FE Modelling of FRP Structures: Influence of Uncertainties

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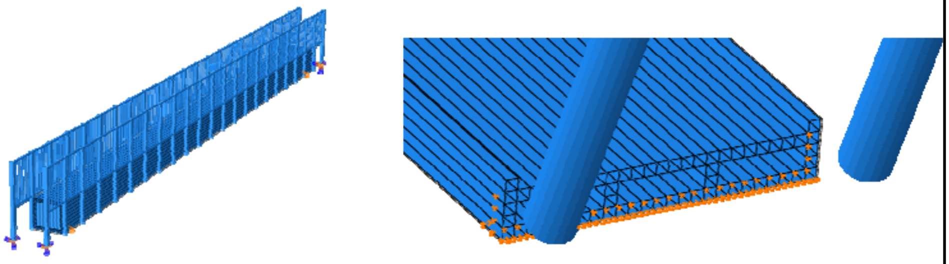

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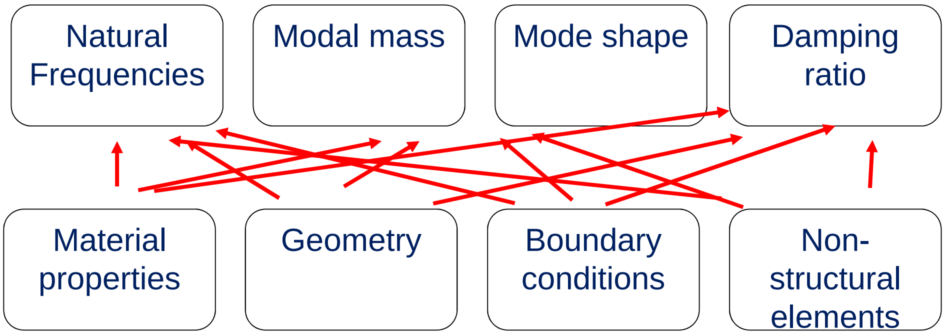
(Very) Brief overview of FRPs

- Fibre Reinforced Polymer
    - Can be pultruded into structural shapes
- Made up of:
    - Bundles of fibre – longitudinal strength
    - Fibre mats – transverse properties
    - Resin matrix – durability

•Key properties
- Light (~1800kg/m^3)
- Flexible (E_1 around 23GPa)
- Anisotropic (E_2 = 20-30% E_1)
- Usually controlled by SLS – vibration a concern

Sources of uncertainty

- Focusing on properties that influence the dynamic response



Sources of uncertainty

- Focusing on properties that influence the dynamic response

Natural frequencies of simply supported beam:

$$f_n = \frac{1}{2\pi}\left(\frac{n\pi}{L}\right)^2\sqrt{\frac{EI}{m}}$$

Elastic material stiffness

Mass per length

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f(t)\}$$

Accelerations controlled by mass matrix

- Therefore, it is clear material properties influence the dynamic response in multiple ways

Sources of uncertainty

- Focusing on properties that influence the dynamic response

Natural Frequencies

Modal mass

Mode shape

Material properties

Longitudinal stiffness

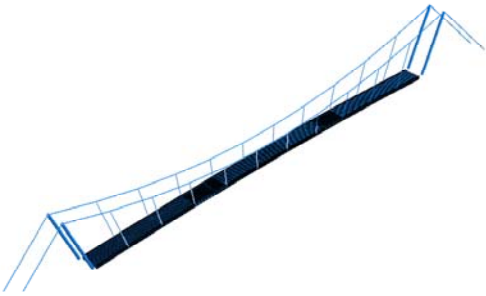
Transverse stiffness

Shear modulus

Density

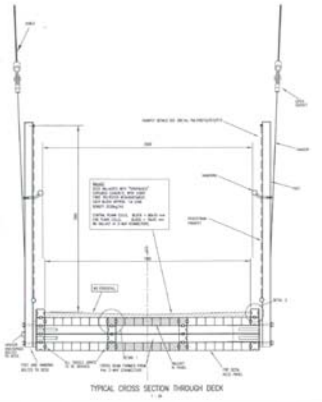
Example 1

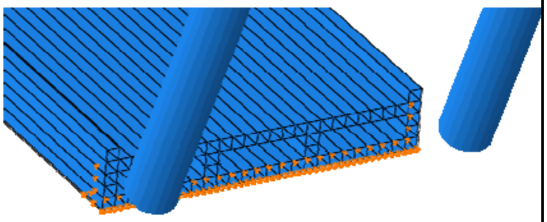
- 50m suspension bridge
- FRP deck, steel cables
- Footbridge over a duel carriage way



