A SimpleFramework for Optimization over Sliding Windows

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Workshop on Data Summarization, University of Warwick

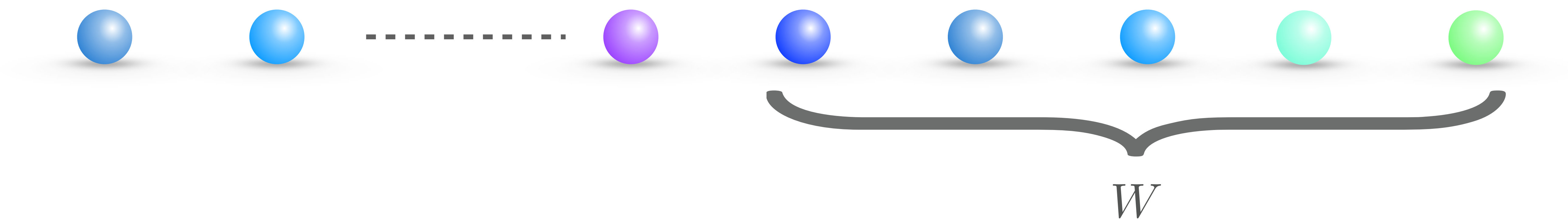
Outline

- Sliding Windows model
  Model, exponential histograms, smooth histograms, limitations
- A Framework for Optimization
  Suffix composability, maximization, minimization, main results
- Applications
  Submodular optimization, k-median
- Conclusions and future works

Sliding Windows model

Sliding Windows model

Elements arrive in a stream:



We are interested in last W elements

Design algorithm that use small memory

General frameworks

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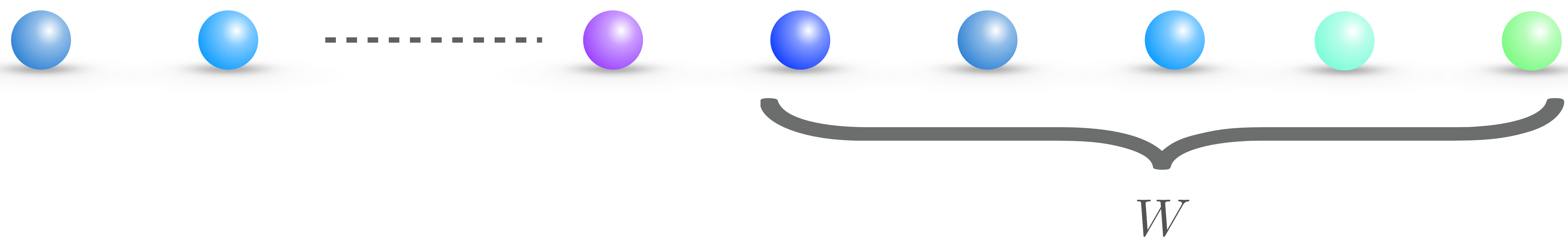

Two main frameworks:

- exponential histograms [DGIM01]
- smooth histograms [BO07]

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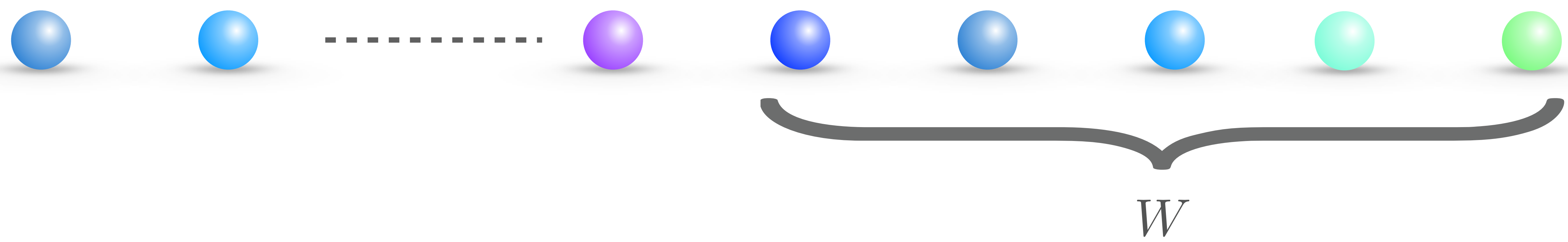

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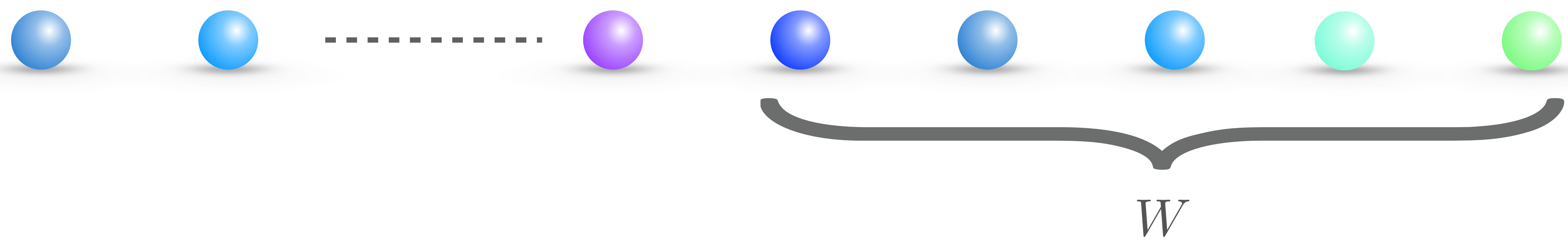

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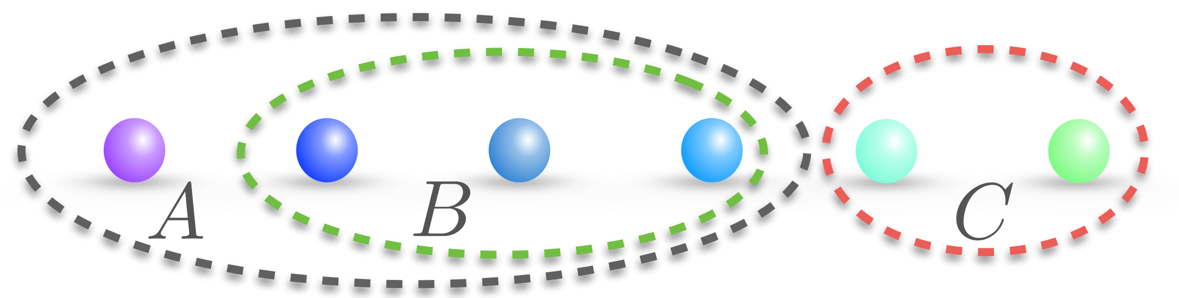
  (α, β)-smooth functions

Smooth histograms

(α, β)-smooth functions

$\forall A, B, C$ with $B \subseteq_r A$



$$(1-\beta)f(A) \leq f(B) \implies (1-\alpha)f(A \cup C) \leq f(B \cup C) \text{ for } 0 < \beta \leq \alpha < 1$$

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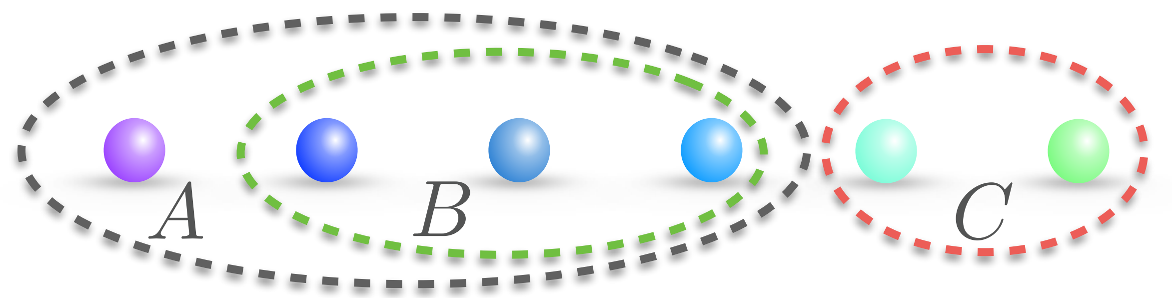

