Estimating Graph Parameters from Random Order Streams

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Joint work with Christian Sohler (TU Dortmund, Germany)

Graph streams

To analyze the structure of massive and dynamic networks/graphs

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Graph streaming algorithms

- Input: a sequence of edge insertions and/or deletions
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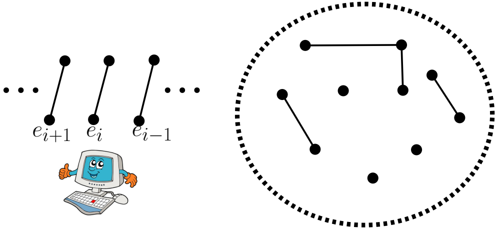

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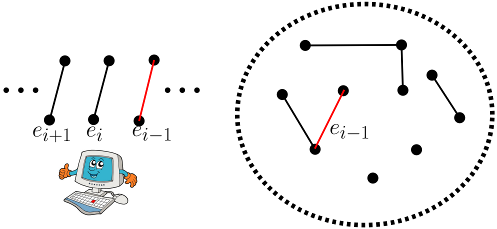

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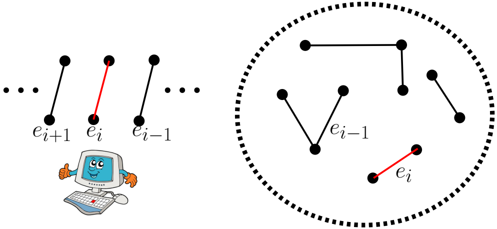

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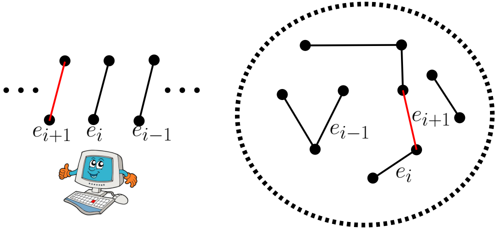

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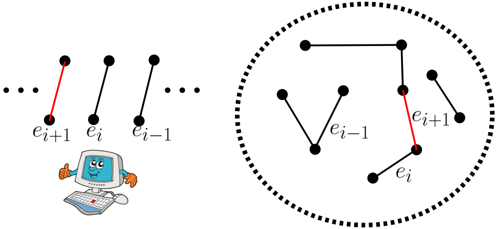

* This work: insertion-only; single pass

Model: adversarial order streams

Edges arrive in arbitrary order: ($n := \#$ vertices)

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– relax the assumption that edges come in arbitrary order

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In general, it is unclear if the random-order assumption leads to more space-efficient algorithms

Our result

A new algorithmic technique:

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$\Downarrow$

(• query access to the adjacency list of the graph
• running time of the algorithm is constant, independent of n)

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approx.	problem	graph class	space
additive εn	number of connected components (CCs)	general	$\left(\frac{1}{\varepsilon}\right)^{O(\frac{1}{\varepsilon^3})}$
$(1+\varepsilon)$	weight of minimum spanning tree (MST)	general connected; edge weights $\{1, \cdots, W\}$	$\left(\frac{1}{\varepsilon}\right)^{\tilde{O}(\frac{W^3}{\varepsilon^3})}$
	size of maximum independent set (MIS)	planar/minor-free	$2^{(\frac{1}{\varepsilon})^{(\frac{1}{\varepsilon})^{\log^{O(1)}(\frac{1}{\varepsilon})}}}$

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Remark: Adversary order: $\Omega(n^{1-O(\varepsilon)})$ for the first two problems [HP16]

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– here "test Π": distinguish if a graph satisfies Π or is "far" from satisfying Π

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generic framework for many constant-time algorithms

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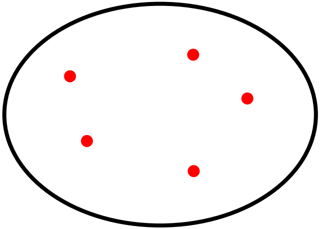

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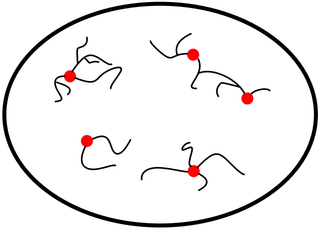

