System Identification

from data to model

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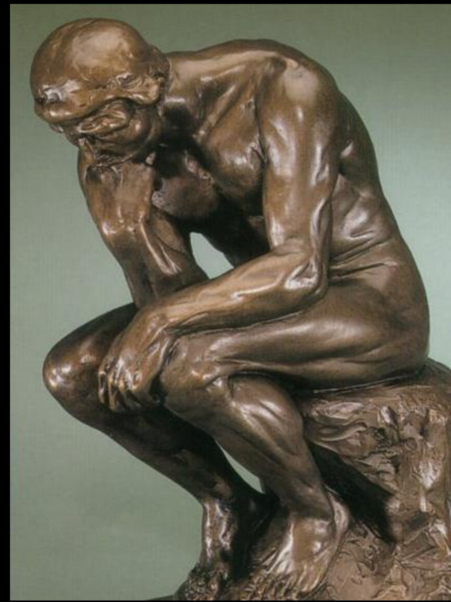

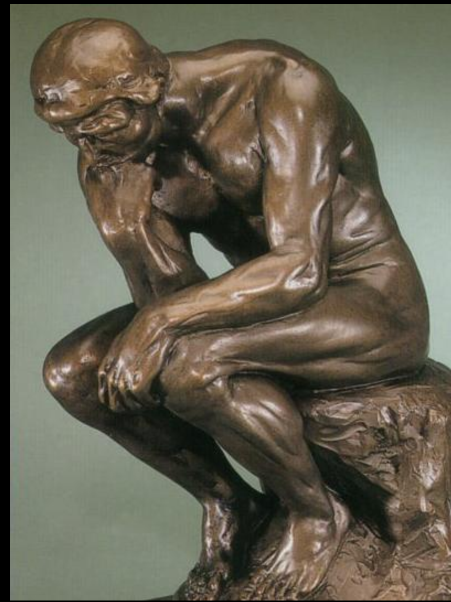




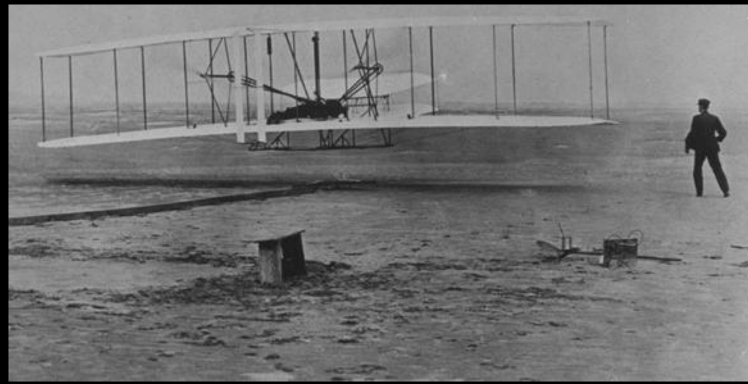


A380
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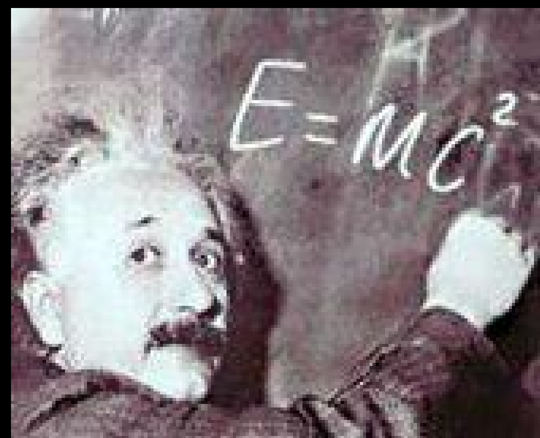
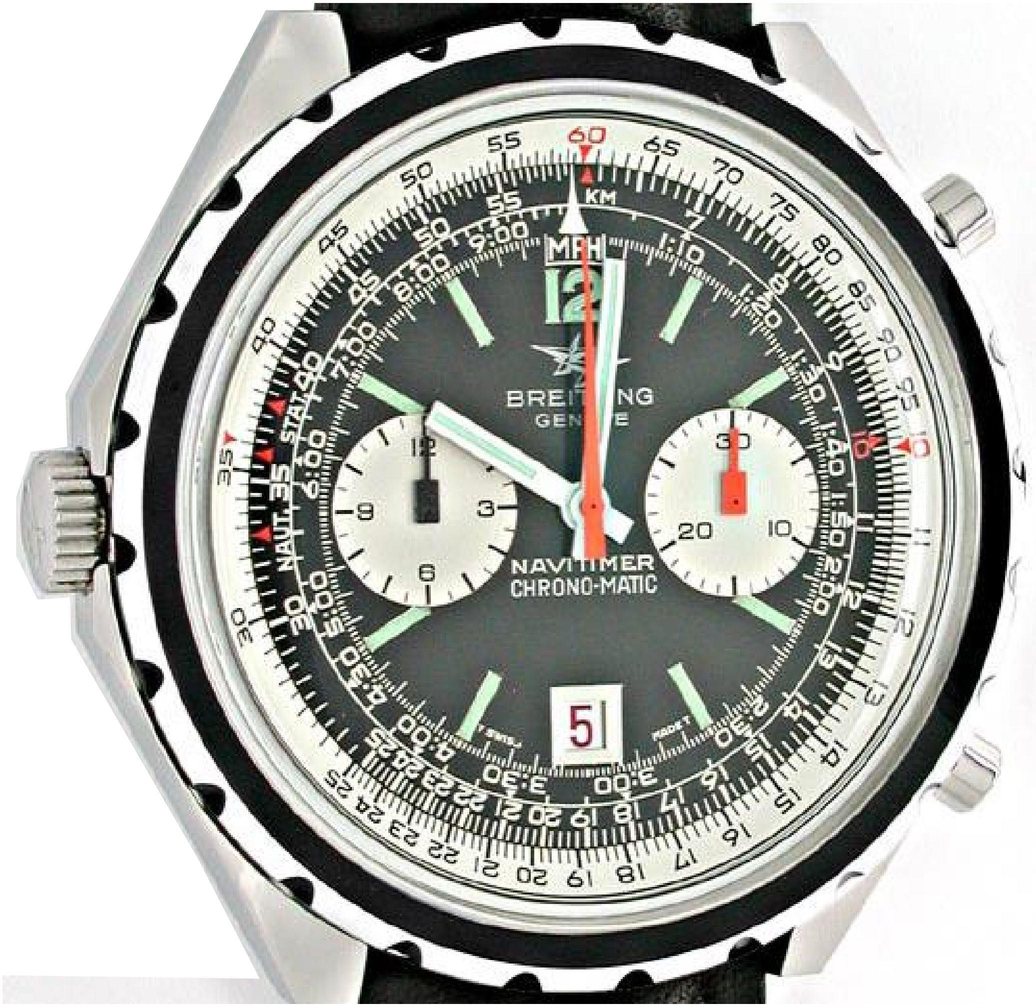

