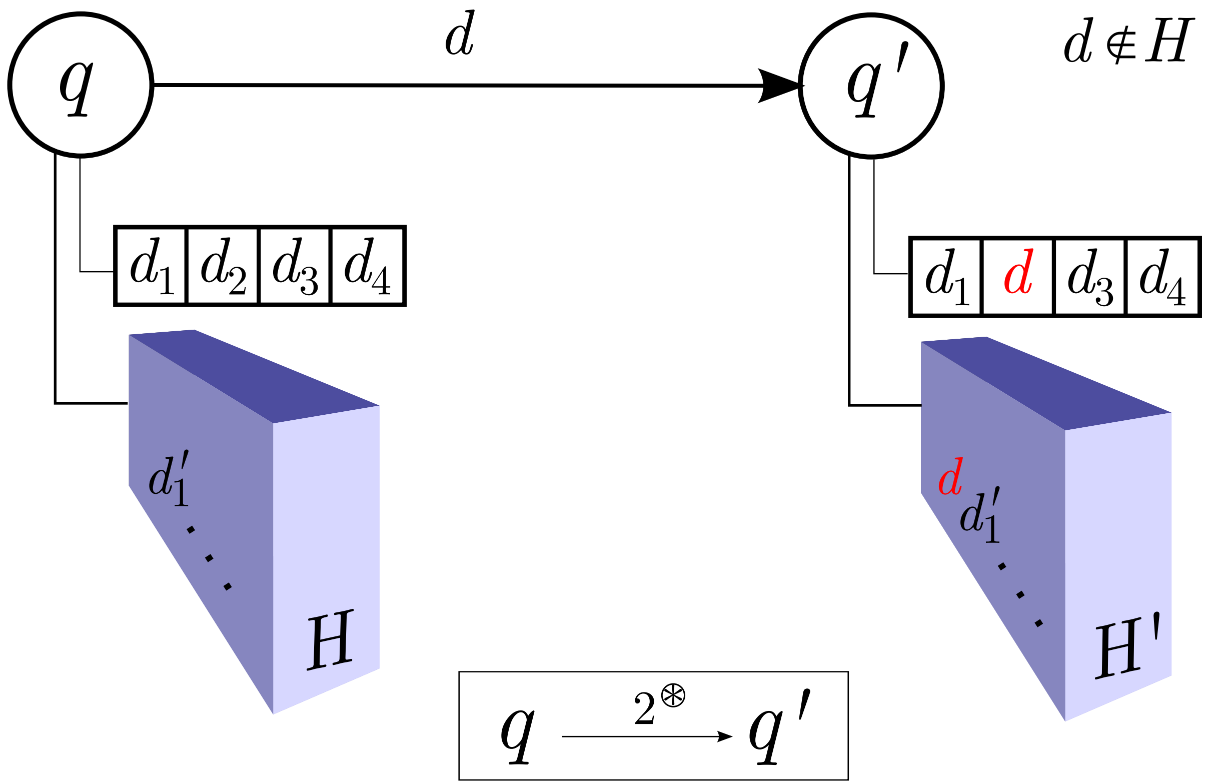
Semantics of FRAs: in pictures



Semantics of FRAs: in pictures



Semantics of FRAs: in pictures

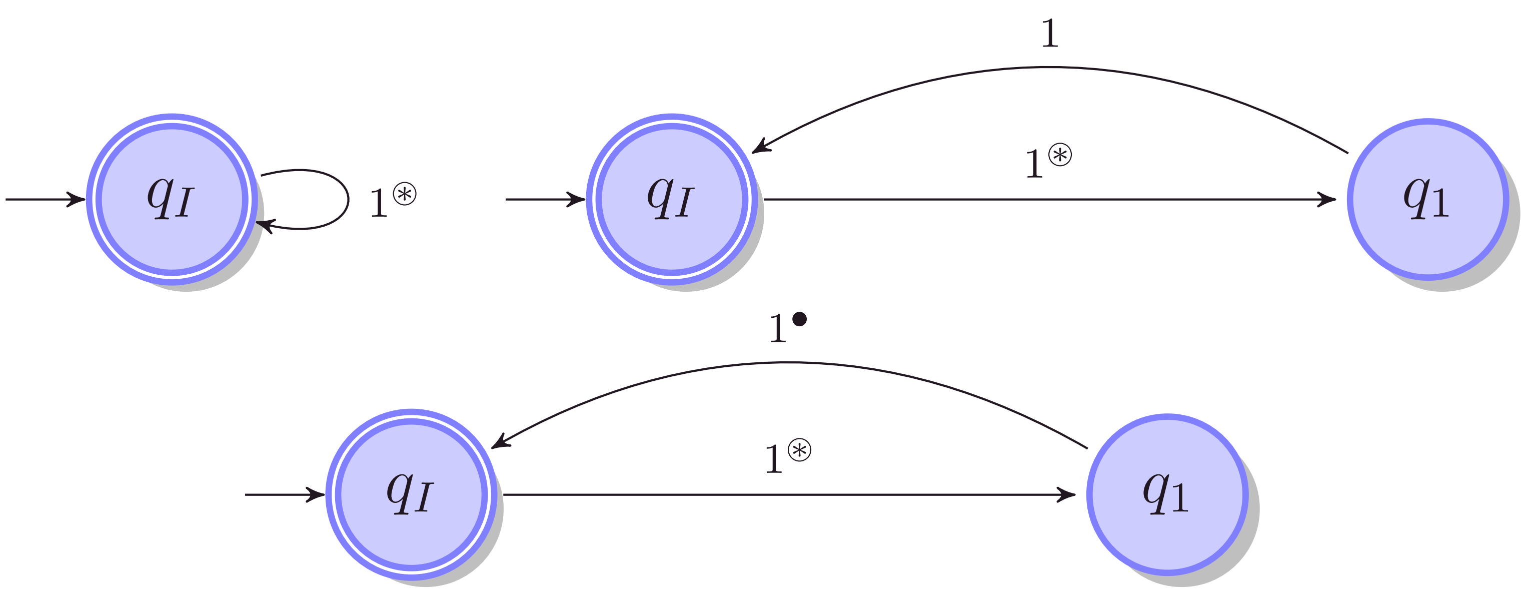


Examples

$$\mathcal{L}_{\text{fresh}} = \{\, d_1 d_2 \cdots d_n \in \mathcal{D}^* \mid n \geq 0 \wedge \forall_{i \neq j} \, (d_i \neq d_j) \,\}$$

$$\mathcal{L} = \{\, d_1 d_1 d_2 d_2 \cdots d_n d_n \in \mathcal{D}^* \mid n \geq 0 \wedge \forall_{i \neq j} \, (d_i \neq d_j) \,\}$$

$$\mathcal{L}' = \{\, d_1 d_1' d_2 d_2' \cdots d_n d_n' \in \mathcal{D}^* \mid n \geq 0 \wedge \forall_{i < j} \, (d_j \neq d_i, d_i', d_j') \,\}$$



Formal semantics of FRAs

Notation:
$\nu(x)$ = the set of names appearing in x

Let $\mathcal{A} = \langle Q, q_I, \tau_I, \delta, F \rangle$ be an r-FRA. To give a semantics to FRAs we need an extended kind of configuration. Let us set:

$$Conf_{\mathcal{A}} = \{ (q, \tau, H) \in Q \times Reg_r^{\mathtt{i}} \times \mathcal{P}_{\text{fin}}(\mathcal{D}) \mid \nu(\tau) \subseteq H \}$$

That is, configurations are triples of a state q, an r-register assignment τ and a *history* H (the set of all names seen so far by the automaton).

