Modal testing and FE model tuning of a lively footbridge structure

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ABSTRACT

Despite huge advances in numerical modelling of civil engineering structures in recent decades, finite element models for footbridges should still be developed and used with caution when evaluating modal properties of these structures. This is due to some inherent modelling uncertainties related to a lack of information on the as-built structure, such as boundary conditions, material properties and the effects of non-structural elements. These are difficult to deal with at the design stage. A common method to rectify this problem is vibration testing of these structures after construction. As footbridges are unique prototype structures, testing at this late stage does not help very much in the design of the actual structure. However, combining testing and analysis improves understanding of its vibration behaviour, helps future designs of similar structures and provides key information for the design of remedial measures, if required.

This paper describes a lively full-scale footbridge, its numerical modelling and dynamic testing. This was done using state-of-the-art procedures available nowadays for finite element modelling and frequency response function based modal testing. The time efficiency of the testing and parameter estimation procedures carried out without formally closing the footbridge is demonstrated as well as good quality of results achieved. The identified vibration parameters compare well with those from an ambient vibration survey where only the bridge responses were measured. Also, it was demonstrated that properly planned testing can be performed successfully even with some limited facilities, such as only two accelerometers available. The correlation between a very detailed finite element model and experimental results is then studied. For this particular structural system, stiffnesses of girder end supports in the longitudinal direction and bending stiffness of inclined columns were identified as the modelling parameters which influenced most strongly the vertical and the horizontal modes of vibration, respectively.

Keywords: **Footbridge; FE modelling; Modal testing; Correlation; Model tuning.**

1 Introduction

Finite element (FE) modelling of footbridges is now common in the normal footbridge design process. With advances in numerical modelling, it is often expected that FE models based on technical design data and best engineering judgement can reliably simulate both the static and dynamic behaviour of the bridge. However, because of modelling uncertainties (such as stiffness of supports and non-structural elements, material properties and so on) as well as inevitable differences between the properties of the designed and as-built structure, these FE models often cannot predict natural frequencies and mode shapes with the required level of accuracy. This raises the need for verification of the FE models of footbridges after their construction. This is especially so when they are required to study further the structural behaviour, for example when designing vibration suppression measures (e.g. tuned mass dampers). Moreover, the modal damping, a very important dynamic parameter which governs the footbridge dynamic response near resonance, varies from structure to structure and can only be determined experimentally after the particular structure is built.

A possible approach nowadays for establishing a feedback between the real structural performance and design FE models is to employ some form of modal testing [1] on footbridges in service. The aim of modal testing is to determine as-built natural frequencies, mode shapes and damping ratios. Currently, the dynamic testing and experimental verification of FE models are not part of the normal design procedure in civil structural engineering. This is contrary to the mechanical and aerospace engineering disciplines where studying of prototype models of the structure and its correlation with a corresponding FE model has become a part of everyday design practice [2]. This is often justified by the fact that mechanical and aerospace engineering disciplines deal with the production of large numbers of identical structural units (e.g. cars, airplanes, etc.) as opposed to civil structural engineering where almost every structure is actually a unique prototype.

As dynamic testing is not a part of the normal design process in civil structural engineering and it can also be expensive, it is common not to do it. Additionally, there are serious legal and cultural issues related to it, such as a fear of legal liability, possibly leading to additional expense, if the testing proves that the structure does not perform as predicted during design. As a result, there is currently a serious lack of information about the as-built dynamic performance of civil engineering structures, leading also to a lack of information about the reliability of FE modelling in civil structural engineering design. In other words, by avoiding testing as-built structures and correlating the measured data with the numerical models, we do not know how good the created dynamic FE models are even if best engineering judgement and practice is employed in the modelling.

On the other hand, there are multiple benefits of dynamic testing. In the case of footbridges it adds to the body of knowledge about their as-built performance. As mentioned above, this knowledge is currently very limited, which is not satisfactory considering that vibration serviceability is becoming the governing design criterion for footbridges [3]. Also, testing enables retrofitting measures, such as installation of tuned mass dampers. Moreover, it helps future design of structures of similar configurations. Finally, dynamic testing is nowadays an important part of research into vibration serviceability of footbridges. Namely, good quality experimental data are required for both manual (model tuning) and automatic updating of the initial design FE models [1] and [2]. These models are crucial for studying vibration serviceability of as-built footbridges.

Bearing all this in mind, this paper presents a case study related to FE modelling, modal testing and FE model tuning of a lively footbridge in Podgorica, Montenegro. It demonstrates the reliability of a very detailed FE model, which was firstly developed by employing best engineering judgement and available design data. Then, the lowest modes of vibration in both the vertical and horizontal lateral directions were identified using a state-of-the-art frequency response function (FRF) based testing procedure, using shaker excitation with in-situ data processing and analysis. Also, to verify the shaker test results, an ambient vibration survey (AVS) was conducted to examine the (lively) vertical direction only. Based on the experimental results, the initial FE model was then revised and manually tuned to match the actual natural frequencies of the footbridge more closely. Manual tuning is required when developing an FE model suitable for later implementation of the automatic updating procedure [4]. Otherwise, automatic updating may not work as the starting model is too far away from the measured targets. Using the manually tuned FE model, a sensitivity-based automatic updating was also conducted successfully, but the results of that study are beyond the scope of this paper.

The first part of this paper reviews briefly the existing literature concerning the numerical and experimental investigations of dynamic performance of footbridges. Then, the steel box-girder footbridge investigated is described followed by a description of an initial FE model and the main assumptions made during its development. Then an FRF-based modal testing of the bridge is described, together with an AVS testing exercise. After this, the as-built modal properties of the footbridge are identified and compared with the numerical results. Finally, the FE model is manually tuned and key results of this interesting exercise are discussed.

2 Background Review

There are only a few articles dealing with both vibration testing and FE modelling specifically related to footbridges. Probably the first extensive work related to vibration testing of footbridges was conducted by the UK Transport and Road Research Laboratory in the 1970s [5], [6] and [7]. Their work concentrated on presenting and discussing the experimentally identified modal properties [5] and [6]. The testing equipment used was described separately [7] and from today's point of view it was quite limited. This is particularly so regarding the type of exciters used, very limited excitation frequency range, the slow data collection and limited vibration parameter estimation techniques. A common shortcoming related to the 1970s reporting of tests was that the digital data acquisition parameters (sampling rate, type of filtering, etc.) were regularly omitted from written reports. Knowing these parameters is very important to judge the quality of the experimental data collected and, consequently, the quality of the vibration parameter estimation.

In the last 10–15 years, a number of authors have started to pay more attention to providing complete data acquisition parameters in their work [8] and [9]. Together with this trend some work discussing correlation between modal testing and FE model results has also emerged [10], [11], [12], [13] and [14]. The reported discrepancies between the initial model predictions and experimental results were usually highly significant, typically due to inaccurate modelling of boundary conditions, neglected influence of non-structural elements, such as deck and handrails, and uncertain material properties. This has led to improved structural idealisation (boundary conditions, non-structural elements, etc.) and revised estimates of numerical modelling parameters to be used in creation of the FE models [4] and [15].

The modal testing procedures used in all these papers were either FRF-based testing (using either instrumented shaker or hammer excitation) or ambient vibration survey. In the former both the input force and output response are measured. In the latter only the response is measured while the input force due to environmental excitation (wind, traffic, etc.) is assumed to be a stationary random process (that is white noise excitation having an approximately flat frequency spectrum within the frequency range of interest). A brief but well presented review of modal testing methods for bridges explaining their advantages and limitations was presented by Salawu and Williams [16].