Example 1

- Finite Element modelling Abaqus/Standard
- FRP deck modelled with shell elements, lamina properties
- i.e. thickness (*t*), density (*ρ*), elastic moduli (E_1, E_2), Poisson's ratio (*v)* and shear modulus (*G)*





Wei X, Russell J, Živanović S, Mottram JT. Experimental investigation of the dynamic characteristics of a glass-FRP suspension footbridge. In: Caicedo J, Pakzad S, editors. Dynamics of Civil Structures, Volume 2 : Proceedings of the 35th IMAC, A Conference and Exposition on Structural Dynamics 2017. Cham: Springer International Publishing; 2017. p. 37-47.

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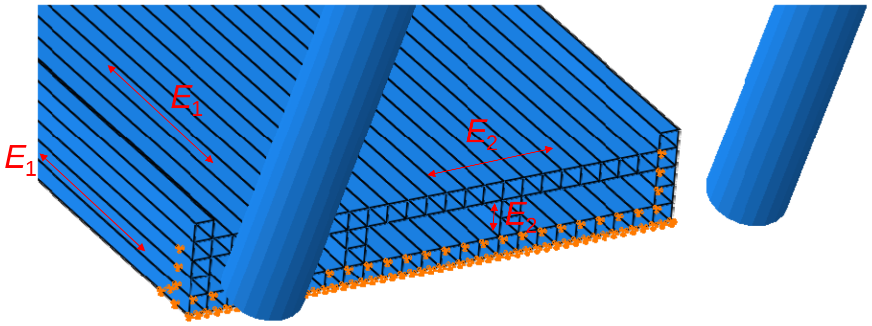
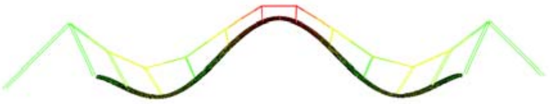


Example 1

- Modal outputs

1st Vertical Mode
Frequency: 0.87Hz
Modal Mass: 15.4t

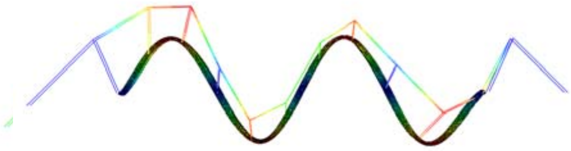


2nd Vertical Mode
Frequency: 1.41Hz
Modal Mass: 8.9t

Example 1

- Modal outputs

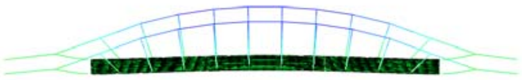
3rd Vertical Mode
Frequency: 2.35Hz
Modal Mass: 14.7t



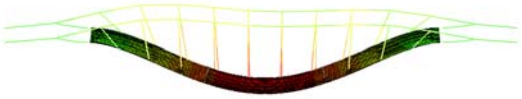
4th Vertical Mode
Frequency: 2.54Hz
Modal Mass: 17.1t

Example 1

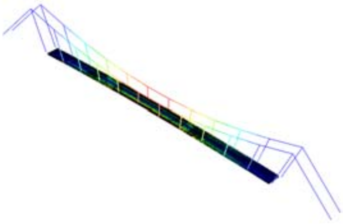
- Modal outputs



1st Cable Mode
Frequency: 1.08 Hz
Modal Mass: 2.4t



1st Lateral Mode
Frequency: 1.56 Hz
Modal Mass: 13.0t



Torsion Mode
Frequency: 3.76 Hz
Modal Mass: 6.4t

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Example 1

- Change longitudinal stiffness (E_1)