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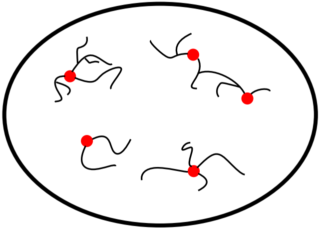

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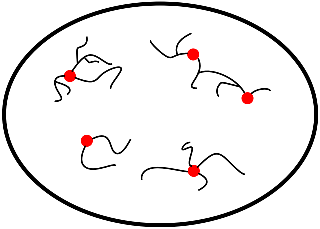

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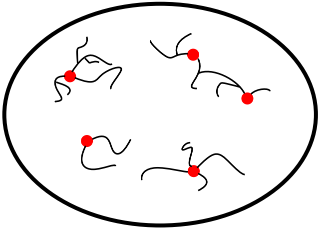

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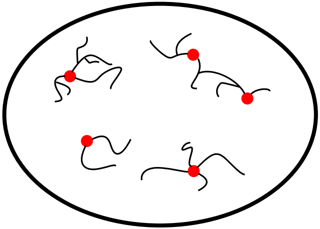

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For random order streams:

- with (small) constant probability to see the right exploration
- challenge: to identify when the graph exploration behaves as in the original graph and when it does not.

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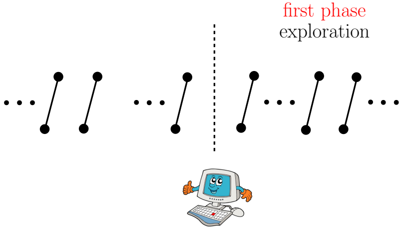

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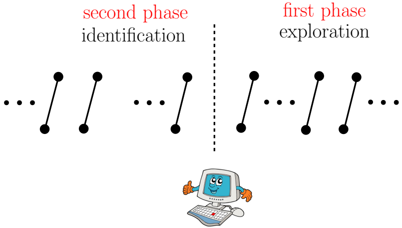

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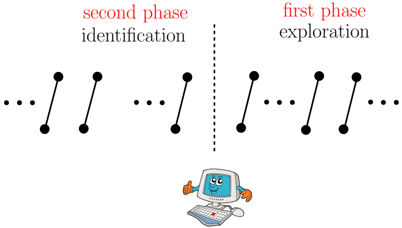

(ii) use of conditional probabilities for the analysis

In the rest

- Approximate #CCs
- Approximate the weight of MST

not in this talk

- Approximate the size of MIS in planar graphs and beyond

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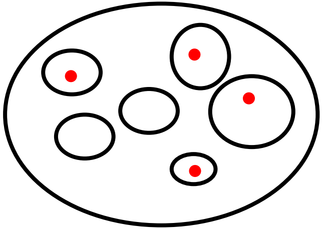

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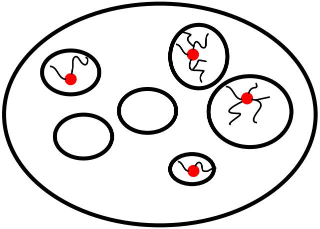

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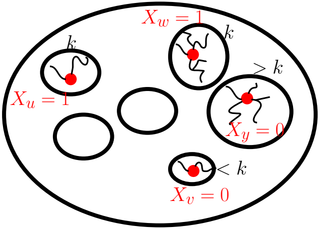

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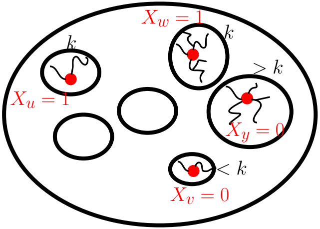

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$$\boxed{\text{BFS}(v) \Longrightarrow \text{StreamBFS}(v)}$$

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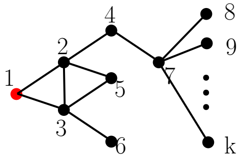

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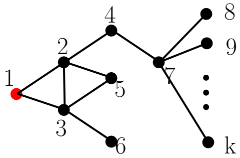