3 Description of Test Footbridge Structure

The investigated footbridge spans 104 m over the Morača River in Podgorica, capital of Montenegro (Fig. 1).

The structural system of the Podgorica footbridge is a steel box girder with inclined supports (Fig. 2). The structure's main span between inclined columns is 78 m and it has two side spans of 13 m each. The top flange of the main girder forms a 3 m wide deck. The depth of the girder varies from 1.4 m in the middle of the central span to 2.8 m at the points where the inclined columns connect to the main box girder (Fig. 2). Along its whole length the box girder is stiffened by longitudinal and transverse stiffeners, as shown in Fig. 2. The connection between the inclined columns and box girder is strengthened by vertical stiffeners visible in Fig. 1. Water supply and drainage pipes pass through the steel box section and they are suspended from the top flange of the main girder (Fig. 2).

In the early 1970s, when the footbridge was designed and constructed, design guidelines for vibration serviceability of footbridges did not exist in the former Yugoslavia. This is not surprising considering that the first modern footbridge vibration serviceability guidelines appeared in BS5400 [17] in the UK in 1978.

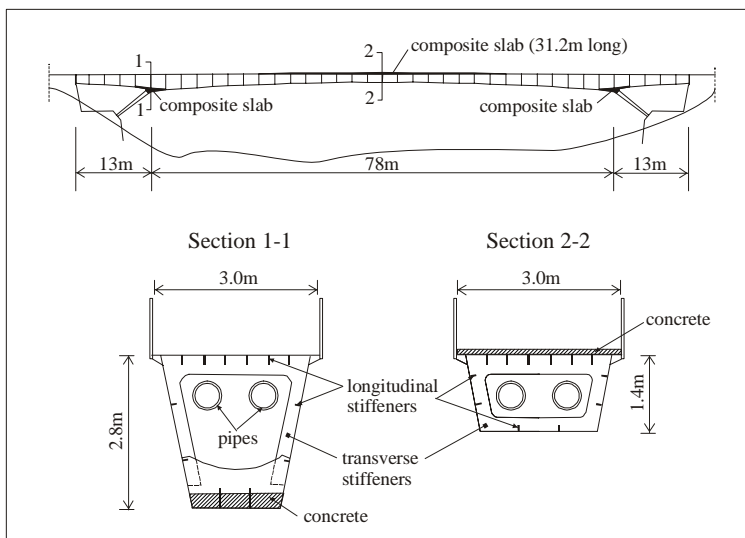Figure 1: Photograph of the Podgorica footbridge.

Figure 2: General arrangement drawing (not to scale) of Podgorica footbridge.

Since its construction, under pedestrian walking excitation, the footbridge has been very lively in the vertical direction. Also, a review of its design calculations found that a particular design load combination induces stresses close to allowable limits in some of its cross sections. This was the reason for designers to try to strengthen the structure to increase its carrying capacity. The plan for this strengthening was to also stiffen the structure and shift its natural frequencies above the frequency region of normal human walking which is 1.6–2.4 Hz [18]. The steel box girder was strengthened by a concrete slab cast over the bottom steel flange in the region around the columns (Fig. 2). The aim was to enhance this part of the box section so that it can resist large column reactions and compression due to hogging bending moments. A similar steel–concrete composite slab was cast around the mid-span of the footbridge, but this time over the top flange (Fig. 2). However, this added not only stiffness but also a significant mass to the dynamic system. Consequently the natural frequencies did not change very much and the footbridge remained very lively. Sometimes, perceptible vibrations are excited by just a single pedestrian crossing over the bridge. The vibrations are perceptible to the person generating them and also to other pedestrians, either stationary or walking, who are present on the footbridge deck. At first sight this is somewhat surprising considering how massive the 260 tonne structure is (Fig. 1 and Fig. 2).

This liveliness, together with the fact that information about vibration behaviour of the described structural layout can seldom be found in the published literature, makes the Podgorica footbridge interesting for investigation of its as-built dynamic characteristics.

4 Pre-Test Finite Element Modelling

A good practice for modal testing of an as-built structure requires a development of reasonably detailed FE model before the testing [1]. This first insight into dynamic behaviour of the footbridge helps the test planning and preparation.

A 3D FE model for the Podgorica footbridge was developed (Fig. 3) using the ANSYS FE code [19]. The aim was to construct a detailed model which would be able to simulate the dynamic behaviour of the structure as well as possible. This was based on the limited technical data available and best engineering judgement. The key modelling assumptions were as follows:

- The main steel box girder and its longitudinal and transverse stiffeners and box section columns were modelled using orthotropic SHELL63 elements assuming isotropic properties. These elements are capable of transferring both in-plane and out-of-plane loads.

- The composite steel–concrete slabs (Fig. 2) were modelled using an equivalent steel thickness and, again, SHELL63 elements with isotropic property.

- The handrails were modelled using 3D BEAM4 elements.

- Water and drainage pipes were modelled as distributed mass along the lines connecting points at which the pipes were suspended from the bridge deck. The mass was calculated by assuming that water filled a half of the pipes' volume.

- Inclined column supports were modelled as fully fixed considering solid rock foundations.

- Supports at both ends of the main girder were modelled as pinned, but with a possibility to slide freely in the longitudinal direction i.e. along the bridge longitudinal axis (Fig. 3).
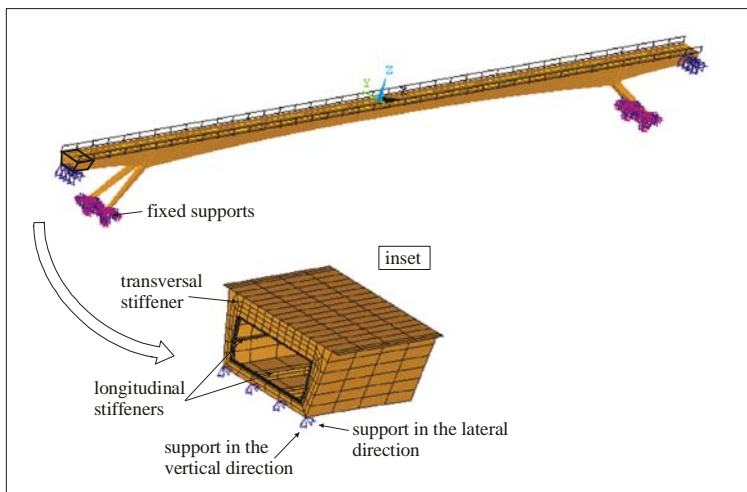

Figure 3: Pre-test FE model.