	Decrease 20%		Increase 20%	
	Δ fn	Δ m	Δ fn	Δ m
1st Vertical	-4.6%	0.0%	3.4%	0.6%
2nd Vertical	-5.0%	1.1%	5.0%	-1.1%
3rd Vertical	-1.7%	2.7%	2.1%	-0.7%
4th Vertical	-7.1%	-1.8%	6.3%	1.2%
Cable Mode	1.9%	0.0%	0.9%	0.0%
Lateral	-7.7%	3.8%	7.1%	-0.8%
Torsion	0.0%	0.0%	0.5%	1.6%

- Some modes affected much more than others
- Significant variations possible in frequency
- Modal mass less affected as mode shape unaffected

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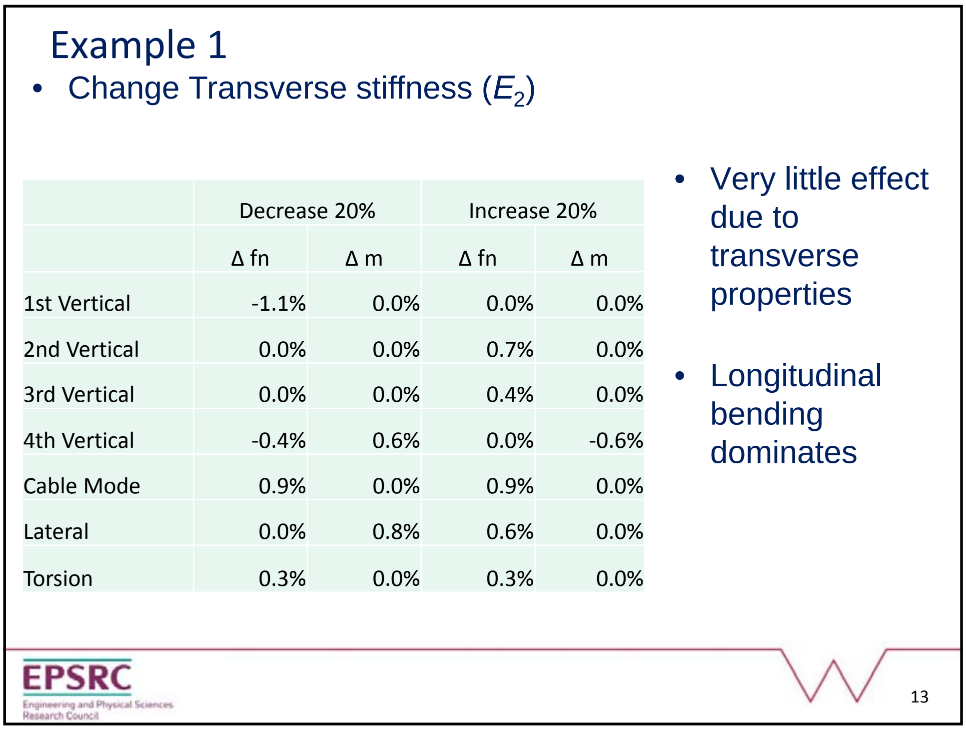
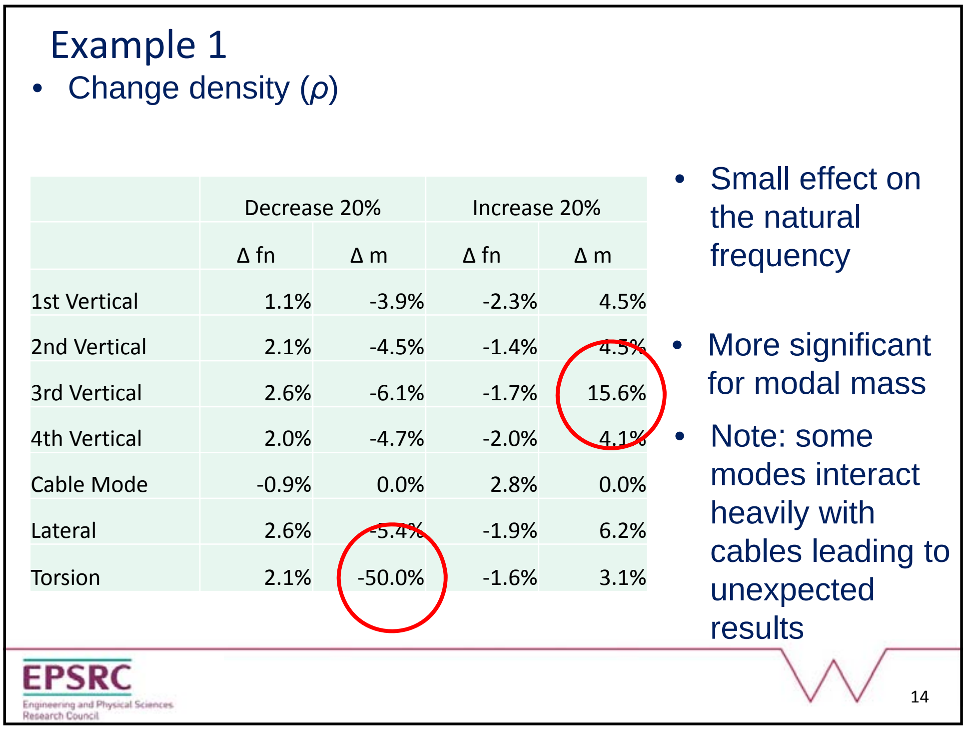