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If there is a streaming algorithm using space g to estimate f than there is a s.w. algorithm that computes an α approximation using space $O\left(\frac{1}{\beta}(g + \log n)\log n\right)$

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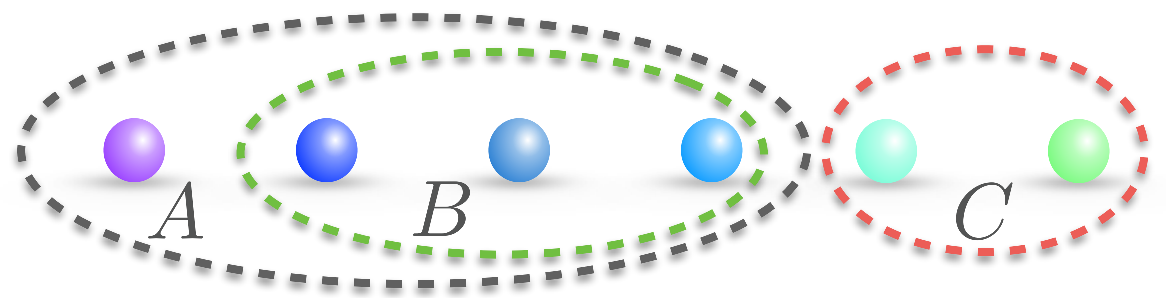

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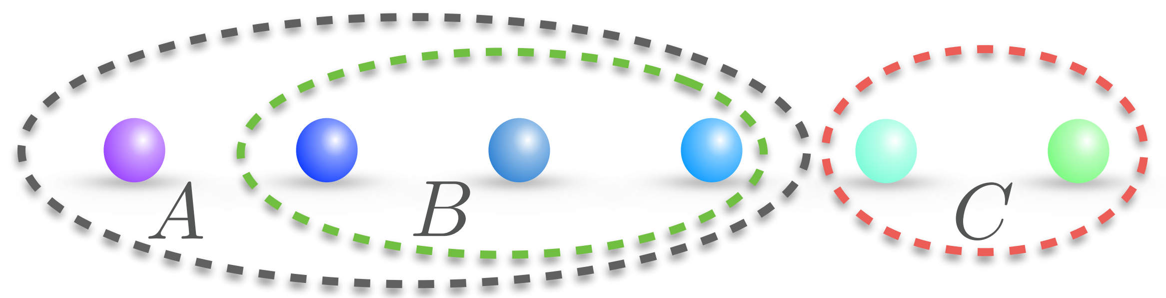

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Algorithms for L_p-norms, frequency moments, geometric mean,…

Limitations

Not all the functions have $(1+\epsilon)$-approximation

Not all the functions are smooth

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Submodular Optimization

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Submodular Optimization [CNZ16,ELVZ17]

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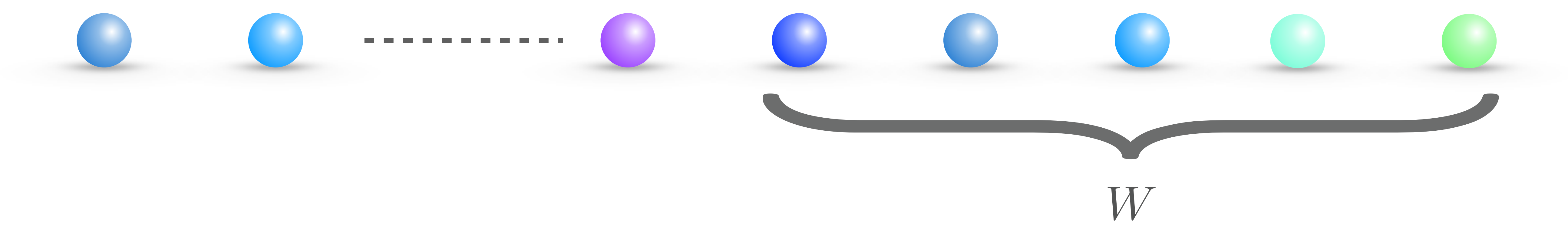
Can we find a framework for optimization?

A Framework for Optimization

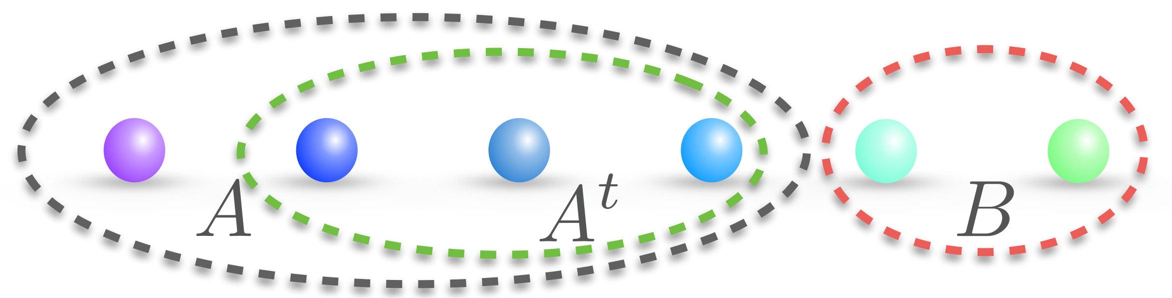
Main idea



Instead of characterize functions for which we have s.w. algorithms, we focus on sketch properties

Suffix composability

f be a monotone function and Z be a sketch function.



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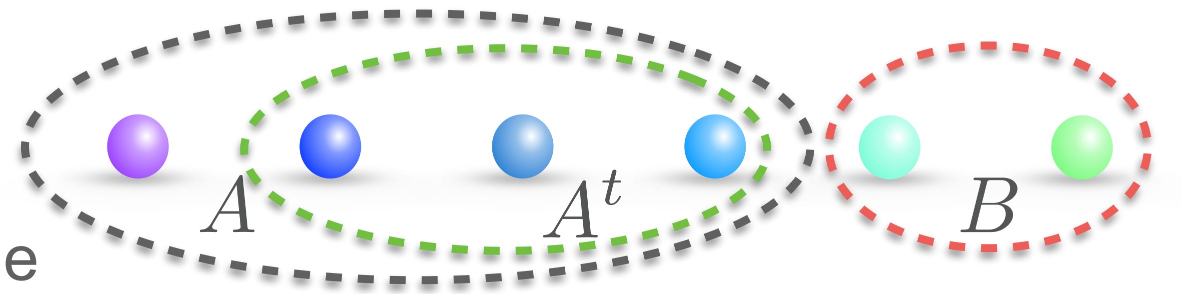

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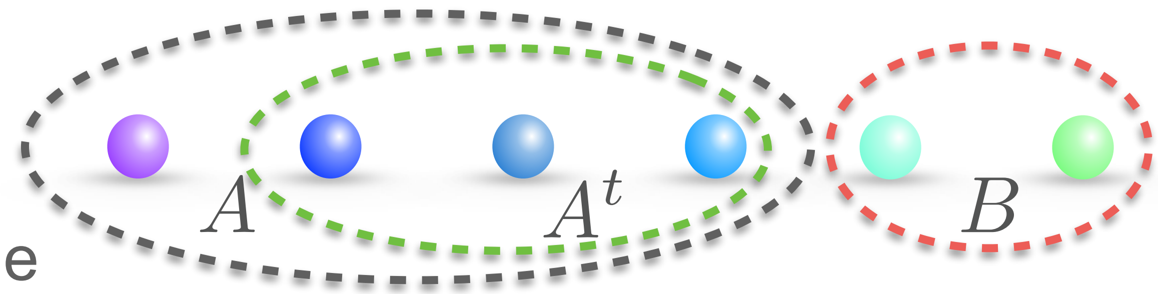