The seven lowest modes of vibration calculated using this model are shown in Fig. 4. Labels H and V stand for the horizontal and vertical modes, respectively. Similarly, S and A stand for the symmetric and anti-symmetric modes, respectively.
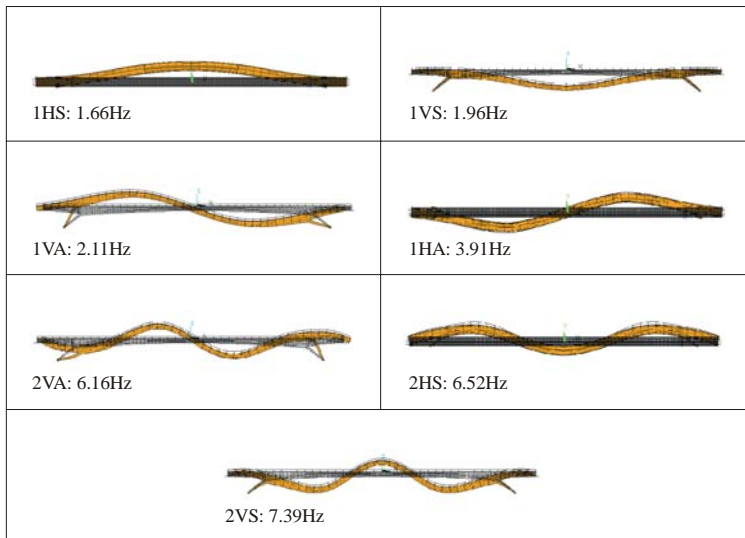
Figure 4: The first seven modes of vibration obtained from the pre-test FE model.

5 Testing

The primary aim of the modal testing was to identify the lowest modes of vibration in both the vertical and horizontal lateral directions. For this purpose an FRF-based modal testing procedure [1] was employed. The second aim was to conduct an ambient vibration survey and compare results with the FRF-based modal testing, to check their consistency. Both types of test required the footbridge to be empty, which means closed and without any pedestrians present during measurements. Due to the importance of this footbridge located in the city centre, the only time when the tests could be conducted was during the night when pedestrian traffic was reduced, although still present. The tests were scheduled to last up to five hours during two nights in October 2004, starting at midnight.

5.1 FRF-Based Modal Testing

Based on mode shapes identified in the FE model (Fig. 4) and the fact that only two accelerometers were available for measurements, it was decided to conduct testing at 14 measurement points, as shown in Fig. 5. This was done to avoid problems with spatial aliasing of mode shapes [1] and enable the identification of the lowest few modes of vibration presented in Fig. 4. The test was first conducted to identify the vertical modes and then repeated for the horizontal lateral modes.

The excitation source for the FRF-based testing was an APS model 113 electrodynamic shaker. The dynamic force induced by the shaker was measured by an accelerometer attached to its armature. The same type of transducer was used for the structural response measurements. Both transducers used were Endevco 7754-1000 piezoelectric accelerometers, having nominal sensitivity 1000 mV/g. They are suitable for low frequency measurements down to less than 1 Hz. The shaker operating in the vertical direction is shown in Fig. 6 and the set-up used for it in the horizontal direction is shown in Fig. 7.
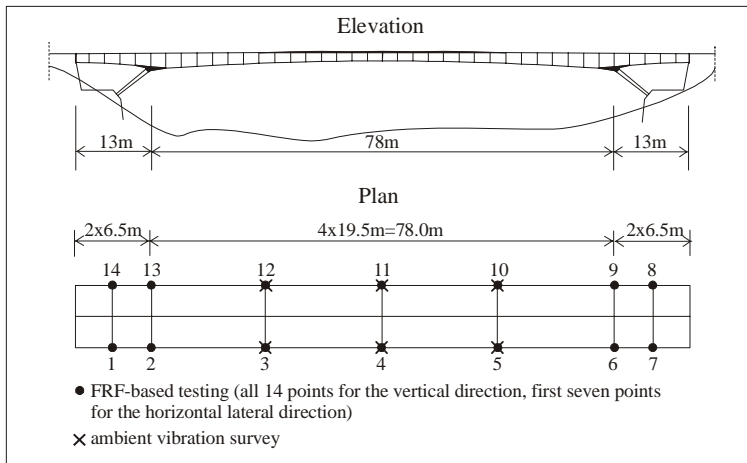
Figure 5: Measurement grid for modal testing. Plan drawn not to scale.

Figure 6: Electrodynamic shaker operating in the vertical mode.

Figure 7: Electrodynamic shaker operating in the horizontal mode.

Test point 3 (TP3), at the quarter of the main span (Fig. 5), was chosen as an excitation point for both directions. For the vertical measurements the vertical response was measured at all 14 points while for the horizontal lateral modes only the first seven points were used to measure the horizontal response.

A dual-channel Data Physics SignalCalc ACE dynamic signal analyser was used to acquire the time domain acceleration data and process them leading to a set of FRFs between the excitation and response points. Based on some very limited previous measurements [20], a damping ratio ζ_1 as low as 0.3% was expected in the vertical fundamental mode of vibration having frequency f_1 of 2.0 Hz. A suggested frequency resolution Δf for data acquisition is therefore approximately [21]:

$$\Delta f = \frac{2\zeta_1 f_1}{6} = \frac{2 \cdot 0.003 \cdot 2.0}{6} = 0.002 Hz. \tag{1}$$

This, in turn, required time-domain data blocks lasting at least 500 s, as calculated in Eq. (2):

$$T = \frac{1}{\Delta f} = 500\, s. \tag{2}$$

With this in mind, and considering the limited time available for keeping the bridge closed during the testing, a compromise had to be made. A 327.68 s data block was chosen using available options in the signal analyser provided that the FRFs were analysed immediately during testing to ensure that the experimental FRF data can lead to a good and believable curve fit. The next step was to decide about the type of force signal capable of exciting the 260 tonne bridge so that its response is measurable in all relevant modes of vibration. A typical time history of a chirp excitation signal at TP3 together with the response measurement at the same point, as well as their spectra are shown in Fig. 8. The resulting point accelerances [1] obtained firstly with three averages and then with no average (i.e. using just one block of data) are shown in Fig. 9. Quite surprisingly the FRF data quality was almost the same. Because it was difficult to close the bridge even for 10 min at a time, an FRF estimation without averaging was adopted as it approximately halved the time required for testing. Also, this choice was supported by the good quality of the FRF that was not averaged as a consequence of high signal to noise ratio. This ratio was high owing to both the good excitation technique and quiet night with almost no wind. Also, the fact that some modes of vibration had very low damping ratio and therefore were easily excitable, contributed to obtaining high quality FRF data without any averaging. This feature is not often seen when testing full-scale civil engineering structures residing in open-space environments. These are naturally quite noisy and therefore create a considerable level of extraneous unmeasured excitation of the test structure. If uncorrelated with the measured excitation the only way to remove the effects of this excitation is by averaging. However, and quite interestingly, on this particular occasion this was not necessary.

The adopted data acquisition parameters are shown in Table 1. However, having in mind that the modal mass is the least reliable parameter in a parameter estimation process, and that it is very sensitive to response magnitude, it was decided to conduct a single point accelerance measurement due to random excitation with 10 averages, each lasting 327.68 s, just to estimate the modal mass. Using a Hanning window with 75% overlapping, the whole exercise took about 15 min. This measurement was used to estimate the modal mass of the second mode of vibration (i.e. the first vertical mode denoted as 1VS in Fig. 4), which dominated the footbridge response under normal pedestrian traffic [20].

For modal testing in the horizontal lateral direction, the shaker as well as the response accelerometer were rotated by 90 (Fig. 7). All other measurement parameters remained unchanged.