An **evolution** $(q_1, \tau_1, H_1) \xrightarrow{d} (q_2, \tau_2, H_2)$ between configurations needs to satisfy one of the following conditions (for some $1 \le i \le r$):

- $(q_1 \xrightarrow{i} q_2) \in \delta$, and $\tau_1(i) = d$, $\tau_2 = \tau_1$ and $H_2 = H_1$;
- $(q_1 \xrightarrow{i^\bullet} q_2) \in \delta$, and $d \notin \nu(\tau_1)$, $\tau_2 = \tau_1[i \mapsto d]$ and $H_2 = H_1 \cup \{d\}$;
- $(q_1 \xrightarrow{i^\circledast} q_2) \in \delta$, and $d \notin H_1$, $\tau_2 = \tau_1[i \mapsto d]$ and $H_2 = H_1 \cup \{d\}$.

The **configuration graph** of $\mathcal{A}$ is formed by all possible configuration evolutions, and $\mathcal{L}(\mathcal{A}) = \{ w \in \mathcal{D}^* \mid (q_I, \tau_I, \nu(\tau_I)) \xrightarrow{w}\!\!\!\!\rightarrow (q, \tau, H) \wedge q \in F \}$.

Non-Examples

$$\mathcal{L}_{\text{palindrome}} = \{\, d_1 d_2 \cdots d_n d_n \cdots d_2 d_1 \in \mathcal{D}^* \mid n \geq 0 \,\}$$
$$\mathcal{L}^2_{\text{fresh}} = \{\, w w' \in \mathcal{D}^* \mid w, w' \in \mathcal{L}_{\text{fresh}} \,\}$$

These follow from the next boundedness result.