We say that Z is (α, β)-suffix composable if there exist a function $h : Z \times Z \times t \rightarrow Z$

$\forall A, A^t, B$ either

$$f(Z(A)) > \beta OPT(A^t \cup B)$$

or

$$f(h(Z(A), Z(B), t)) \geq \alpha OPT(A^t \cup B)$$

Maximization framework

Lemma

f be a monotone function and Z be a γ-approximating sketch function, such that Z is (α, β)-suffix composable. Then there is a s.w. algorithm that computes a $\min\left(\alpha, \frac{\gamma\beta}{1+\epsilon}\right)$ approximation using space $O\left(s_Z \log_{1+\epsilon} \frac{M}{\gamma m}\right)$

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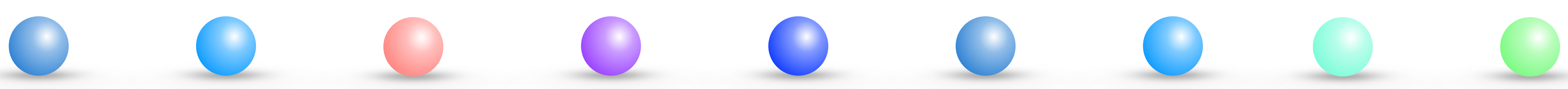
Proof

$$\lambda \in \{\gamma m, (1+\epsilon)\gamma m, (1+\epsilon)^2\gamma m, ...\}$$

$$Z(A_\lambda)$$

$$Z(B_\lambda)$$



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$$\lambda \in \{\gamma m, (1+\epsilon)\gamma m, (1+\epsilon)^2\gamma m, ...\}$$

$$Z(A_\lambda) \leftarrow Z(\emptyset)$$

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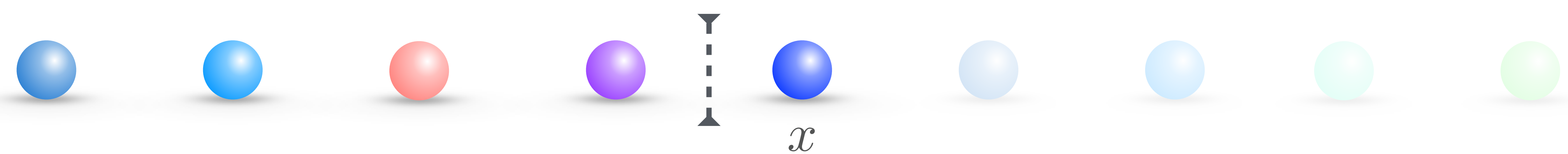

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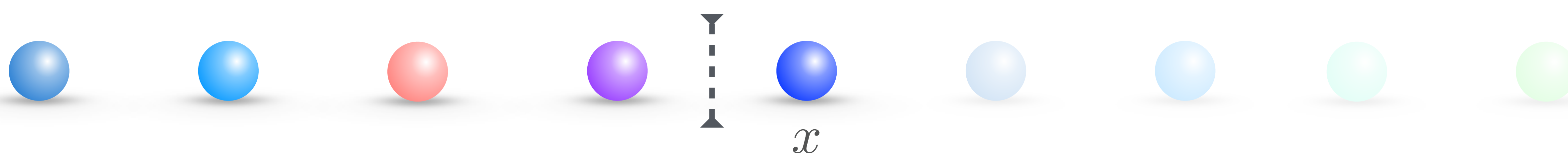

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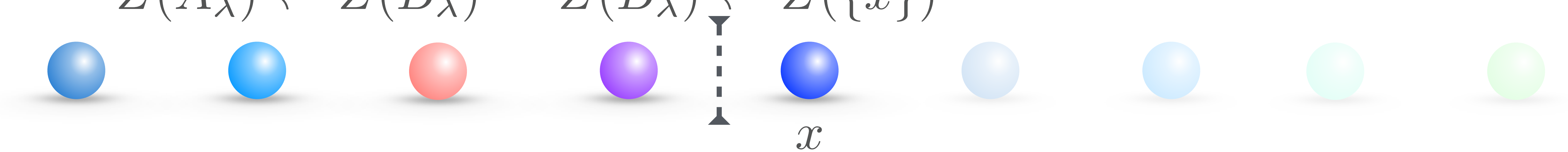
$Z(A_\lambda) \leftarrow Z(B_\lambda)$ $\quad$ $Z(B_\lambda) \leftarrow Z(\{x\})$



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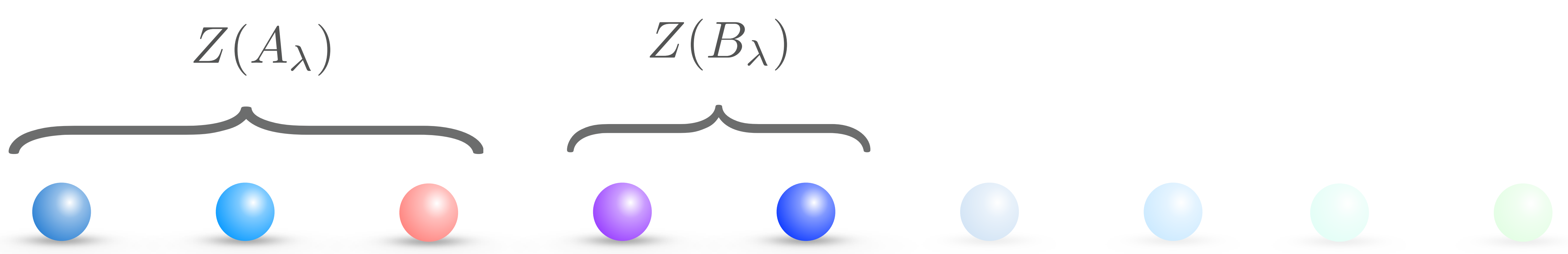

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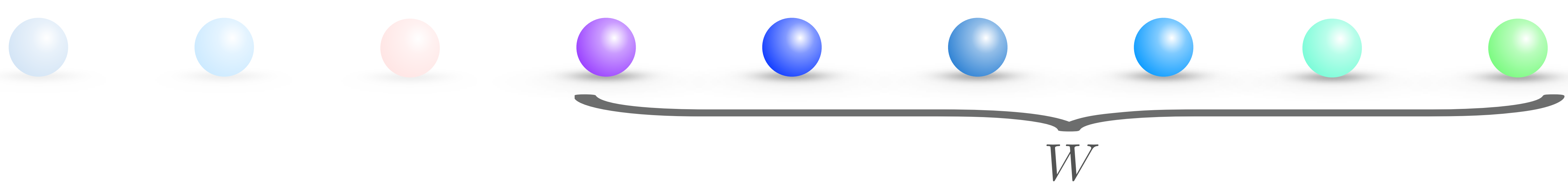