The test in the vertical direction was completed during the first night. The whole exercise, together with (un)packing the equipment and trials took about 5 h, as planned. Although the time required for the data acquisition was less than 6 min per point, the setting up time between two consecutive measurements was about 10 min. This long time was primarily required to allow occasional pedestrians to cross the footbridge. Having in mind the low damping of the structure, the time needed for it to damp out vibrations after these crossings was very long. Sometimes this was as long as 2 min, depending on the level of vibration induced by pedestrians. The testing in the horizontal lateral direction was conducted during the second night together with ambient measurements.
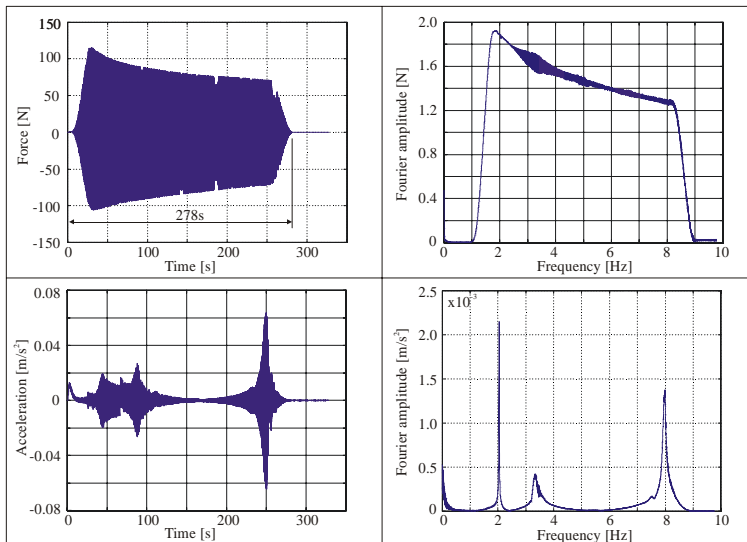
Figure 8: Typical chirp excitation and corresponding acceleration response signals @TP3 in the time and frequency domains.
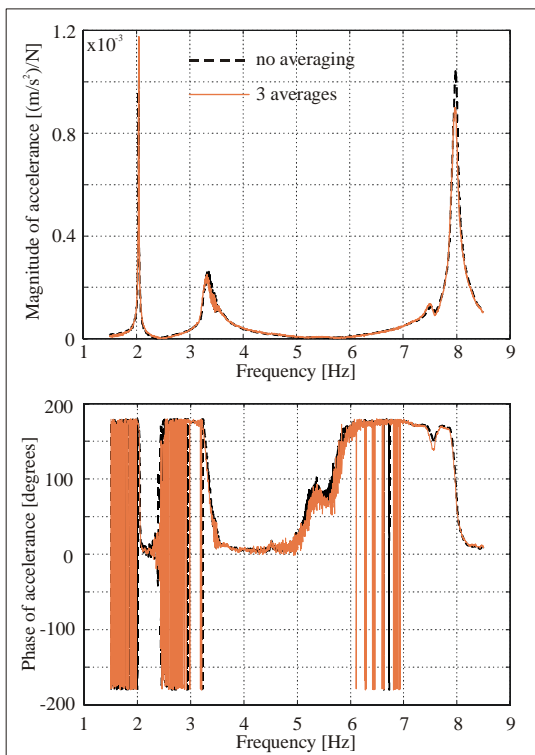

Figure 9: Comparison between point accelerance @TP3 with one (black-dashed line) and three averages (grey line) under the chirp excitation.

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Table 1: Data acquisition parameters used for FRF-based modal testing.

Parameter	Value
Acquisition time [s]	327.68
Frequency resolution [Hz]	0.00305
Time step [s]	0.04
Sampling frequency [Hz]	25
Excitation type	linear chirp
Frequency range of excitation [Hz]	1-9
Window type	rectangular
Number of FRF averages	1
Excitation duration [s]	278

5.2 Ambient Vibration Survey

The ambient vibration survey was conducted after the FRF-based testing. The shaker was removed and only vertical response measurements due to ambient excitation on the empty bridge were made. The aim was to verify the results obtained by FRF-based testing, having in mind its unusual data acquisition approach (i.e. without averaging). TP3 was again chosen as the location for the reference accelerometer while the traveller accelerometer was placed only at points 4, 5, 10, 11 and 12 (Fig. 5). This reduced measurement grid (in comparison with the FRF-based testing) was adopted due to the short testing time available and difficulties in footbridge closures lasting 10 min at a time, as needed for AVS. For this reason the AVS work was focused on the vertical direction only as it was more critical for vibration serviceability.

The data acquisition parameters adopted in these tests are given in Table 2 while typical measured time history at the reference point is shown in Fig. 10.

The complete vertical AVS together with FRF-based testing in the horizontal direction lasted less than 4.5 hours during the second night of testing. For reference, it should be said that tests during both nights were conducted under very pleasant weather with average temperature being around 18°C.

Table 2: Data acquisition parameters used for ambient vibration survey in the vertical direction.

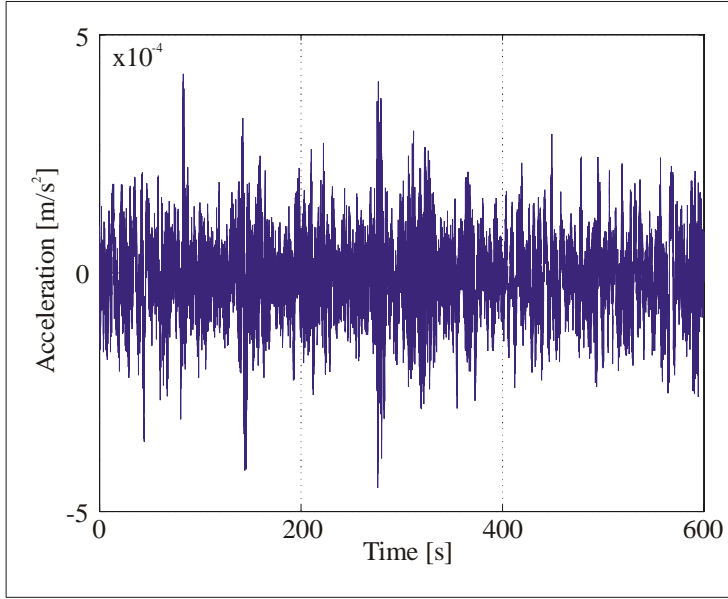| Parameter	Value
Acquisition time [s]	600
Time step [s]	0.04
Sampling frequency [Hz]	25

Figure 10: Typical acceleration signal @TP3 during AVS.

6 Vibration Parameter Estimation

Modal parameter estimation was performed on site for both FRF-based modal testing and AVS.

6.1 FRF-Based Estimates

The FRF data collected were analysed using an MDOF parameter estimation procedure available in the ICATS software [22] under the usual assumption of linear behaviour of the footbridge for the vibration levels measured. In particular, the Non-Linear Least-Squares (NLLS1-M) method [23] was applied. This method produced slightly better fits (judging them visually) than the Rational Fraction Polynomial method [24], called GRF-M in ICATS and the Global Method [25], called GLOBAL-M. The method used is also considered as one of the most accurate single-input multiple-output FRF fitting methods [21, 23]. However, it should be said that the vibration parameter estimates (natural frequencies, mode shapes and modal damping ratios) were similar across the different estimation methods, with the only exception of the damping ratio for mode 1VA which varied most. This is because this mode was quite damped (in comparison with the others) and therefore was difficult to excite.