Theorem. *Let $\mathcal{L}$ be some FRA-recognisable language. There is an $r \in \mathbb{N}$ such that, for any word $w_1 w_2 \in \mathcal{L}$ with $\nu(w_2) \subseteq \nu(w_1)$, there is some $w_1 w_2' \in \mathcal{L}$ with $|w_2'| = |w_2|$ and $|\nu(w_2')| \leq r + 1$.*

Proof. Take r to be the number of registers of an FRA $\mathcal{A}$ accepting $\mathcal{L}$. Given an accepting run ρ of $\mathcal{A}$ on $w_1 w_2$, with $\nu(w_2) \subseteq \nu(w_1)$, it must be the case that there are no global fresh transitions after accepting w_1. But, using only local freshness, $\mathcal{A}$ can at most distinguish between $r + 1$ names, hence ρ can be repeated with no more than $r + 1$ names. □

Closure properties

Following a similar route as for RAs, we can show:

- for any pair of FRAs $\mathcal{A}_1, \mathcal{A}_2$ there is FRA $\mathcal{A}'$ such that $\mathcal{L}(\mathcal{A}') = \mathcal{L}(\mathcal{A}_1) \cap \mathcal{L}(\mathcal{A}_2)$
- for any pair of FRAs $\mathcal{A}_1, \mathcal{A}_2$ there is FRA $\mathcal{A}'$ such that $\mathcal{L}(\mathcal{A}') = \mathcal{L}(\mathcal{A}_1) \cup \mathcal{L}(\mathcal{A}_2)$

However, our previous theorem denies the following closures:

concatenation e.g. $\mathcal{L}^2_{\text{fresh}}$ $(= \mathcal{L}_{\text{fresh}}\mathcal{L}_{\text{fresh}})$ is not FRA-recognisable

Kleene star we can find a similar example as above [exercise]

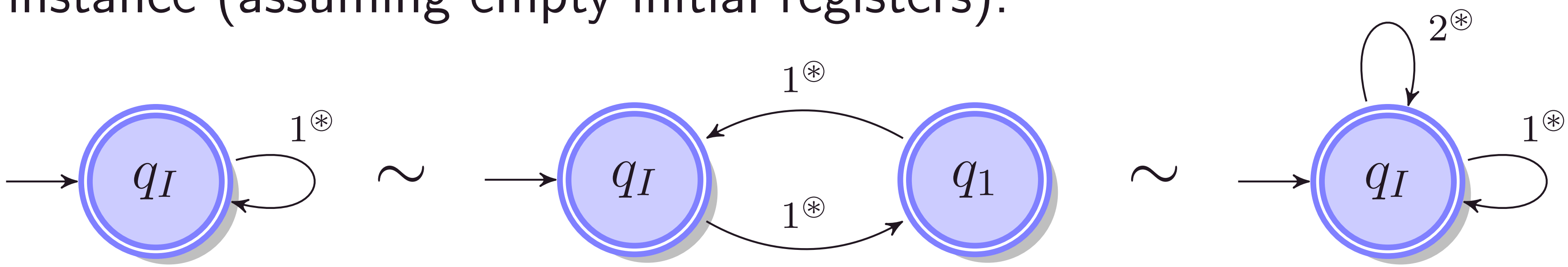
complement e.g. $\overline{\mathcal{L}^2_{\text{fresh}}}$ can be recognised by an (F)RA [exercise]

Another notion of equivalence: Bisimulation

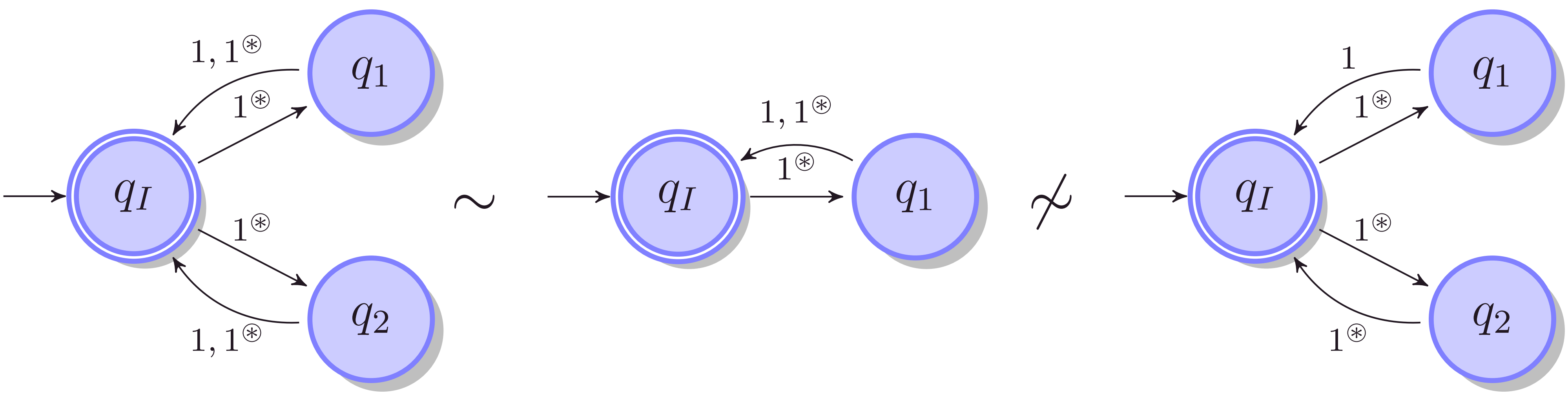
A more *behavioural* notion of equivalence says:

> Two automata are equivalent if they can simulate the operation of one another in a name-by-name manner

For instance (assuming empty initial registers):



but also:



Bisimulation formally

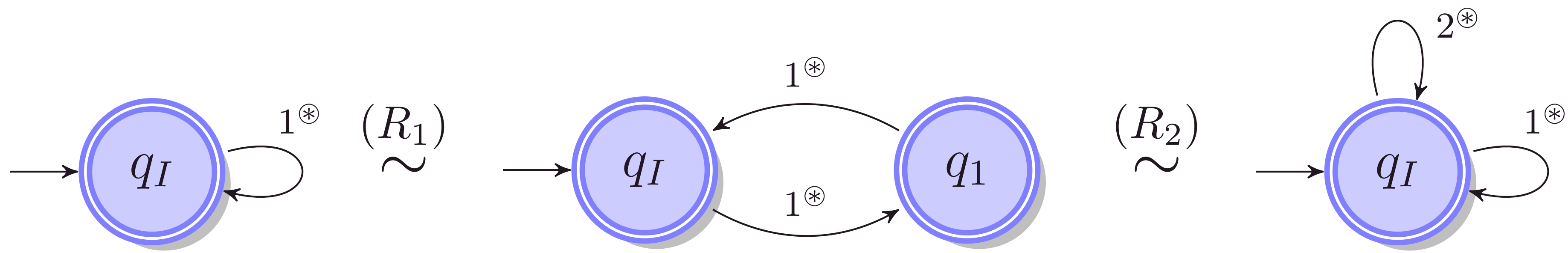
Let $\mathcal{A}$ be an FRA and let $\mathcal{G}$ be its configuration graph. A relation $R \subseteq Conf_{\mathcal{A}} \times Conf_{\mathcal{A}}$ is called a **bisimulation** if, whenever $\kappa_1 \, R \, \kappa_2$:

- for all $\kappa_1 \xrightarrow{d} \kappa_1'$ there is some $\kappa_2 \xrightarrow{d} \kappa_2'$ such that $\kappa_1' \, R \, \kappa_2'$,
- for all $\kappa_2 \xrightarrow{d} \kappa_2'$ there is some $\kappa_1 \xrightarrow{d} \kappa_1'$ such that $\kappa_1' \, R \, \kappa_2'$,
- if $\kappa_i = (q_i, \tau_i, H_i)$, for $i = 1, 2$, then $q_1 \in F \iff q_2 \in F$.

Moreover:

- If $R_1 \cup R_2$ are bisimulations then so is $R_1 \cup R_2$.
- We take $\sim$ to be the union of all bisimulations, called **bisimilarity**. I.e. $\kappa_1 \sim \kappa_2$ if $\kappa_1 \, R \, \kappa_2$ for some bisimulation R (κ_1, κ_2 called *bisimilar*).
- $\mathcal{A}_1 \sim \mathcal{A}_2$ if their initial configurations are bisimilar (in the union configuration graph).
- Bisimilarity is an equivalence (reflexive, symmetric & transitive).

Examples revisited



- $R_1 = \{\, ((q_I, \tau, H), (q_I, \tau, H)) \mid \nu(\tau) \subseteq H \,\}$
  $\cup \{\, ((q_I, \tau, H), (q_1, \tau, H)) \mid \nu(\tau) \subseteq H \,\}$
- $R_2 = \{\, ((q_I, \tau, H), (q_I, \tau', H)) \mid \nu(\tau) \cup \nu(\tau') \subseteq H \,\}$
  $\cup \{\, ((q_1, \tau, H), (q_I, \tau', H)) \mid \nu(\tau) \cup \nu(\tau') \subseteq H \,\}$
- $R_1 ; R_2$ witnesses bisimilarity of first and last automaton

We observe that, in all cases above, the FRAs accept the same languages. Is there a general connection?