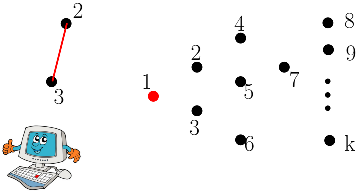

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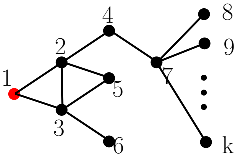

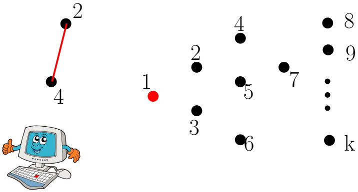

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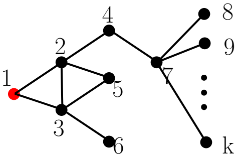

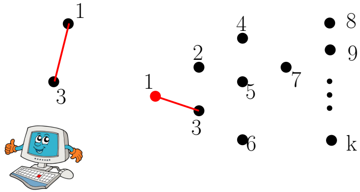

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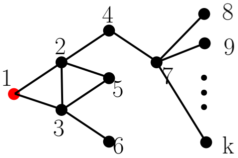

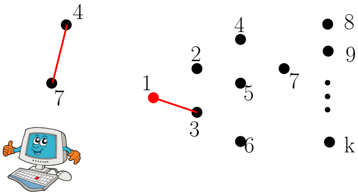

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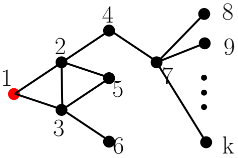

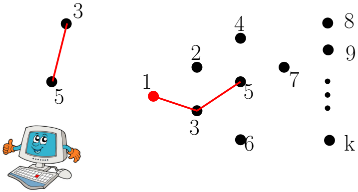

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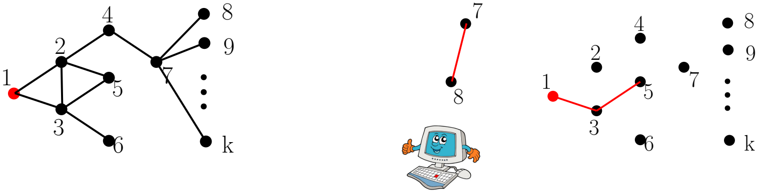

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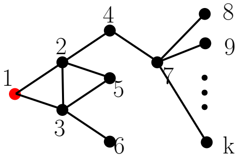

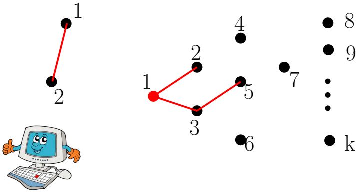

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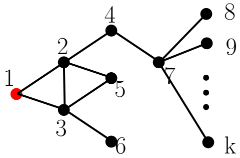

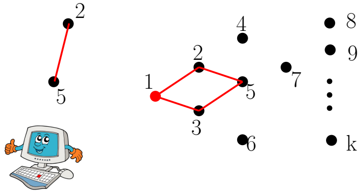

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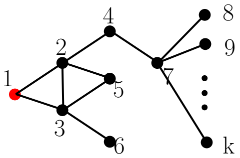

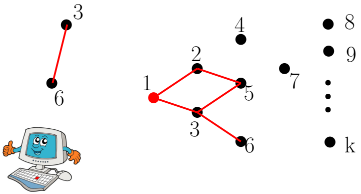

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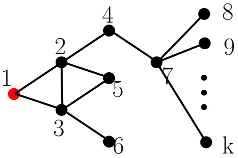

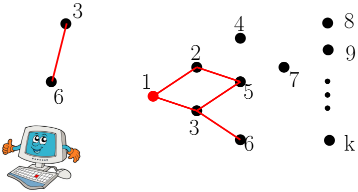

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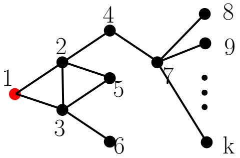

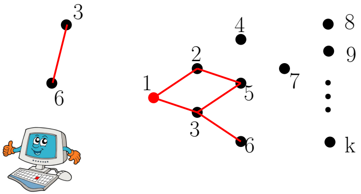

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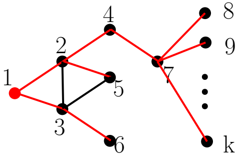