Basically, the NLLS1-M method fits every receptance function $\alpha(\omega)$ by summing N modes as follows [21, 23]:

$$\alpha(\omega) = \alpha_0 + \left[\sum_{r=1}^{N} \frac{A_r + i\omega B_r}{\omega_r^2 - \omega^2 + i2\omega\omega_r\zeta_r} \right] e^{j\phi} \qquad (3)$$

with the aim to minimise an error function er, given in the form:

$$er = \sum_{i=1}^{L} \left(\hat{\alpha}_i^* - \alpha_i^* \right)\left(\hat{\alpha}_i - \alpha_i \right), \qquad (4)$$

over L measured data points, each corresponding to a different discrete (forcing) frequency ω within the frequency range of interest. In equations (3) and (4), α_i and $\hat{\alpha}_i$ represent receptance as mathematically modelled using equation (3) and measured, respectively, while α_i^* and $\hat{\alpha}_i^*$ are their complex conjugates. Moreover, ω_r and ζ_r are the natural frequency and damping ratio for mode r,

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respectively, while A_r and B_r are constants containing information about the modal mass and mode shape amplitudes related to mode r. Furthermore, α_0 and ϕ are introduced to compensate for measurement errors. α_0 is a complex (constant) number which covers errors due to translation of the true origin for presenting $\alpha(\omega)$ in the Nyquist plane relative to the measurement origin, while angle ϕ compensates for the rotational shifts in the same Nyquist plane. One example of the Nyquist plane fitting of an isolated mode of vibration in the horizontal direction is given in Fig.11.
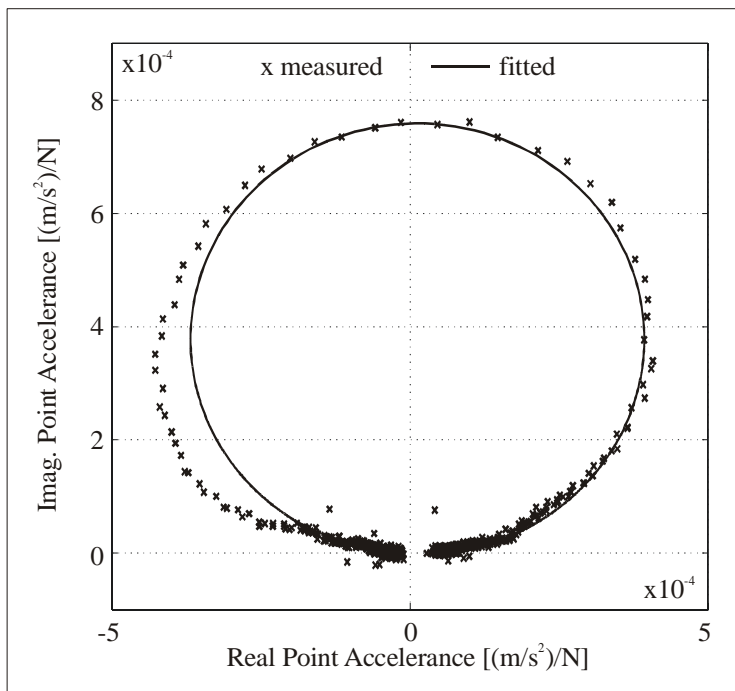

Figure 11: Comparison between fitted model and measured data for point accelerance in the horizontal direction in the form of a Nyquist plot. Frequency range [3.5-4.5Hz] relevant for 1HA mode is only presented.

By curve fitting modal frequencies ω_r, damping ratios ζ_r and mode shapes were identified. The seven modes identified are shown in Table 3 and Figure 12 (black dashed line).

Table 3: Modal frequencies and damping ratios according to different models.

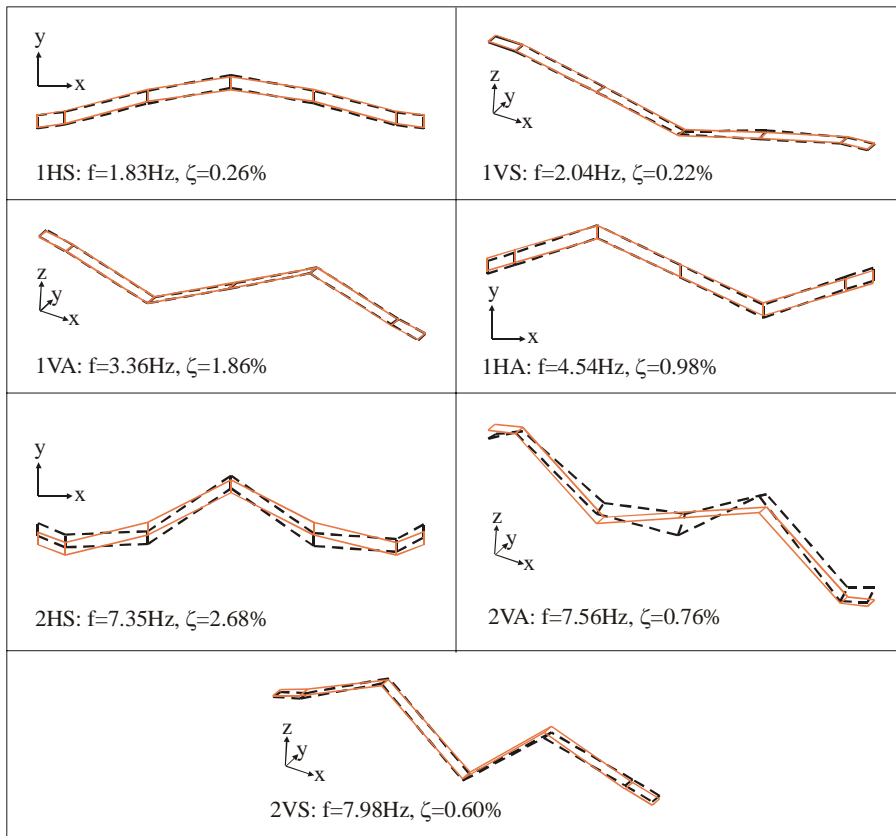| mode	pre-test FEM	FRF based		AVS		model_2	final FEM
#	f [Hz]	f [Hz]	ζ [%]	f [Hz]	ζ [%]	f [Hz]	f [Hz]
1	1.66 (1HS)	1.83 (1HS)	0.26	-	-	1.82 (1HS)	1.82(1HS)
2	1.96 (1VS)	2.04 (1VS)	0.22	2.05 (1VS)	0.29	2.02 (1VS)	2.02 (1VS)
3	2.11 (1VA)	3.36 (1VA)	1.86	3.42 (1VA)	1.04	2.36 (2VA)	3.47 (1VA)
4	3.91 (1HA)	4.54 (1HA)	0.98	-	-	4.35 (1HA)	4.36 (1HA)
5	6.16 (2VA)	7.35 (2HS)	2.68	-	-	6.52 (2VA)	7.15 (2HS)
6	6.52 (2HS)	7.56 (2VA)	0.76	7.55 (2VA)	0.76	7.13 (2HS)	7.34 (2VA)
7	7.39 (2VS)	7.98 (2VS)	0.60	8.00 (2VS)	0.44	7.56 (2VS)	7.74 (2VS)

Figure 12: Modes identified in the FRF-based testing (black dashed line). Mode shapes obtained from tuned FE model are presented in grey (see Section 7).