Bisimilarity vs language equivalence

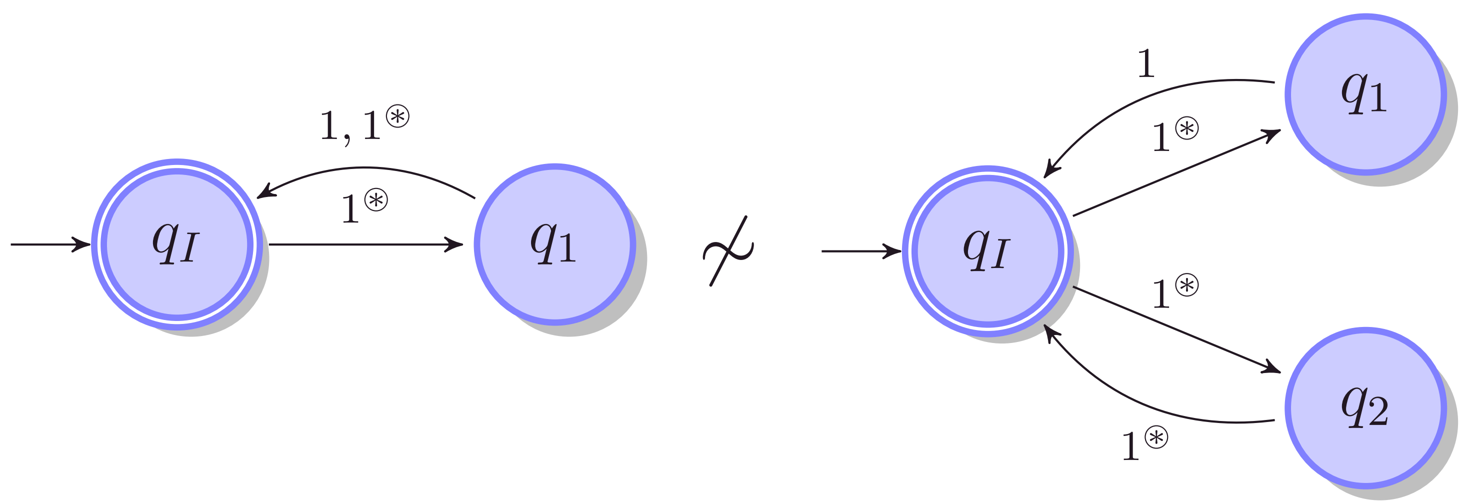
Theorem. *If* $\mathcal{A}_1 \sim \mathcal{A}_2$ *then* $\mathcal{L}(\mathcal{A}_1) = \mathcal{L}(\mathcal{A}_2)$.

Proof idea. Given an FRA-configuration graph $\mathcal{G}$ and a configuration κ, let $\mathcal{L}(\kappa)$ be the language of all paths from κ to some final configuration:

$$\mathcal{L}(\kappa) = \{\, w \in \mathcal{D}^* \mid \text{there is a } w\text{-labelled path in } \mathcal{G} \text{ from } \kappa \text{ to a final } \kappa_F \,\}$$

It suffices then to show that $\kappa \sim \kappa'$ implies $\mathcal{L}(\kappa) = \mathcal{L}(\kappa')$. □

The converse does not hold in general, for example:



Application: equivalence with M-automata

We can define M-type automata similarly to RA(M)'s by simply extending transition labels to: $Op_r = (\mathcal{P}([r]) \cup \{\circledast\}) \times \mathcal{P}([r])$

e.g. in $q \xrightarrow{\circledast, Y} q'$ we *read a globally fresh name and store it in registers* Y.

Theorem. *For any* r*-FRA(M)* $\mathcal{A}$ *there is an* $(r{+}1)$*-FRA* $\mathcal{A}'$ *such that* $\mathcal{A} \sim \mathcal{A}'$.

Application: equivalence with M-automata

Theorem. *For any r-FRA(M) $\mathcal{A}$ there is an $(r+1)$-FRA $\mathcal{A}'$ s.t. $\mathcal{A} \sim \mathcal{A}'$.*

Proof. Given an r-FRA(M) $\mathcal{A} = \langle Q, q_I, \tau_I, \delta, F\rangle$, we define:

- $Q' = Q \times Part_r$, $q'_I = (q_I, \pi_I)$ for some $\pi_I : [r+1] \to \mathcal{P}([r])$ partitioning τ_I (cf. [RA: sl.28]), $F' = F \times Part_r$ and:
  - for $q \xrightarrow{X,Y} q'$ in δ and $\pi_1(i) = X$, add $(q, \pi_1) \xrightarrow{i} (q', \pi_1[Y \hookrightarrow i])$ in δ',
  - for $q \xrightarrow{\emptyset,Y} q'$ in δ and $\pi_1(i) = \emptyset$, add $(q, \pi_1) \xrightarrow{i^\bullet} (q', \pi_1[Y \hookrightarrow i])$ in δ',
  - for $q \xrightarrow{\circledast,Y} q'$ in δ and $\pi_1(i) = \emptyset$, add $(q, \pi_1) \xrightarrow{i^\circledast} (q', \pi_1[Y \hookrightarrow i])$ in δ',

and take $\mathcal{A}' = \langle Q', q'_I, \tau'_I, \delta', F'\rangle$.