E=MC²

System identification

Building models from experimental data

System identification

a simple example



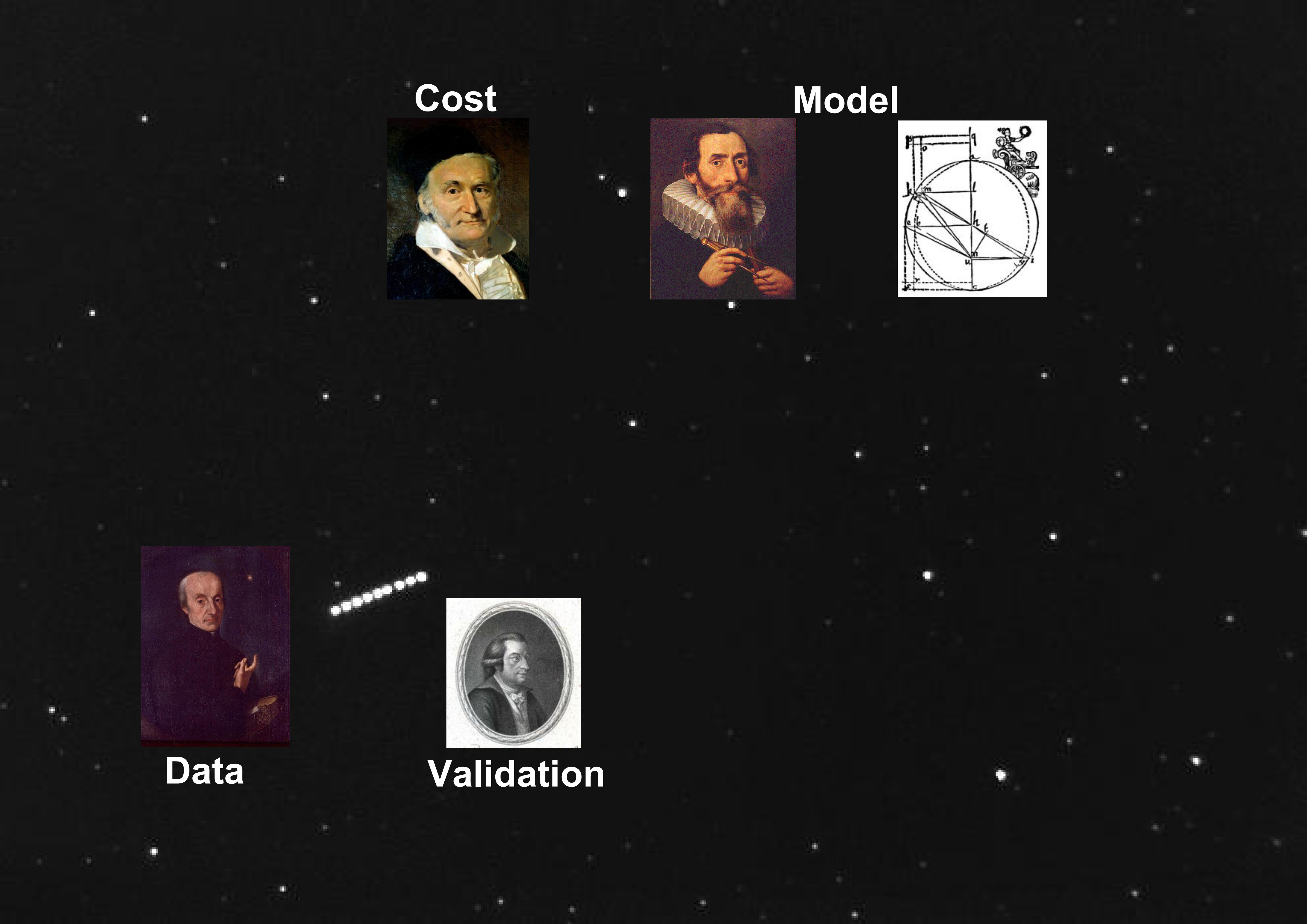
The story of Ceres


Piazzi

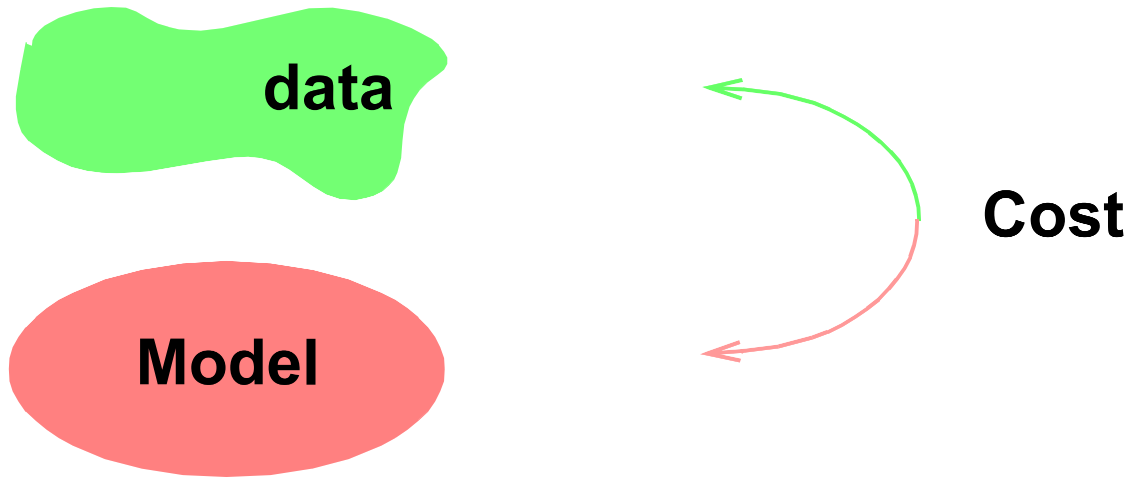