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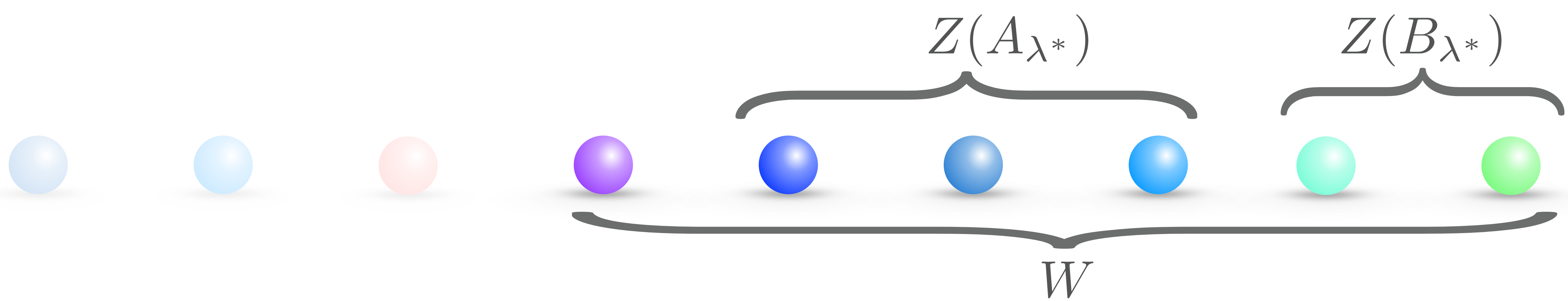

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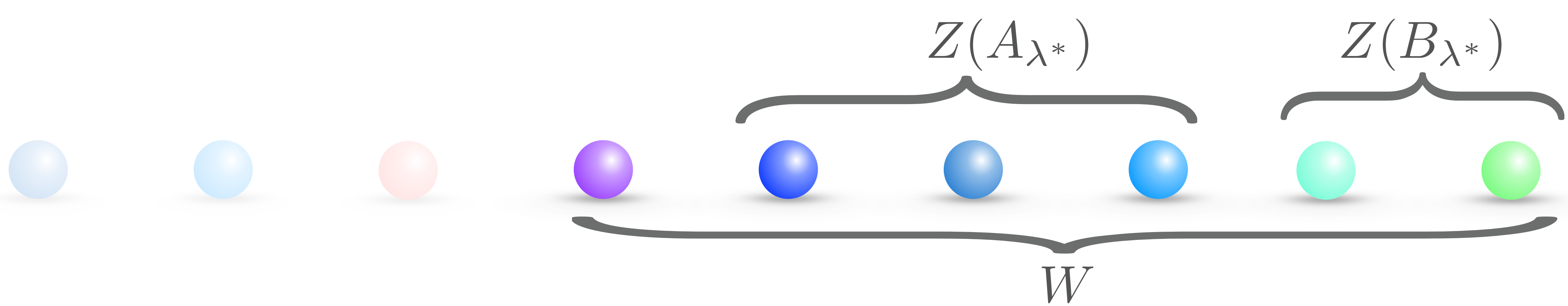

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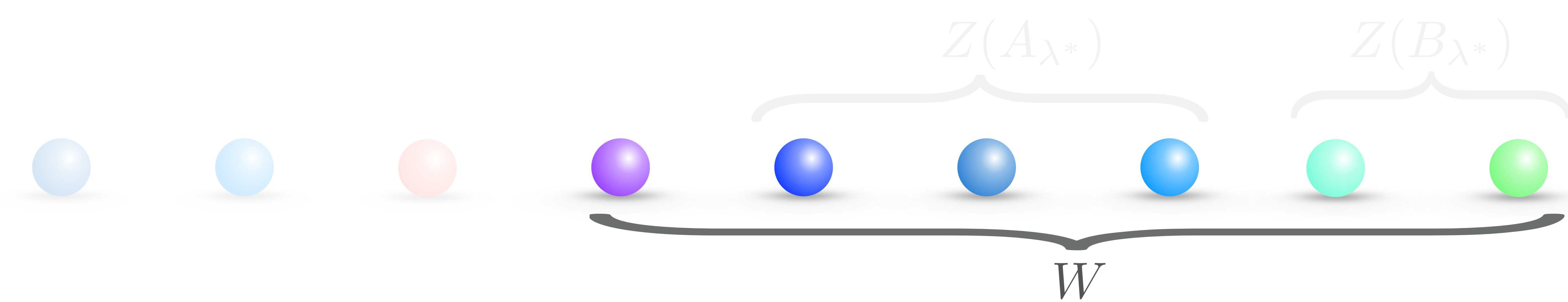

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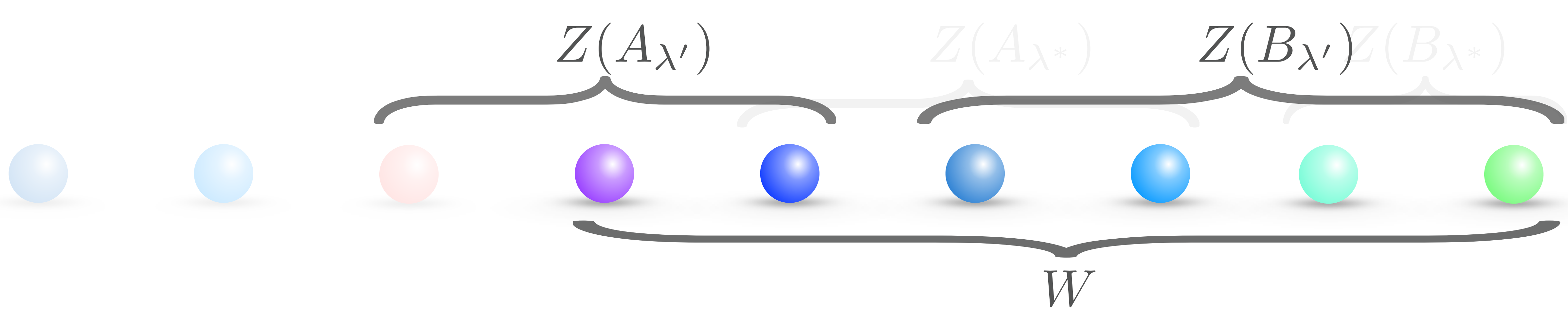

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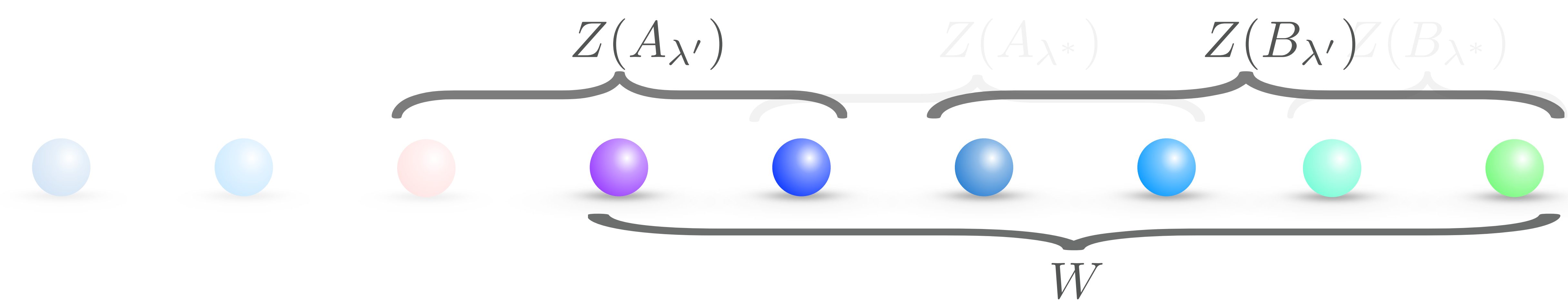