Now, taking R to contain all pairs $((q,\tau,H),((q,\pi),\tau',H))$ of $\mathcal{A}/\mathcal{A}'$ configurations such that (π,τ') represent τ:

$$\nu(\tau) \subseteq \nu(\tau') \wedge \forall_{i,d}\, (\tau'(i) = d \implies \tau^{-1}(d) = \pi(i))$$

we can show that R is a bisimulation [Exercise]. $\square$

Main result: bisimilarity is decidable

As FRAs extend RAs, language equivalence for FRAs in undecidable.
However, we are going to show the following.

Theorem. *The following problem, called* BISIMILARITY, *is decidable.*

INPUT: *An r-FRA $\mathcal{A}$ and configurations κ_1, κ_2 with common history.*
QUESTION: *Is it the case that $\kappa_1 \sim \kappa_2$?*

Some notes:

- Important for applicability of FRAs as a modelling paradigm.
- The restriction to common history is not essential (but easier).
- Strictly speaking, the input $(\mathcal{A}, \kappa_1, \kappa_2)$ is not finite as κ_1, κ_2 may contain names, which come from an infinite domain.

  However, in this case it suffices to think of the domain as being just H, which is finite.

Proof idea: symbolic reasoning

Consider a pair (κ_1, κ_2) of configurations with $\kappa_i = (q_i, \tau_i, H)$.

To check whether $\kappa_1 \sim \kappa_2$ are bisimilar, we do not need to know τ_1, τ_2, H in full detail. Rather, it suffices to know:

- what are the common names of τ_1, τ_2 (and in which positions),
- what are the private names of τ_1, τ_2 (and in which positions),
- what is the size of H with respect to the names in $\nu(\tau_1) \cup \nu(\tau_2)$.

Given the above, we can then reason *symbolically*, by looking directly at $\mathcal{A}$ (which is finite) rather than its configuration graph (that is infinite).

Symbolic reasoning

Consider a the following situation:

$$\kappa_1 = (q_1, [d_1, d_2, d_3, d_4, d_5], H) \qquad \kappa_2 = (q_2, [d'_1, d'_2, d'_3, \#, \#], H)$$

where $d_2 = d'_2$, $d_3 = d'_1$, $d_5 = d'_3$ and $|H| = 8$.

In order for $\kappa_1 \sim \kappa_2$ to hold:

- if $q_1 \xrightarrow{3} q'_1$ then there must be $q_2 \xrightarrow{1} q'_2$,
- if $q_1 \xrightarrow{1} q'_1$ then there must be $q_2 \xrightarrow{j^\bullet} q'_2$,
- if $q_2 \xrightarrow{1^\bullet} q'_2$ then there must be $q_1 \xrightarrow{i^\bullet} q'_1$,
- if $q_2 \xrightarrow{1^\bullet} q'_2$ then there must be $q_1 \xrightarrow{1} q'_1$ and $q_1 \xrightarrow{4} q''_1$,
- if $q_2 \xrightarrow{1^\circledast} q'_2$ then there must be $q_1 \xrightarrow{i^\circledast} q'_1$ or and $q_1 \xrightarrow{i^\bullet} q'_1$.

In the last case we use the fact that local freshness is more general than global freshness (i.e. it can accept more names).

Global freshness can (sometimes) be as general as local

Consider a the following situation:

$$\kappa_1 = (q_1, [d_1, d_2, d_3, d_4, d_5], H) \qquad \kappa_2 = (q_2, [d'_1, d'_2, d'_3, \#, \#], H)$$

where $d_2 = d'_2$, $d_3 = d'_1$, $d_5 = d'_3$ and $|H| = 5$.

In order for $\kappa_1 \sim \kappa_2$ to hold:

- $\ldots$
- if $q_2 \xrightarrow{1^\bullet} q'_2$ then there must be $q_1 \xrightarrow{1} q'_1$ and $q_1 \xrightarrow{4} q''_1$,
- if $q_2 \xrightarrow{1^\bullet} q'_2$ then there must be $q_1 \xrightarrow{i^\bullet} q'_1$ or $q_1 \xrightarrow{i^\circledast} q'_1$

  This is because $q_2 \xrightarrow{1^\bullet} q'_2$ can accept:

  - d_1 and d_4 (taken care of by previous case)
  - any name in $H \setminus (\{d_1, \cdots, d_5\} \cup \{d'_1, d'_2, d'_3\})$ (empty!)
  - any name not in H (taken care by either of $q_1 \xrightarrow{i^\bullet / i^\circledast} q'_1$)