Gauss
Piazzi


Gauss
Keppler
Piazzi


Gauss
Keppler
Piazzi
Von Zach


Cost
Model
Data
Validation

System identification



Outline

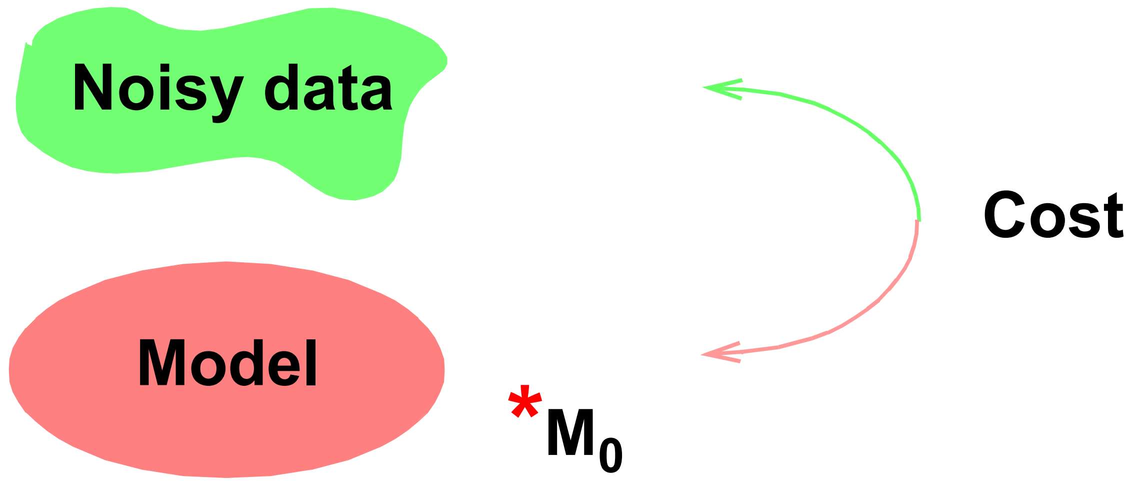
- Introduction

- Intuitive solutions?