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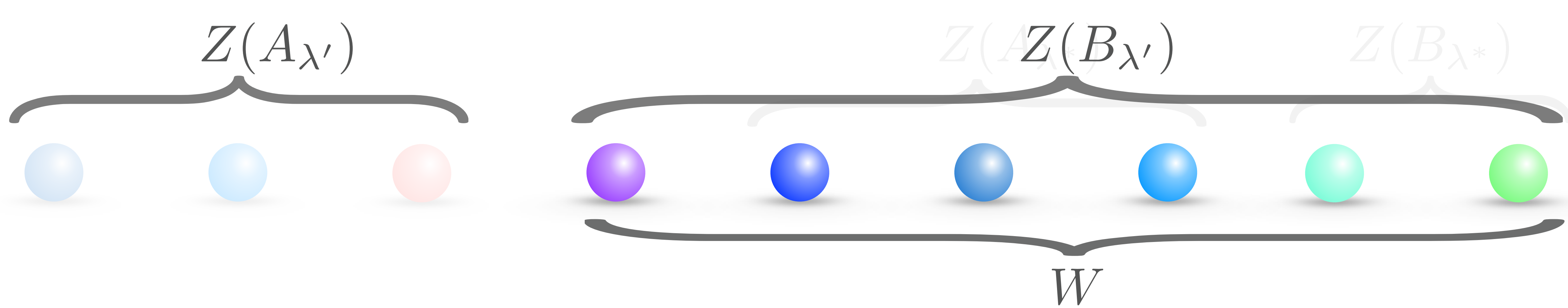

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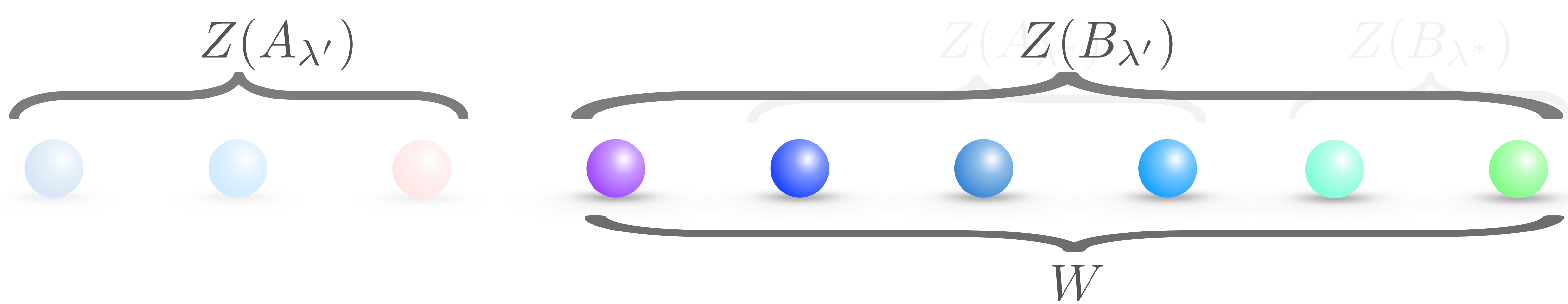

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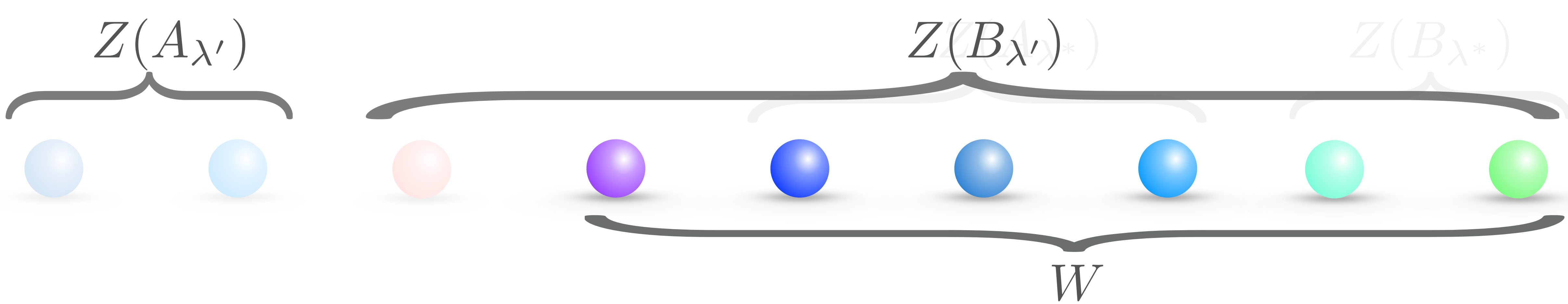

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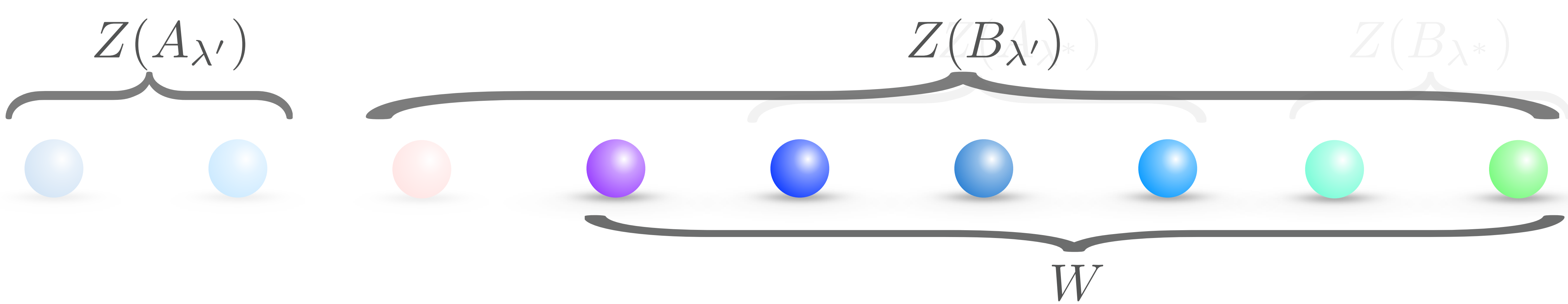

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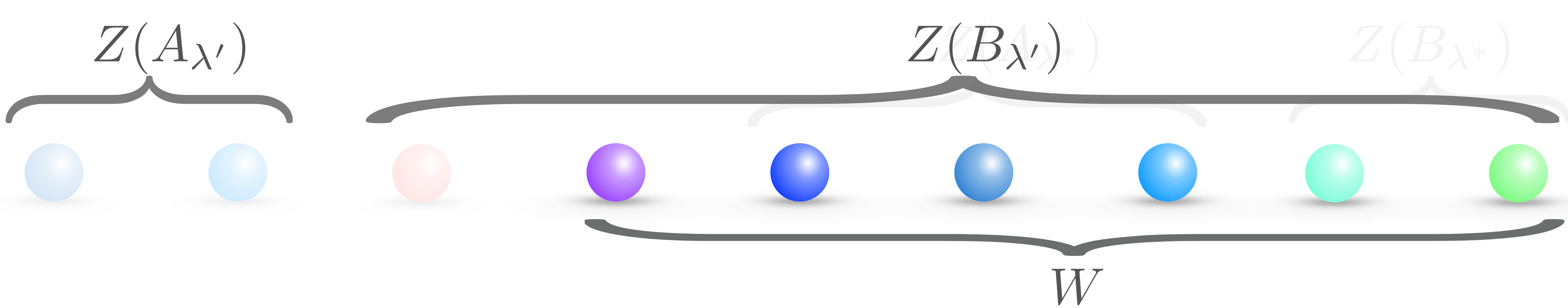
contradiction



General framework

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f be a monotone function

$$X' \subseteq X \implies f(X) \geq f(X')$$

Minimization

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if:

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then:

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We get sliding windows algorithms

General framework

Problem	Space	Approx
Sub. Optimization	$O(k \log n)$	$1/3 - \epsilon$
Diversity max.	$\tilde{O}(k)$	$\gamma/5 - \epsilon$
k-median / k-means	$\tilde{O}(k)$	$O(1)$
k-center	$\tilde{O}(k)$	$24 + \epsilon$

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Applications

Submodular Optimization

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Let $h(Z(A), Z(B), t) = Z(B)$

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$$f(Z(B)) \geq \gamma OPT(B)$$