A comparison between the measured and synthesised (via the experimentally estimated modal properties) point accelerance for the vertical direction at TP3 is shown in Fig. 13. It suggests that the estimated modal properties represented the measured structural behaviour very well indeed. The mismatching of the measured and curve-fitted FRF phase around 5–6 Hz happened due to the presence of a weak mode which was not identified. This is a common problem which clearly does not affect the accuracy of the curve-fitting around strongly excited modes. The same applies to the obtained quality of analytical model representing horizontal modes which is not elaborated here further.
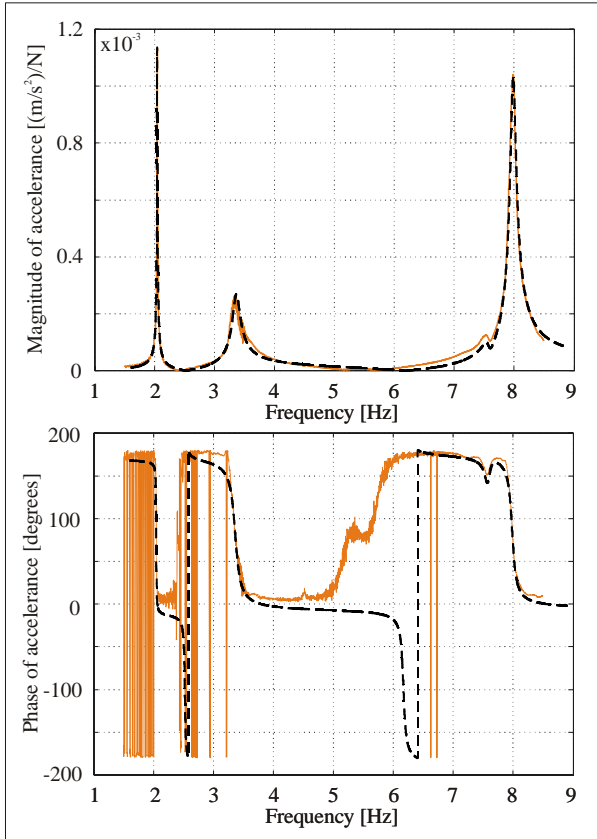
Figure 13: Comparison between the point accelerance for the vertical direction as measured (grey line) and as modelled (black dashed line).

At the end of parameter estimation process, the modal mass of the first mode of vibration in the vertical direction was calculated. This was done based on the magnitude of acceleration response measured under random excitation. The point accelerance is shown in Fig. 14 (black dashed line), with the magnitude of 0.001036 m/s^2 under 1 N unit harmonic force. This measurement was conducted at TP3 which is an important detail considering that modal mass is usually defined with regard to the modal amplitude corresponding to an antinode (i.e. maximum amplitude in absolute sense) of a particular mode. In this case the antinode of the 1VS mode was at TP4. Therefore, the ratio between amplitudes at TP3 and TP4, obtained from experimental mode shapes shown in Fig. 12, was used as a scaling factor for both the force and response magnitude at TP4. In this way the modal mass m_{1VS} can be calculated from the formula for steady state response under sine resonant excitation [26]:

$$m_{1VS} = \frac{F_4}{2\zeta_{1VS}a_4} = \frac{\dfrac{\phi_{3,1VS}}{\phi_{4,1VS}} F_3}{2\zeta_{1VS}\left(\dfrac{\phi_{4,1VS}}{\phi_{3,1VS}} a_3\right)} = \frac{\dfrac{0.002492}{0.005061}\cdot 1}{2\cdot 0.0022\cdot\left(\dfrac{0.005061}{0.002492}\cdot 0.001036\right)} = 53,188\,\text{kg}. \qquad (5)$$

Here, $\phi_{3,1VS}$ and $\phi_{4,1VS}$ are mode shape ordinates for mode 1VS at TP3 and 4, respectively, as obtained from curve-fitting, F_3 and F_4 are resonant sine force amplitudes at points 3 and 4, while a_3 and a_4 are the corresponding measured acceleration amplitudes. ζ_{1VS} is the estimated damping ratio for mode 1VS.

If the modal mass is estimated in the same way, but using the point acceleration FRF obtained via the unaveraged chirp excitation (grey line in Fig. 14), then a mass of 48,763 kg is obtained. This indicates the high level of sensitivity of the estimate of modal mass.
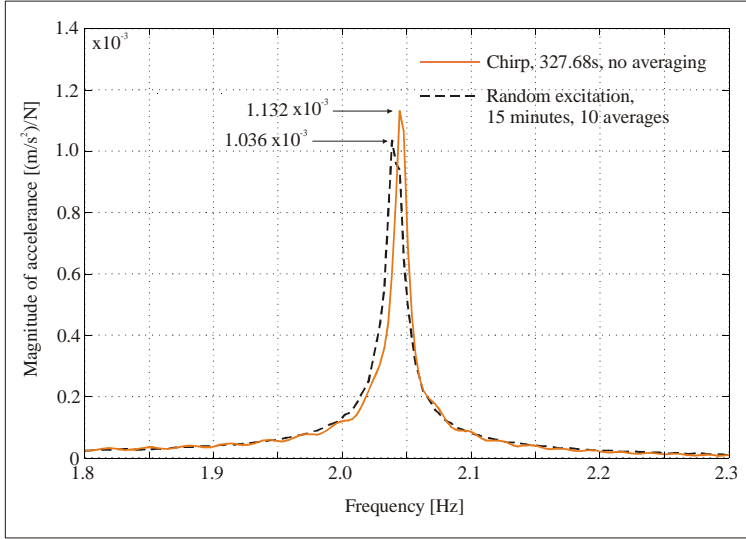

Figure 14: Magnitude of accelerance in the first vertical mode of vibration due to two different types of excitation.

6.2 AVS-Based Estimates

For parameter estimation using the AVS data, the ARTeMIS software [27] was used. The Canonical Variate Analysis (CVA) identification method [28], which is a time-domain parameter estimation technique, was chosen. This is one of the so called 'data-driven' stochastic subspace identification methods. This means that the method fits the raw measured time-domain data directly, instead of fitting the covariances [28] produced using data from different test points. The starting point in the method is to present the dynamics of a structure under unknown environmental excitation in the discrete-time state-space form, assuming structural linearity and time-invariability, as follows [28]:

$$\{x_{k+1}\}_{n\times 1} = [A]_{n\times n}\{x_k\}_{n\times 1} + \{w_k\}_{n\times 1} \tag{6}$$

$$\{y_k\}_{p\times 1} = [C]_{p\times n}\{x_k\}_{n\times 1} + \{v_k\}_{p\times 1} \tag{7}$$

where Eq. (6) is typically called the state equation while (7) is called the observation/output equation. The state equation is sufficient to describe dynamics of the system (instead of using the well-known equation of motion) with $n/2$ degrees of freedom, where n is called the model order. Here, $\{x_k\} = \{x(k\Delta t)\}$ is the discrete time state vector containing $n/2$ displacements and $n/2$ velocities describing the *state* of the system at time instant $t_k = k\Delta t$, $\{x_{k+1}\}$ is the same vector defined at time $t_{k+1} = (k+1)\Delta t$, $[A]$ is the discrete state matrix dependent on the mass, stiffness and damping properties of the structure, while $\{w_k\}$ represents the inevitable but unmeasured noise input as well as the noise present due to modelling inaccuracies at time t_k. The observation equation calculates the response of the structure $\{y_k\}$ at p measurement locations at time t_k via the state vector $\{x_k\}$. Here, $[C]$ is the discrete output matrix which main purpose is to map the state vector into the

measured output, while $\{v_k\}$ represents the noise due to measurements and unmeasured noise input at time t_k.