Example 1

- Change Transverse stiffness (E_2)

	Decrease 20%		Increase 20%	
	Δ fn	Δ m	Δ fn	Δ m
1st Vertical	-1.1%	0.0%	0.0%	0.0%
2nd Vertical	0.0%	0.0%	0.7%	0.0%
3rd Vertical	0.0%	0.0%	0.4%	0.0%
4th Vertical	-0.4%	0.6%	0.0%	-0.6%
Cable Mode	0.9%	0.0%	0.9%	0.0%
Lateral	0.0%	0.8%	0.6%	0.0%
Torsion	0.3%	0.0%	0.3%	0.0%

- Very little effect due to transverse properties
- Longitudinal bending dominates



Example 1

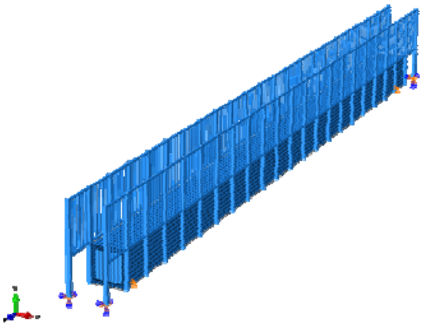
- Change density (ρ)

	Decrease 20%		Increase 20%	
	Δ fn	Δ m	Δ fn	Δ m
1st Vertical	1.1%	-3.9%	-2.3%	4.5%
2nd Vertical	2.1%	-4.5%	-1.4%	4.5%
3rd Vertical	2.6%	-6.1%	-1.7%	15.6%
4th Vertical	2.0%	-4.7%	-2.0%	4.1%
Cable Mode	-0.9%	0.0%	2.8%	0.0%
Lateral	2.6%	-5.4%	-1.9%	6.2%
Torsion	2.1%	-50.0%	-1.6%	3.1%

- Small effect on the natural frequency
- More significant for modal mass
- Note: some modes interact heavily with cables leading to unexpected results