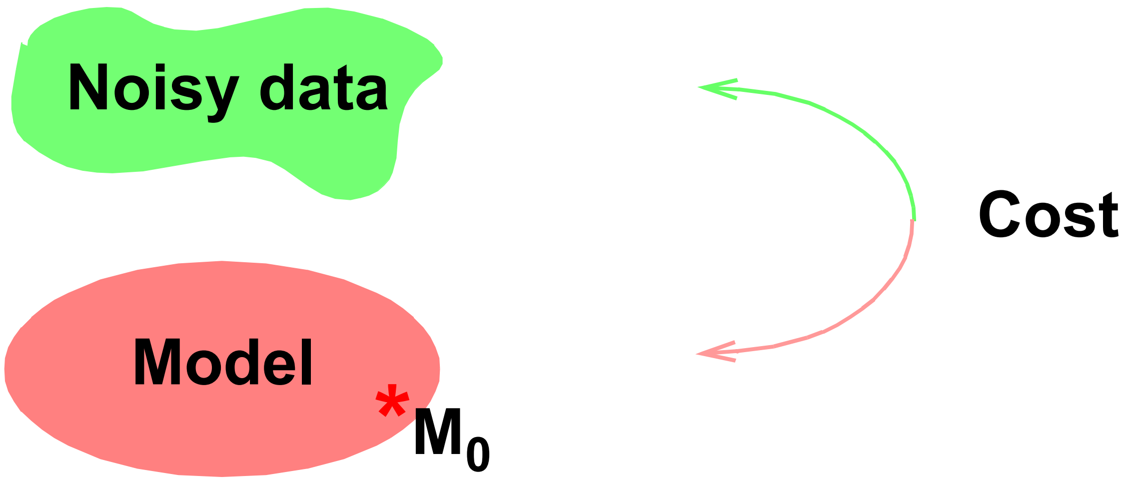
- Linear system identification

- Impact nonlinear distortions on the linear framework

System identification



System identification: noise errors dominate



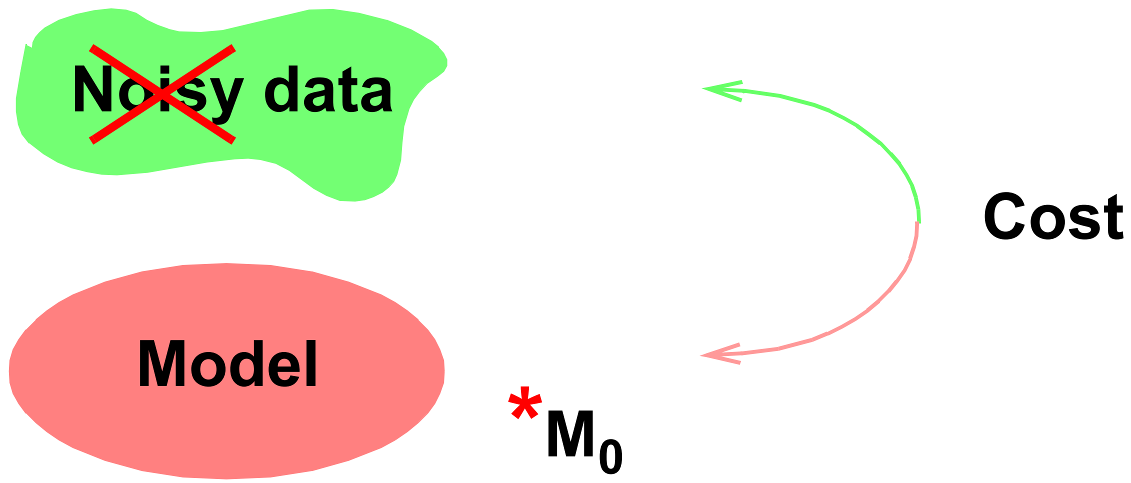
- noise errors dominate

- Cost set by noise properties --> Maximum Likelihood framework

- model and excitation only weakly coupled

Example: identification of linear systems

System identification



- model errors dominate

- model and excitation closely linked

- design of the experiment!

Example: identification of a linear model in the presence of NL distortions