Symbolic bisimulations

First, let us represent each pair:

$$(q_1, [d_1, d_2, d_3, d_4, d_5], H) \qquad (q_2, [d'_1, d'_2, d'_3, \#, \#], H) \qquad |H| = 5$$

with: $\quad (q_1, q_2, (\{1,2,3,4,5\}, \{(2,2), (3,1), (5,3)\}, \{1,2,3\}), 5).$

Thus, we define *symbolic configurations* as:

$$\begin{aligned} \mathit{Conf}_{\mathsf{s}} = \{\, (q_1, q_2, \rho, h) \in Q \times Q \times \mathit{Span}_r \times [2r+1] \\ \mid \mathit{Span}_r = \mathcal{P}([r]) \times ([r] \stackrel{\cong}{\rightharpoonup} [r]) \times \mathcal{P}([r]) \\ \wedge\, \rho = (S_1, \hat{\rho}, S_2) \wedge \mathsf{dom}(\hat{\rho}) \subseteq S_1 \wedge \mathsf{cod}(\hat{\rho}) \subseteq S_2 \,\} \end{aligned}$$

Notes:

- Symbolic configurations describe pairs of concrete configurations.
- We call the third component of a symbolic configuration a *span*. It describes how the two registers assignments are related.
- We only need to count the size of H up to $2r+1$.

Definition of symbolic bisimulation

We represent each pair:

$$(q_1, [d_1, d_2, d_3, d_4, d_5], H) \qquad (q_2, [d'_1, d'_2, d'_3, \#, \#], H) \qquad |H| = 5$$

as: $(q_1, q_2, \rho, h) \ = \ (q_1, q_2, (\{1,2,3,4,5\}, \{(2,2), (3,1), (5,3)\}, \{1,2,3\}), 5)$.

Given r-FRA $\mathcal{A}$, a relation $R \subseteq Conf_{\mathsf{s}}$ is called a **symbolic bisimulation** if, when $(q_1, q_2, \rho, h) \in R$, with $\rho = (S_1, \hat{\rho}, S_2)$, $q_1 \in F \iff q_2 \in F$ and:

- for all $q_1 \xrightarrow{i} q'_1$ with $i \in \mathrm{dom}(\hat{\rho})$ there is $q_2 \xrightarrow{\hat{\rho}(i)} q'_2$ with $(q'_1, q'_2, \rho, h) \in R$,
- for all $q_1 \xrightarrow{i} q'_1$ with $i \in S_1 \setminus \mathrm{dom}(\hat{\rho})$ there is $q_2 \xrightarrow{j^\bullet} q'_2$ with $(q'_1, q'_2, \rho[i \mapsto j], h) \in R$,
- for all $q_1 \xrightarrow{i^\circledast} q'_1$ there is $q_2 \xrightarrow{j^\bullet} q'_2$ or $q_2 \xrightarrow{j^\circledast} q'_2$ with $(q'_1, q'_2, \rho[i \mapsto j], h \oplus 1) \in R$,

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where: $\rho[i \mapsto j] = (S_1 \cup \{i\}, \hat{\rho}[i \mapsto j], S_2 \cup \{j\})$

$$\hat{\rho}[i \mapsto j] = (\hat{\rho} \setminus (\{i\} \times [r]) \setminus ([r] \times \{j\})) \cup \{(i,j)\}$$

$h \oplus 1 = h + 1$ if $h \leq 2r$, and $(2r+1) \oplus 1 = 2r + 1$

Definition of symbolic bisimulation (ctd)

We represent each pair:

$$(q_1, [d_1, d_2, d_3, d_4, d_5], H) \qquad (q_2, [d'_1, d'_2, d'_3, \#, \#], H) \qquad |H| = 5$$

as: $(q_1, q_2, \rho, h) = (q_1, q_2, (\{1,2,3,4,5\}, \{(2,2), (3,1), (5,3)\}, \{1,2,3\}), 5)$.

Given r-FRA $\mathcal{A}$, a relation $R \subseteq Conf_{\mathsf{s}}$ is called a **symbolic bisimulation** if, when $(q_1, q_2, \rho, h) \in R$, with $\rho = (S_1, \hat{\rho}, S_2)$, $q_1 \in F \iff q_2 \in F$ and:

- for all $q_1 \xrightarrow{i^\bullet} q'_1$:
  - for all $j \in S_2 \setminus \mathsf{cod}(\hat{\rho})$ there is $q_2 \xrightarrow{j} q'_2$ with $(q'_1, q'_2, \rho[i \mapsto j], h) \in R$,
  - if $\|\rho\| < h$ then there is $q_2 \xrightarrow{j^\bullet} q'_2$ with $(q'_1, q'_2, \rho[i \mapsto j], h) \in R$,
  - there is $q_2 \xrightarrow{j^\bullet} q'_2$ or $q_2 \xrightarrow{j^\circledast} q'_2$ with $(q'_1, q'_2, \rho[i \mapsto j], h \oplus 1) \in R$; (because, in every case, $i^\bullet$ can capture some globally fresh name)

where $\|\rho\| = |S_1| + |S_2| - |\mathsf{dom}(\hat{\rho})|$ is the number of all names in the two simulated assignments (removing repetitions)