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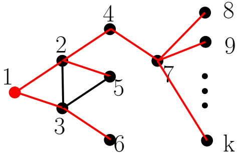

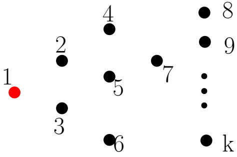

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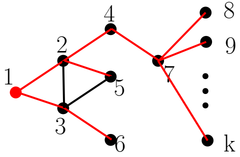

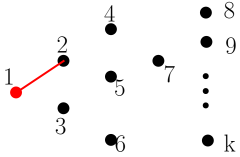

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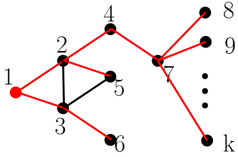

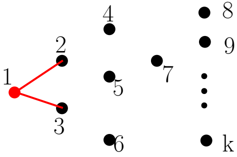

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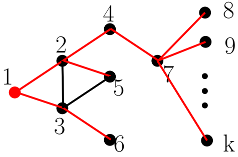

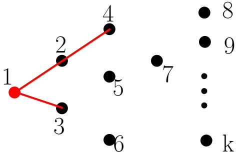

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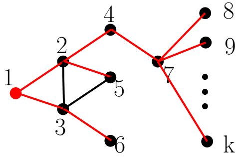

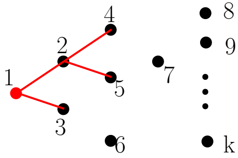

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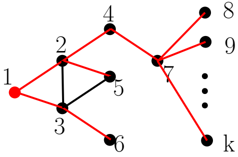

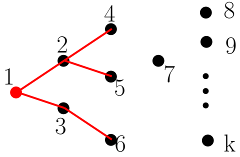

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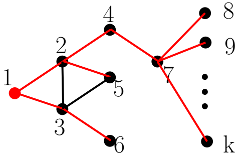

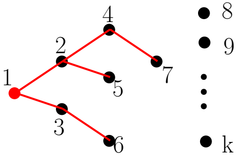

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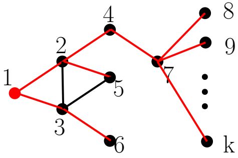

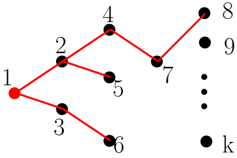

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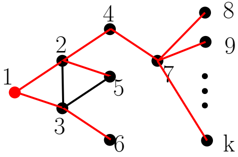

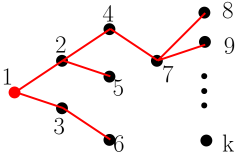

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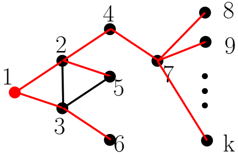

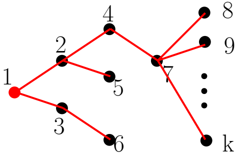

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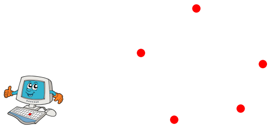

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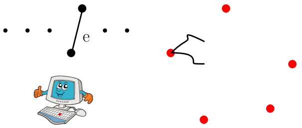

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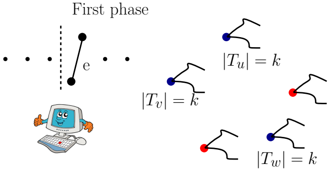

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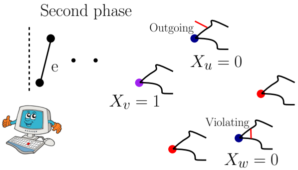

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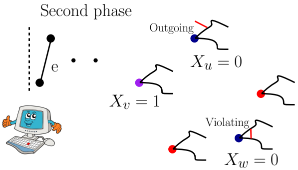

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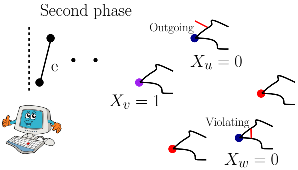

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γ_k:=Pr[any set T of $k-1$ edges appears in the lexico. order in the first phase]

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Conclusions and open problems

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