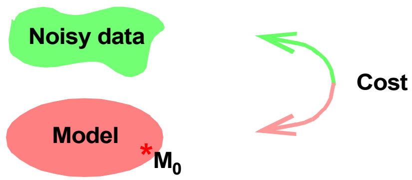
Example 1
Noise errors dominate

Data

$$\begin{aligned} \text{Voltage} \quad u(k) &= u_0(k) + n_u(k) \\ \text{Current} \quad i(k) &= i_0(k) + n_i(k) \end{aligned}, \; k = 1, \ldots, N$$

Model

$$u_0(k) = R_0 i_0(k)$$



3 different estimators

$$\hat{R}_{\mathrm{SA}}(N) = \frac{1}{N}\sum_{k=1}^{N}\frac{u(k)}{i(k)}$$

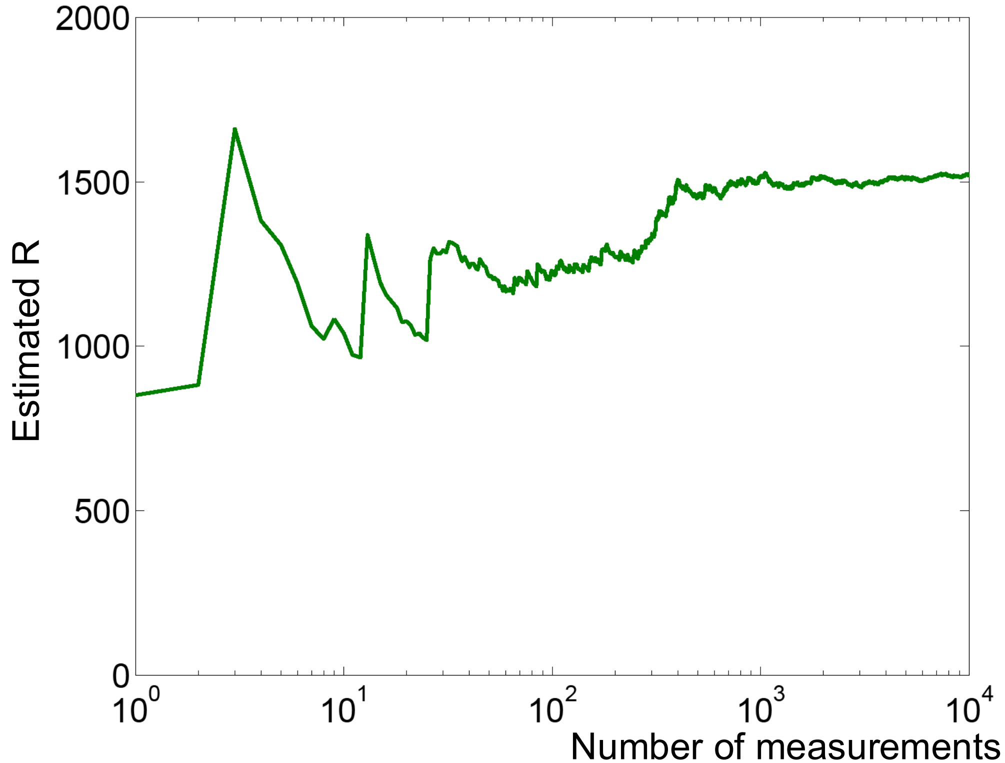
$$\hat{R}_{\mathrm{LS}}(N) = \frac{\frac{1}{N}\sum_{k=1}^{N}u(k)i(k)}{\frac{1}{N}\sum_{k=1}^{N}i(k)^2}$$

$$\hat{R}_{\mathrm{EV}}(N) = \frac{\frac{1}{N}\sum_{k=1}^{N}u(k)}{\frac{1}{N}\sum_{k=1}^{N}i(k)}$$