It can be shown that modal properties (natural frequencies, mode shapes and damping ratios) of a structure under white-noise excitation can be identified by relying only on the measured output responses $\{y_k\}$ [28]. However, models of different order will typically produce more or less different estimates. This is the reason to consider several models of different order and to choose the one with the lowest level of error obtained when comparing the measured and analytically predicted outputs. This is how the optimum models were selected for data sets at all five test points while analysing the Podgorica footbridge ambient responses. An example of this process is shown in Fig. 15 which shows that there are four stable modes identified in all five measurement sets corresponding to the five test points (connected by lines in Fig. 15). These identified modes are listed in Table 3. The identified natural frequencies compared well with those from the FRF-based modal testing procedure. Regarding damping ratios, the largest difference was for mode 1VA. However, the reliability of this damping ratio as identified by AVS is not very high as the scattering across the data sets, expressed as the standard deviation, is as high as 0.57% in comparison with the average (i.e. identified) damping ratio of 1.04% (Fig. 15Figure 15). As mentioned earlier the estimation of the damping ratio for this mode was least reliable in the FRF based-testing too. The agreement between the mode shapes in the two methods was good as checked by visual inspection. The signal processing parameters used as input in the ARTeMIS software are given in Table 4.

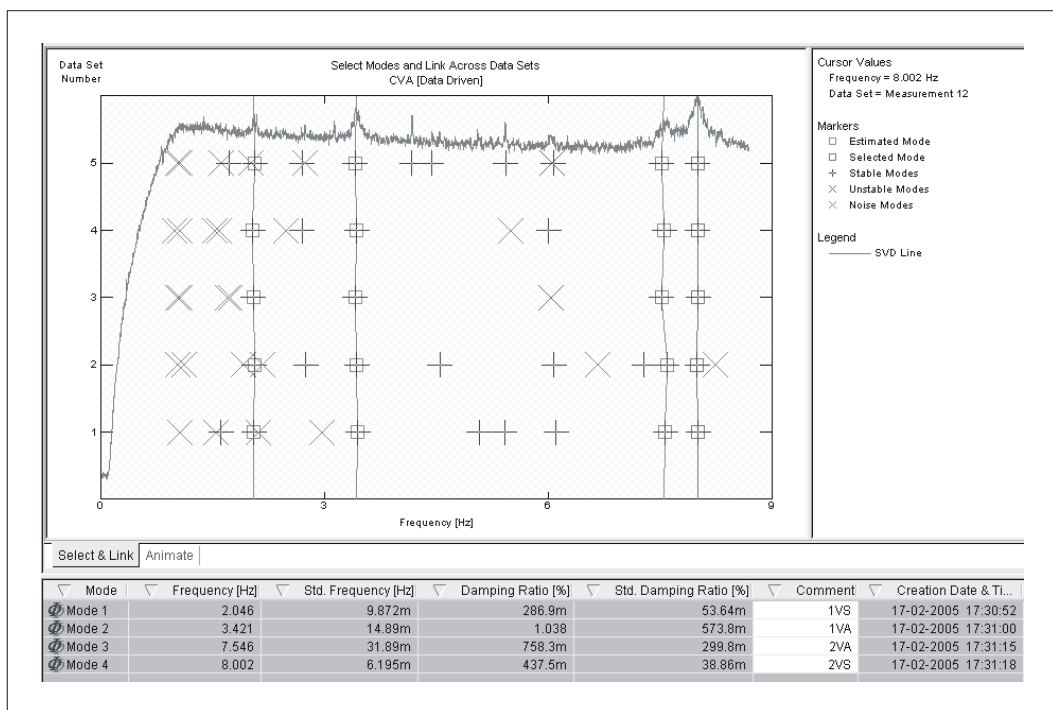| Mode	Frequency [Hz]	Std. Frequency [Hz]	Damping Ratio [%]	Std. Damping Ratio [%]	Comment	Creation Date & Ti...
Mode 1	2.046	9.872m	286.9m	53.64m	1VS	17-02-2005 17:30:52
Mode 2	3.421	14.89m	1.038	573.8m	1VA	17-02-2005 17:31:00
Mode 3	7.546	31.89m	758.3m	299.8m	2VA	17-02-2005 17:31:15
Mode 4	8.002	6.195m	437.5m	38.86m	2VS	17-02-2005 17:31:18

Figure 15: Stable modes identified in AVS. Natural frequencies and damping ratios are listed below the diagram and in Table 3.

Also, it should be said that differences in damping ratios in the two experimental methods may be due to amplitude dependent damping as the level of vibration during the AVS was up to two orders of magnitude lower than during the FRF measurements. However, it was interesting to additionally check the damping value for a lively mode 1VS, since this value is particularly important for vibration serviceability assessment of the footbridge. For this, the vertical acceleration response to a pedestrian crossing the bridge with controlled step frequency at approximately 2.04 Hz (metronome set at 122 beats per minute), was measured at the middle of the bridge. After the pedestrian crossed the bridge, the free decay of the response was also recorded. The measured response (bandpass filtered with

centre frequency of 2.04 Hz) is shown in Fig. 16(a). Using the logarithmic decrement method [1], the damping ratio was estimated for a number of data blocks, each containing eight subsequent cycles of decaying vibration. Each of these blocks corresponds to slightly different amplitude of the first cycle and can be used to estimate damping as a function of this amplitude (Fig. 16(b)). It can be seen that the damping was weakly dependent on the vibration amplitude, but its average value of 0.26% agreed well with those measured during modal testing.

Table 4: Signal processing setup as adopted in data analysis in ARTeMIS.

Parameter	Value
Acquisition time per point [s]	600
Time step [s]	0.04
Filter type	High-pass Butterworth
Filter order	8
Filter cut-off frequency [Hz]	1
Frequency resolution [Hz][*]	0.003052
Window type[*]	Hanning
Overlap [%][*]	66.67
Number of averages[*]	3

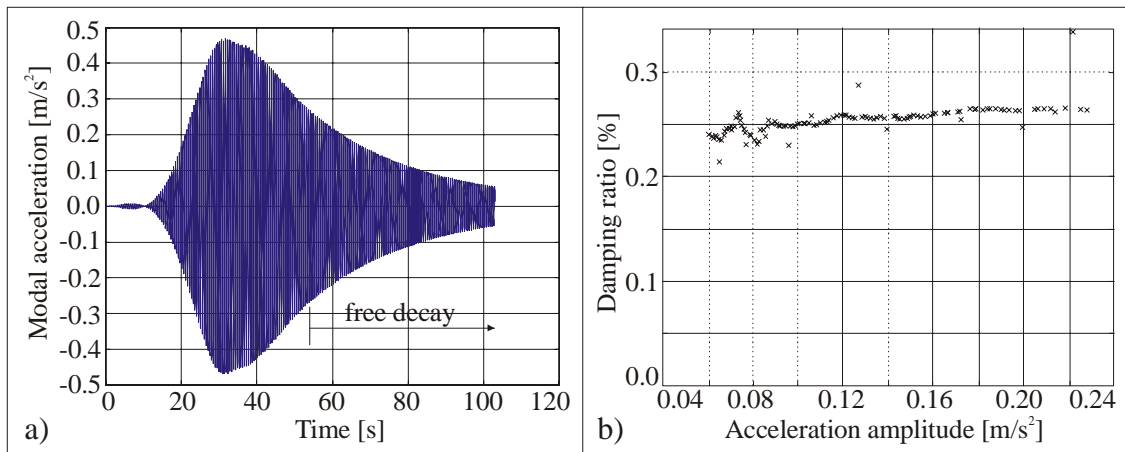[*] parameters used for validation of results from CVA method.