Definition of symbolic bisimulation (ctd ctd)

We represent each pair:

$$(q_1, [d_1, d_2, d_3, d_4, d_5], H) \qquad (q_2, [d'_1, d'_2, d'_3, \#, \#], H) \qquad |H| = 5$$

as: $(q_1, q_2, \rho, h) \;=\; (q_1, q_2, (\{1,2,3,4,5\}, \{(2,2), (3,1), (5,3)\}, \{1,2,3\}), 5)$.

Given r-FRA $\mathcal{A}$, a relation $R \subseteq Conf_{\mathsf{s}}$ is called a **symbolic bisimulation** if, when $(q_1, q_2, \rho, h) \in R$, with $\rho = (S_1, \hat{\rho}, S_2)$, $q_1 \in F \iff q_2 \in F$ and:

- the symmetric conditions apply for all $q_2 \xrightarrow{x} q'_2$.

Given κ_1, κ_2 with $\kappa_i = (q_1, \tau_i, H)$, these are **symbolic bisimilar**, written $\kappa_1 \sim_{\mathsf{s}} \kappa_2$, if there is symbolic bisimulation R such that

$$(q_1, q_2, \tau_1 \asymp \tau_2, |H|_{2r+1}) \in R$$

where $\tau_1 \asymp \tau_2 = (\mathsf{dom}(\tau_1), \tau_1; \tau_2^{-1}, \mathsf{dom}(\tau_2))$

and $|H|_{2r+1} = \begin{cases} |H| & \text{if } |H| < 2r+1 \\ 2r+1 & \text{otherwise} \end{cases}$.

Bisimilarity is decidable

We can show the following.

Theorem. *For any pair of configurations* κ_1, κ_2 *with common history,* $\kappa_1 \sim \kappa_2$ *iff* $\kappa_1 \sim_{\mathsf{s}} \kappa_2$.

and therefore:

Corrolary. $\textsc{Bisimilarity}$ *is decidable.*

Proof. Given $(\mathcal{A}, \kappa_1, \kappa_2)$, it suffices to check whether $\kappa_1 \sim_{\mathsf{s}} \kappa_2$, that is, whether there is symbolic bisimulation $R \subseteq \mathit{Conf}_{\mathsf{s}}$ such that $(q_1, q_2, \tau_1 \asymp \tau_2, |H|_{2r+1}) \in R$.
But note that $\mathit{Conf}_{\mathsf{s}}$ is bounded, so we can exhaustively search in it for an R satisfying the required conditions. □

Correspondence

The proof of the Theorem relies on two correspondences.

Lemma. *Suppose R is a bisimulation. Then, the relation*

$$R' = \{ (q_1, q_2, \rho, h) \mid \exists (q_i, \tau_i, H) \in R.\, \rho = \tau_1 \asymp \tau_2 \wedge h = |H|_{2r+1} \}$$

is a symbolic bisimulation.

Lemma. *Suppose R is a symbolic bisimulation. Then, the relation*

$$R' = \{ ((q_1, \tau_1, H), (q_2, \tau_2, H)) \mid (q_1, q_2, \tau_1 \asymp \tau_2, |H|_{2r+1}) \in R \}$$

is a bisimulation.

Summary and References

Fresh-Register Automata

- Definitions
- Example languages and non-examples
- Closure properties
- Bisimilarity (aka bisimulation equivalence)
- Bisimilarity and language equivalence
- Symbolic methods and decidability

References and further directions

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- A. S. Murawski, S. J. Ramsay, N. Tzevelekos: Bisimilarity in fresh-register automata. LICS 2015: to appear
- A. S. Murawski, N. Tzevelekos: Algorithmic Nominal Game Semantics. ESOP 2011: 419-438
- N. Tzevelekos: Fresh-register automata. POPL 2011: 295-306

Exercises

1. Taking $\mathcal{L}_d = \{\, dw \in \mathcal{D}^* \mid w \in \mathcal{L}_{\text{fresh}} \wedge d \notin \nu(w) \,\}$ for some fixed $d \in \mathcal{D}$, show that $\mathcal{L}_d^*$ is not FRA-recognisable.
2. Design an RA recognising the complement of $\mathcal{L}^2_{\text{fresh}}$.
3. Show that bisimilarity is an equivalence relation.
4. Complete the proof of the reduction from FRA(M) to FRA by showing that the constructed R is a bisimilarity.