Example 1
Noise errors dominate

$$\hat{R}(k) = \frac{u(k)}{i(k)}$$

mean value



Example 1
Noise errors dominate

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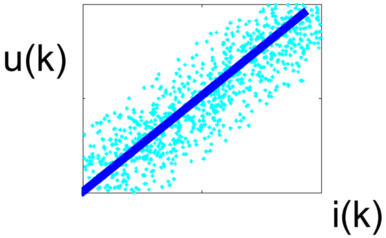
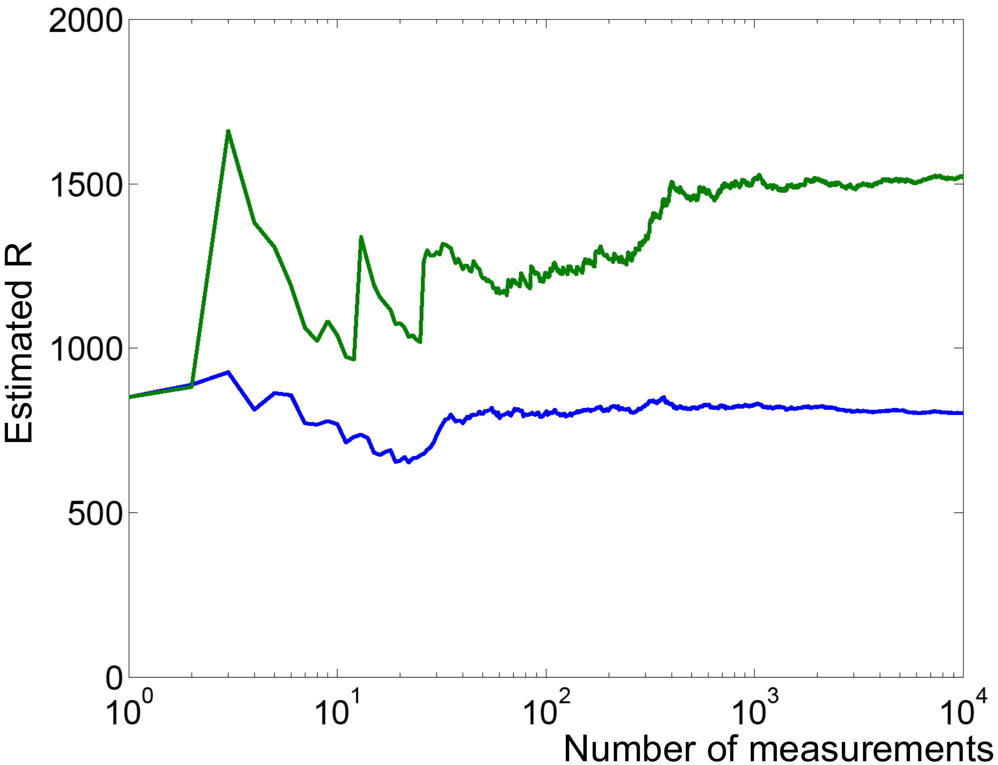
mean value