Submodular Optimization

Maximization

f be a monotone function

$$X' \subseteq X \implies f(X) \geq f(X')$$

Z is (α, β)-suffix composable

either:

$$f(Z(A)) > \beta OPT(A^t \cup B)$$

or:

$$f(h(Z(A), Z(B), t)) \geq \alpha OPT(A^t \cup B)$$

Monotone by definition

Let $h(Z(A), Z(B), t) = Z(B)$

$$f(Z(B)) \geq \gamma OPT(B)$$

$$\geq \gamma(OPT(A^t \cup B) - OPT(A^t))$$

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$$\begin{aligned} f(Z(B)) &\geq \gamma OPT(B) \\ &\geq \gamma(OPT(A^t \cup B) - OPT(A^t)) \\ &\geq \gamma(OPT(A^t \cup B) - OPT(A)) \end{aligned}$$

k-median

Minimization

f be a δ-monotone function

$$X' \subseteq X \implies \delta f(X) \geq f(X')$$

Z is (α, β)-suffix composable

if:

$$f(Z(A)) \leq (1+\beta)\delta OPT(A^t \cup B)$$

then:

$$f(h(Z(A), Z(B), t)) \leq (1+\alpha) OPT(A^t \cup B)$$

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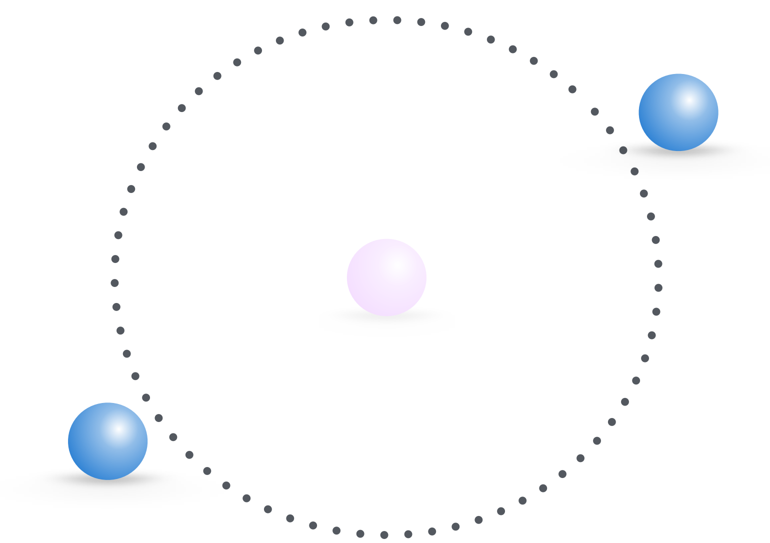
δ -monotone function



k-median

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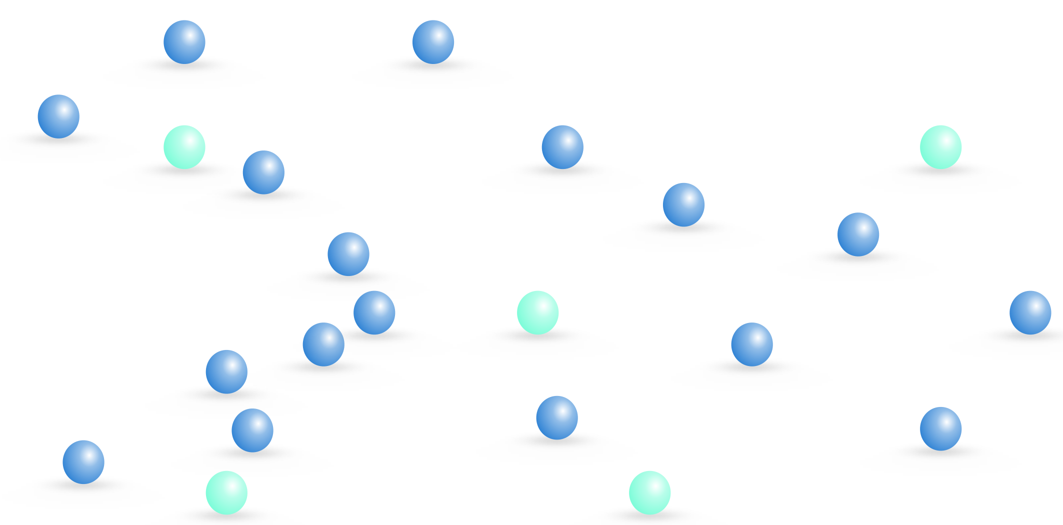
Meyerson sketch



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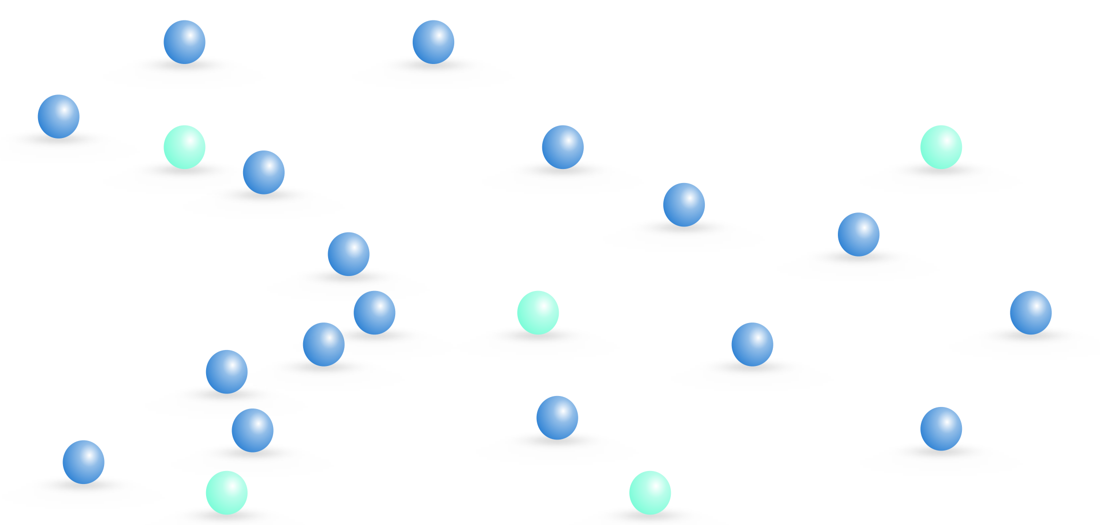

The $\tilde{O}(k)$ points have 2 properties:

- mapping has cost $O(OPT)$
- weighted instance has cost $O(OPT)$

k-median

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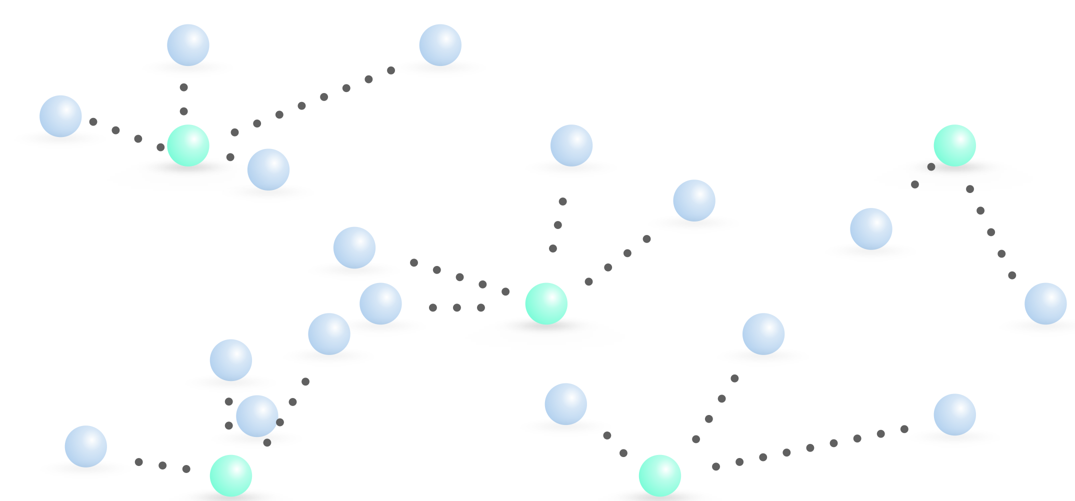