Figure 16: a) Acceleration response due to a pedestrian crossing at step frequency of 2.04Hz. b) Damping estimate from the free decay.

7 FEM Tuning

A comparison between vibration modes obtained experimentally and analytically revealed that all seven mode shapes measured (Fig. 12) had their counterparts in the initial FE model (Fig. 4). However, the sequence of 2VA and 2HS FE modes was reversed compared with their experimental counterparts (Table 3). Also, the FE model underestimated frequencies for all vibration modes, the maximum difference being 37% for mode 1VA. On the other hand, the correlation between mode shapes was good with the modal assurance criterion (MAC) values [1] higher than 0.90, except for modes 2HS (0.82) and 2VA (0.84). This implied that there was a problem with inadequate modelling which underestimated the stiffness consistently throughout the whole footbridge. This was despite the

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fact that the FE model was as detailed as possible using all available design information and based on best engineering judgement.

Some of the differences in the numerical and experimental natural frequencies were significant and somewhat surprising. They prompted the authors to do an additional search for information about the properties of the as-built bridge. It was discovered that during the previously described strengthening of the footbridge, the plates which constitute the box cross section of the inclined columns were also strengthened. These data were missing from the design documentation available for developing the initial FE model. As a consequence, the initial FE model could not simulate the real behaviour of the footbridge accurately. It should be said that this situation with missing technical information is quite typical when retrofitting structures that are relatively old.

The information about the reinforcement of the column cross section was discovered by going beyond what would normally be done in practice to find information about an existing structure. However, apart from the fact that the strengthening took place no precise information was available about the actual thickness of the additional plates. Therefore, it was assumed that they were two times thicker than in the initial FE model (6.0 and 4.0 cm instead of 3.0 and 2.0 cm, respectively). This improved considerably the correlation between the FE model and experimental results, especially for horizontal modes ('Model_2' in Table 3). However, the differences in some of the vertical modes were still quite high. This was the reason to consider the sensitivity of the vertical modes to some other uncertain FE modelling parameters. These were: the thickness of the composite slabs, their dynamic modulus of elasticity, mass of the water pipes with water in them and stiffness of additional elastic supports at both ends of the bridge girder in the longitudinal direction (horizontal springs instead of free edge, see Fig. 17). It was found that the stiffness of the horizontal support springs in the longitudinal direction was the only parameter which increased the frequencies of vertical modes as required. Elastic springs (ANSYS COMBIN14 element), having stiffness of 108 N/m per metre length of the width of the bridge deck, were adopted. The physical explanation of this is the restraint condition caused by the expansion joint between the bridge deck and the walking path used to approach the bridge. For low-level vibration this expansion joint was very much 'stuck' introducing an additional support condition.
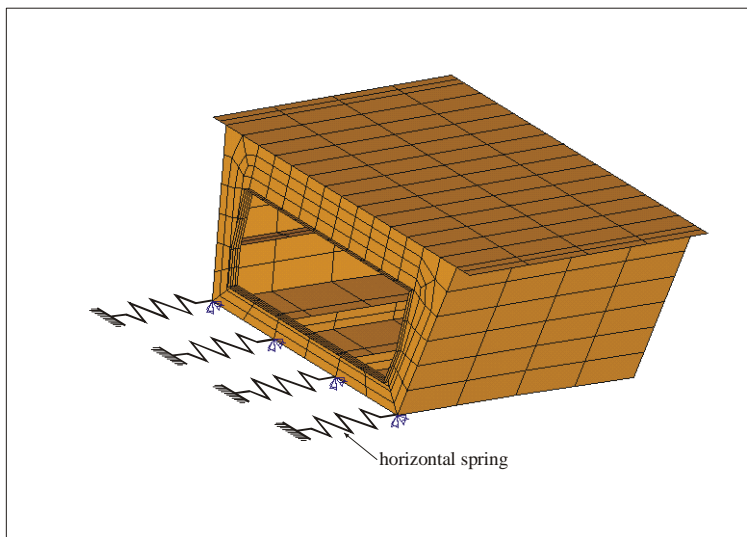

Figure 17: The horizontal springs at the girder ends added to the initial FE model.

In this way the correlation with the experimental data was significantly improved, with the sequence of modes now being the same as obtained experimentally ('Final FEM' in Table 3). The maximum difference in natural frequencies was reduced to a reasonable 4% for mode 1HA. The MAC values increased for all modes except for 2HS, which was now 0.81—still quite acceptable. All FE mode shapes are presented as a grey line in Fig. 12. Clearly, this tuned FE model simulated the footbridge vibration much better than the initial FE model. It was subsequently possible to update automatically this 'manually tuned' FE model, contrary to the initial FE model. The initial FE model had modes that were simply too far away from their measured counterparts for the automatic updating to be

successful. This is another key benefit of the manual tuning, apart from learning about the actual as-built structural behaviour. The results of automatic updating of this FE model are beyond the scope of this paper.

8 Conclusions

An FRF-based modal testing conducted to identify dynamic properties of a lively steel box girder footbridge successfully identified the seven lowest modes of vibration in both the vertical and horizontal–lateral directions. The testing procedure, based on chirp excitation by an electrodynamic shaker, was efficient and sufficiently accurate, even though only two channels were used during the FRF measurements and the traditional signal averaging was not performed. These results compared well with those obtained in an ambient vibration survey conducted in the vertical direction only. It was found that the footbridge had very low damping ratio associated with the lowest horizontal–lateral and vertical modes of vibration of only about 0.26% for both modes. This low damping, together with the natural frequency of 2.04 Hz for the vertical mode, being in the frequency range of normal human walking, contributed to the footbridge liveliness.

A detailed initial FE model, which was developed based on the design data available and best engineering judgement, significantly underestimated some of the measured natural frequencies of the structure. This was somewhat surprising and the experimental results proved to be crucial in identifying drawbacks in the FE modelling. It was found that missing design information about the strengthening of the columns was responsible for the general underestimation of natural frequencies, mainly for the horizontal–lateral modes. Moreover, the stiffness of additional horizontal spring supports (in the direction along the footbridge) at the ends of the girder had a significant influence on the vertical modes of vibration. The effect and presence of this stiffness was not anticipated during the initial FE modelling, which assumed that the expansion joints introduced no stiffness in the horizontal direction. This type of modelling error is hard to avoid due to difficulties in simulating the real boundary conditions. Manual tuning by trial and error of these two modelling parameters improved significantly the FE model and its correlation with the experimental model, reducing the maximum difference in the natural frequencies from 37% to only 4%. Without this tuning it was impossible to perform automatic updating due to large differences between the measured and analytically predicted natural frequencies using the initial FE model. After the tuning, automatic updating was successfully carried out, but its outcomes are beyond the scope of this paper. In conclusion, if required after construction, FE models used in design should be critically evaluated, parametrically studied and, if possible, manually updated based on experimental data.

Acknowledgements

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