linear regression



Example 1
Noise errors dominate

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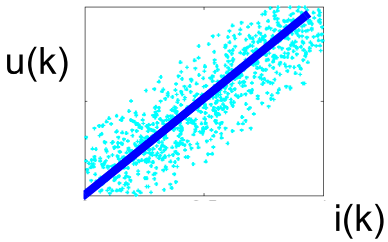
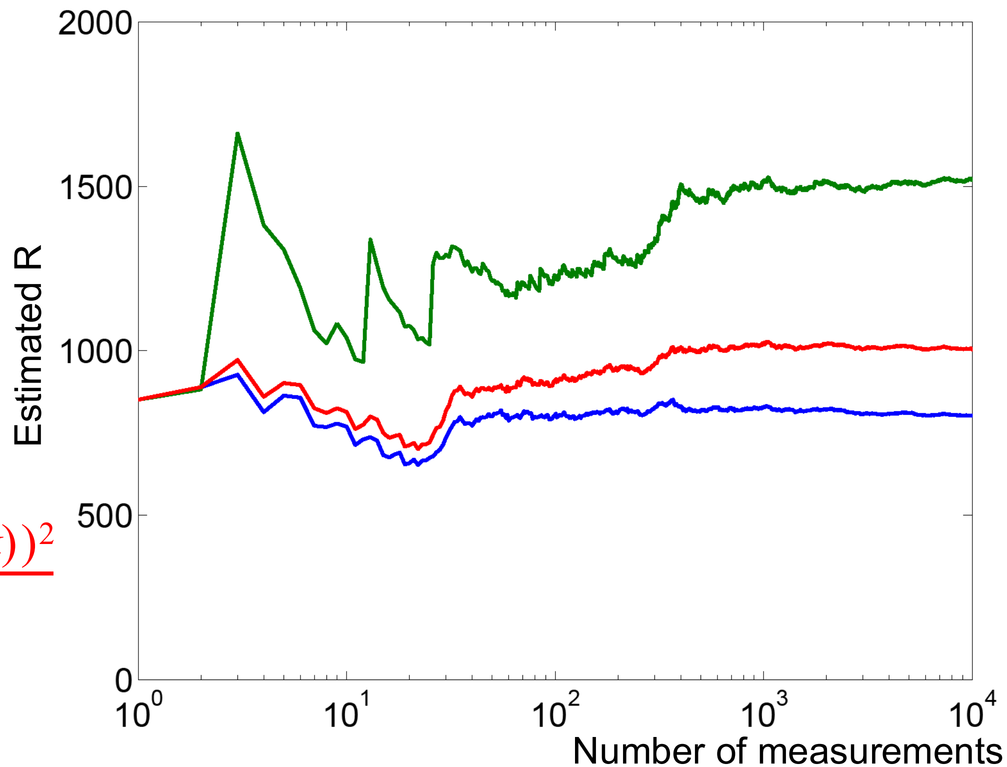
mean value



linear regression

$$V_{EIV} = \sum \frac{(u(k) - u_{\mathrm{p}}(k))^2}{\sigma_u^2} + \sum \frac{(i(k) - i_{\mathrm{p}}(k))^2}{\sigma_y^2}$$



3 different estimators

$$\hat{R}_{\mathrm{SA}}(N) = \frac{1}{N}\sum_{k=1}^{N}\frac{u(k)}{i(k)}$$

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$$\hat{R}_{\mathrm{EV}}(N) = \frac{\frac{1}{N}\sum_{k=1}^{N}u(k)}{\frac{1}{N}\sum_{k=1}^{N}i(k)}$$

Asymptotic value of $\hat{R}_{\mathrm{LS}}$

$$\lim_{N\to\infty}\hat{R}_{\mathrm{LS}}(N) = \lim_{N\to\infty}\left(\sum_{k=1}^{N} u(k)i(k)\right)\Big/\left(\sum_{k=1}^{N} i^2(k)\right)$$

$$= \lim_{N\to\infty}\frac{\frac{1}{N}\sum_{k=1}^{N}(u_0+n_u(k))(i_0+n_i(k))}{\frac{1}{N}\sum_{k=1}^{N}(i_0+n_i(k))^2}$$

Or

$$\lim_{N\to\infty}\hat{R}_{\mathrm{LS}}(N) =$$

$$\lim_{N\to\infty}\frac{u_0 i_0+\frac{u_0}{N}\sum_{k=1}^{N}n_i(k)+\frac{i_0}{N}\sum_{k=1}^{N}n_u(k)+\frac{1}{N}\sum_{k=1}^{N}n_u(k)n_i(k)}{i_0^2+\frac{1}{N}\sum_{k=1}^{N}n_i^2(k)+\frac{2i_0}{N}\sum_{k=1}^{N}n_i(k)}$$

And finally

$$\lim_{N\to\infty}\hat{R}_{\mathrm{LS}}(N) = \frac{u_0 i_0}{i_0^2+\sigma_i^2} = R_0\frac{1}{1+\sigma_i^2/i_0^2}$$

It converges to the wrong value!!!

Asymptotic value of $\hat{R}_{\text{EV}}$

$$\lim_{N \to \infty} \hat{R}_{\text{EV}}(N) = \lim_{N \to \infty} \left(\sum_{k=1}^{N} u(k) \right) \Big/ \left(\sum_{k=1}^{N} i(k) \right)$$

$$= \lim_{N \to \infty} \frac{\frac{1}{N} \sum_{k=1}^{N} (u_0 + n_u(k))}{\frac{1}{N} \sum_{k=1}^{N} (i_0 + n_i(k))}$$

$$= \left(\lim_{N \to \infty} \frac{u_0 + \frac{1}{N} \sum_{k=1}^{N} n_u(k)}{i_0 + \frac{1}{N} \sum_{k=1}^{N} n_i(k)} \right)$$

$$= R_0$$

It converges to the exact value!!!