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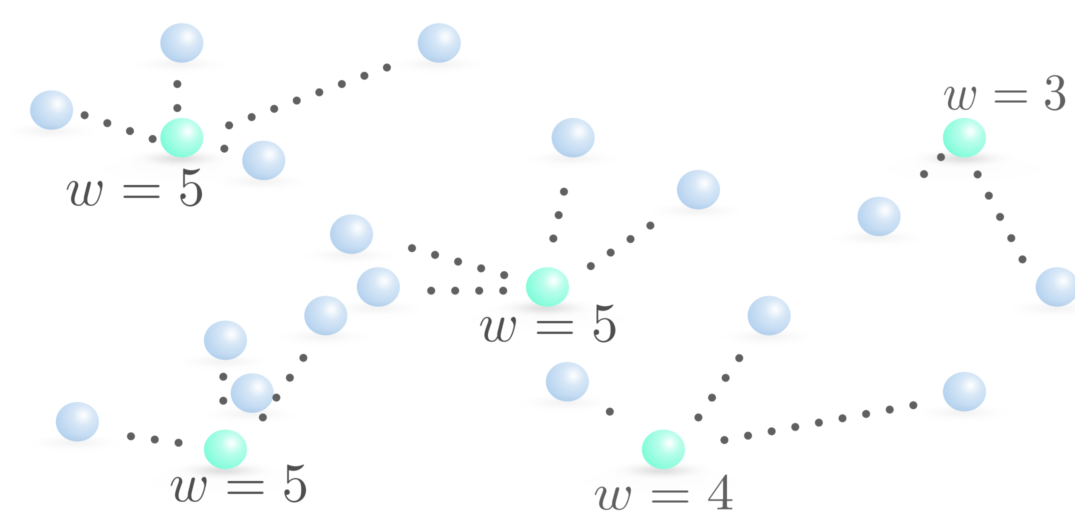

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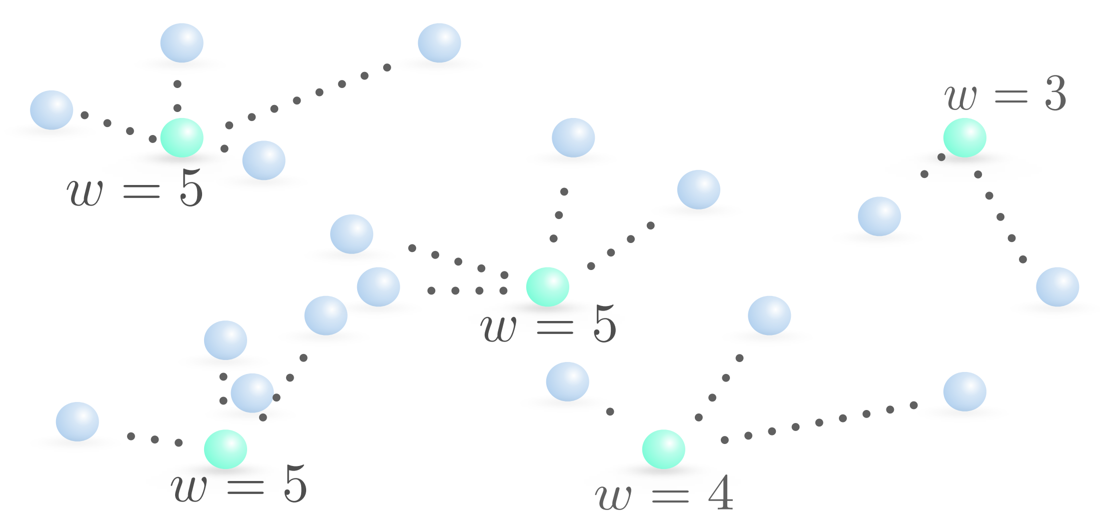

Now suppose $Z(A)$ is a Meyerson sketch,

k-median

Minimization

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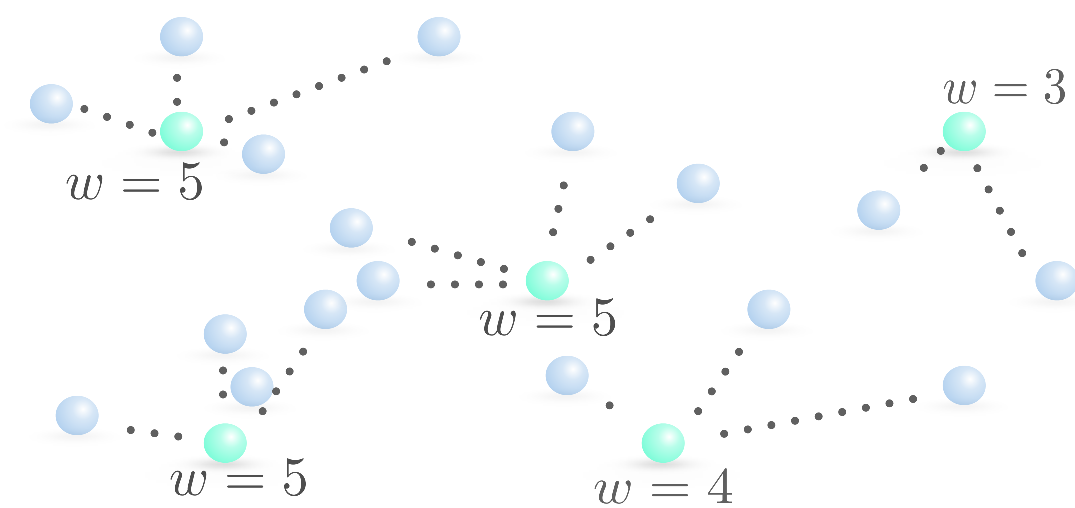
Meyerson sketch



Now suppose $Z(A)$ is a Meyerson sketch, a good sketch for A^t is $Z(A)$ with updated weights.

k-median

Minimization

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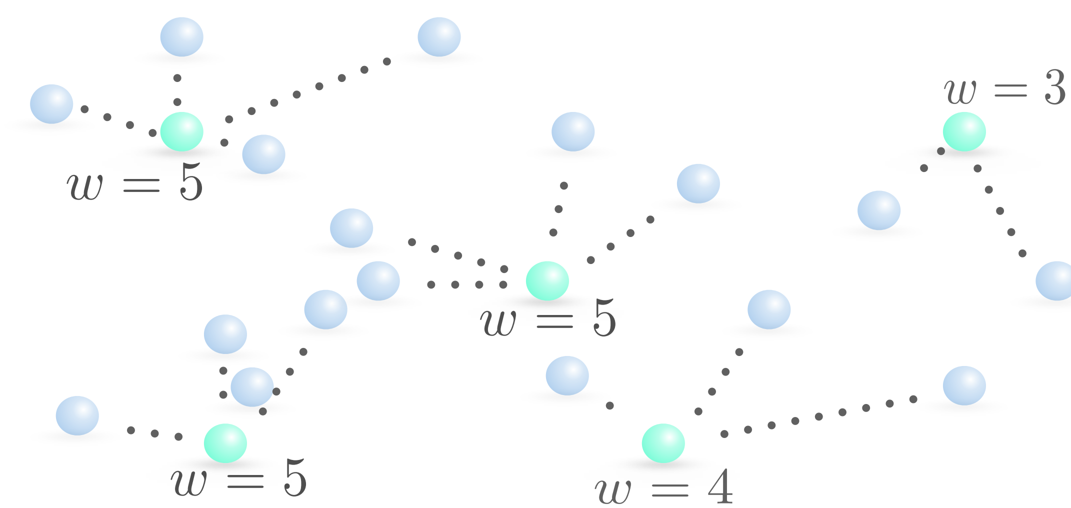

Now suppose $Z(A)$ is a Meyerson sketch, a good sketch for A^t is $Z(A)$ with updated weights.