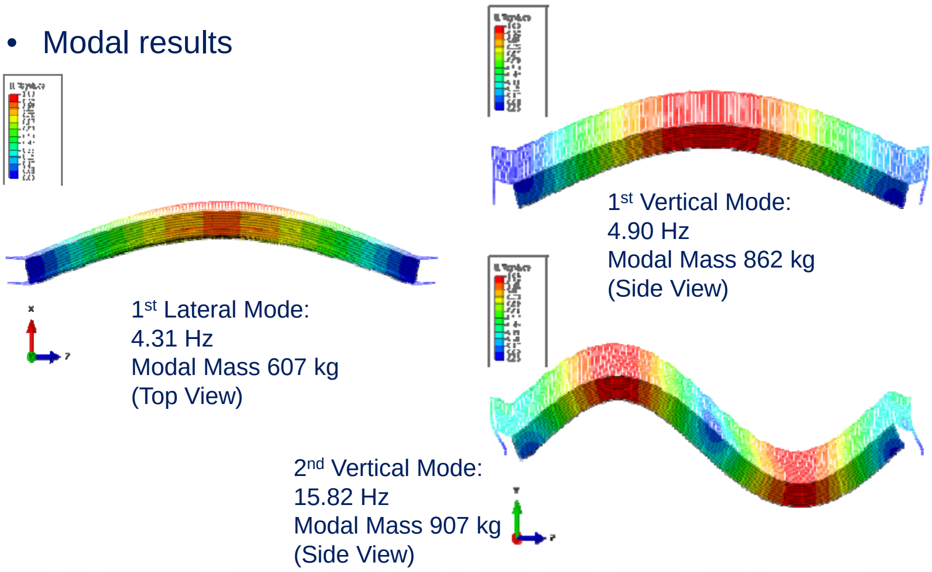
Example 2

- 16.9m girder bridge
- All FRP
- Modelled in Abaqus – similar approach



Example 2

- Modal results



1st Lateral Mode:
4.31 Hz
Modal Mass 607 kg
(Top View)

1st Vertical Mode:
4.90 Hz
Modal Mass 862 kg
(Side View)

2nd Vertical Mode:
15.82 Hz
Modal Mass 907 kg
(Side View)

Example 2

- Conducted an uncertainty quantification analysis
- Based on 10,000 Monte Carlo simulations
- Vary elastic moduli and conduct modal analysis
- Also simulate a walker to obtain variation in peak acceleration

Component	Mechanical property	Distribution	Mean (GPa)	COV(%)
Panel	Longitudinal elastic modulus	Weibull	21.6	10
	Transversal elastic modulus	Weibull	10	10
	Shear modulus	Weibull	3	10
3-way connector	Young’s modulus	Weibull	32	10

Work part of deeper analysis:
X. Wei, H. Wan, J. Russell, S. Zivanovic, J.E. Mottershead, Stochastic characterisation of dynamic responses of an actual all-FRP footbridge, Composite Structures, (2018). (in preparation).

Example 2

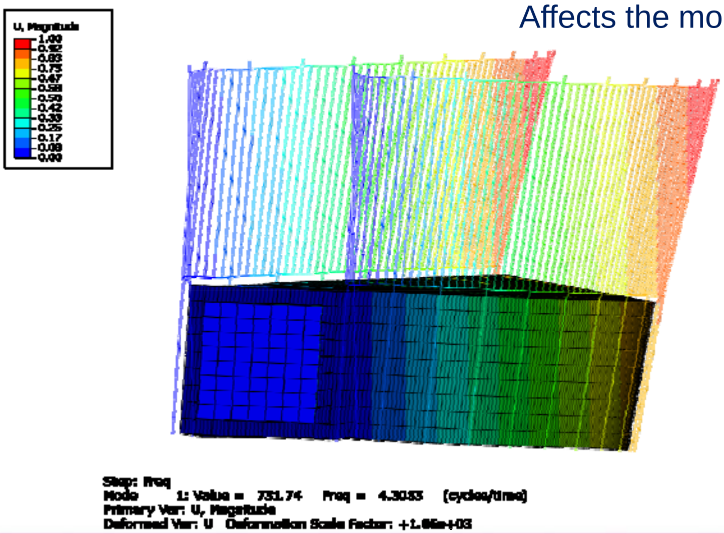
- COV of 10% for E_1, E_2 and G:

Natural frequency COV :
Mode 1 (1st lateral) = 3.37%
Mode 2 (1st vertical) = 2.92%
Mode 3 (2nd vertical) = 3.07%

Modal mass COV:
Mode 1 (1st lateral) = 2.98%
Mode 2 (1st vertical) = 0.04%
Mode 3 (2nd vertical) = 0.61%

Example 2

Note twisting of the deck
Large motion of handrails
Affects the modal mass



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Response to a human, assumed to excite 1st vertical mode:
Peak acceleration COV = 16%

Conclusions

- FRP bridges (like all structures) have uncertainties in their properties
- Dynamic response is based on many parameters within the model
- If vibration serviceability is a concern it is important to obtain suitable models
  - Underestimating stiffness or overestimating mass may lead to unconservative dynamic behaviour
- Variations in material properties can a have significant effect on the modal properties
- Acceleration response due to pedestrians may therefore also be incorrectly estimated

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Thank you

Any questions?