We can keep track using $\log_{1+\epsilon} W$ buckets

k-median

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k-median

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δ -monotone function

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k-median

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$$f(h(Z(A), Z(B), t)) = f(Z(A^t) \cup Z(B))$$

k-median

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$$h(Z(A), Z(B), t) = Z(A^t) \cup Z(B)$$

$$f(h(Z(A), Z(B), t)) = f(Z(A^t) \cup Z(B))$$

$$\leq O(OPT) + O(f(Z(A))) + O(f(Z(B)))$$

Conclusions and future works

Conclusions

- New framework for optimization
- Better results for maximization
- Better results for minimization

Future works

- Can we recover k-center result?
- Can we apply the framework to other problems?
- Can we simplify the framework?

Thanks

k-median

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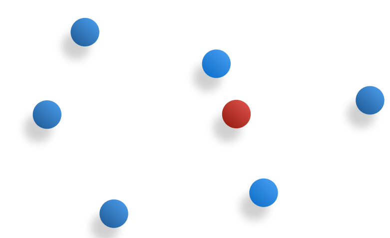

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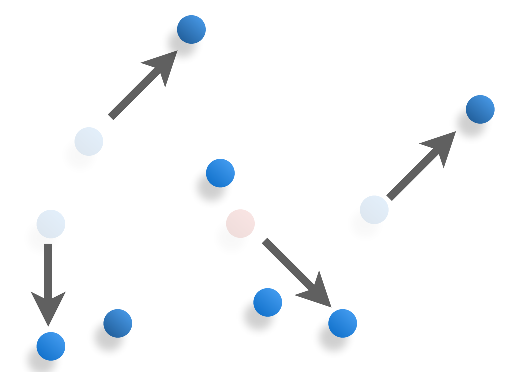

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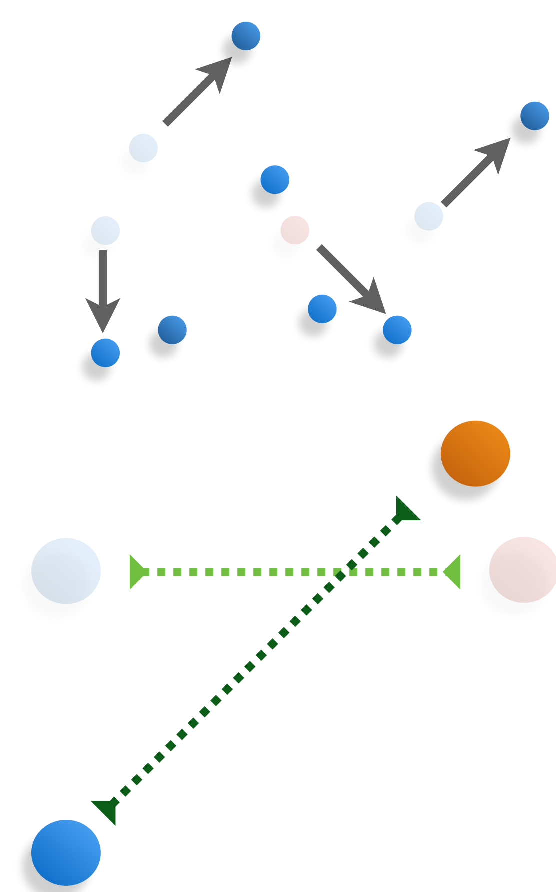

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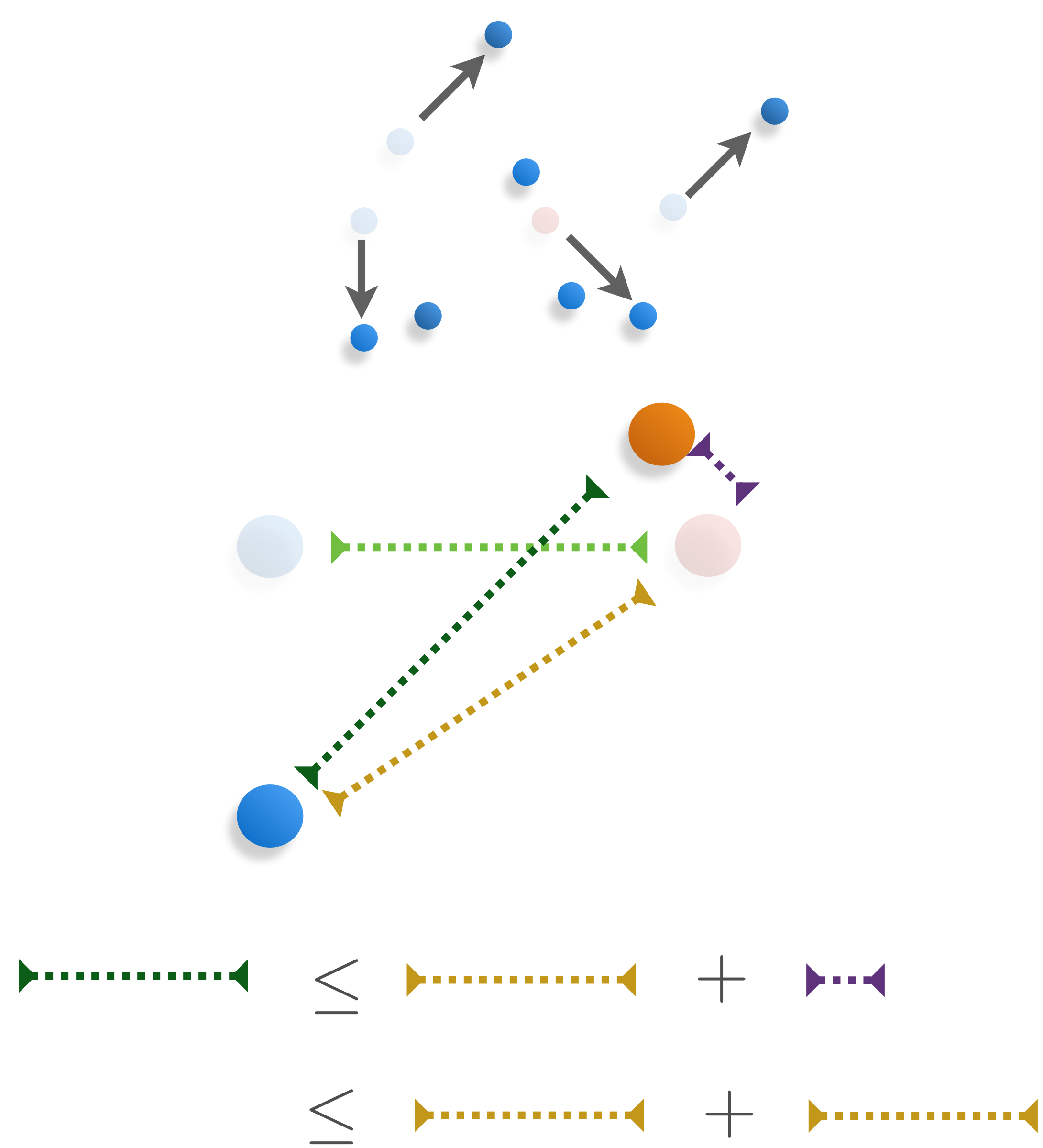

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