Asymptotic value of $\hat{R}_{\mathrm{LS}}$

$$\hat{R}_{\mathrm{SA}}(N) = \frac{1}{N}\sum_{k=0}^{N}\frac{u(k)}{i(k)} = \frac{1}{N}\sum_{k=0}^{N}\frac{u_0 + n_u(k)}{i_0 + n_i(k)} = \frac{1}{N}\frac{u_0}{i_0}\sum_{k=0}^{N}\frac{1 + n_u(k)/u_0}{1 + n_i(k)/i_0}$$

The series expansion exist only for small noise distortions

$$\frac{1}{1+x} = \sum_{l=0}^{\infty}(-1)^l x^l \text{ for } |x| < 1$$

$$\lim_{N\to\infty}\hat{R}_{\mathrm{SA}}(N) \approx R_0\left(1 + \frac{\sigma_i^2}{i_0^2}\right)$$

The estimator converges to the wrong value!!

Variance expressions

First order approximation

$$\sigma^2_{R_{\mathrm{LS}}}(N) \approx \sigma^2_{R_{\mathrm{EV}}}(N) \approx \sigma^2_{R_{\mathrm{SA}}}(N) \approx \frac{R_0^2}{N}\left(\frac{\sigma_u^2}{u_0^2} + \frac{\sigma_i^2}{i_0^2}\right)$$

- variance decreases in $1/N$

- variance increases with the noise

- for low noise levels, all estimators have the same uncertainty

---> **Experiment design**

Example: noise dominates

Conlusions

- A simple problem

- Many solutions

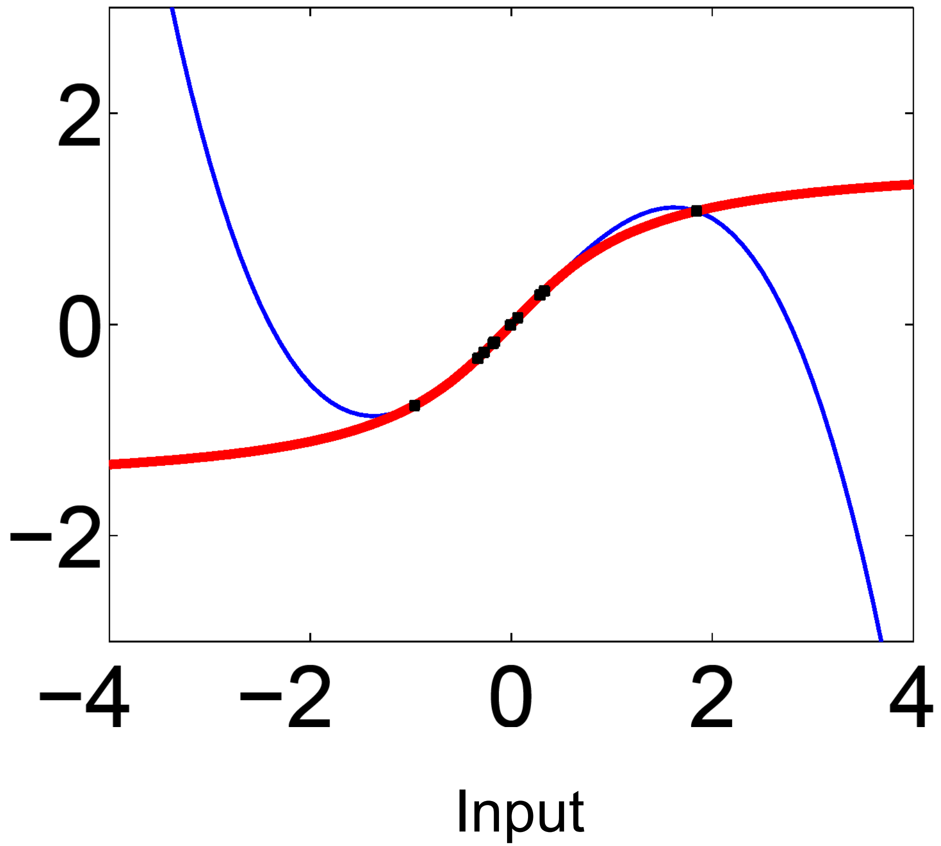
- How to select a good estimator?

- Can we know the properties in advance?

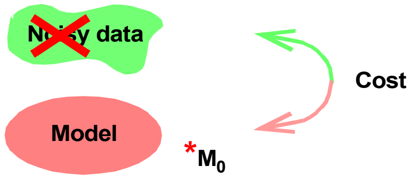
Need for a general framework !!

Intuition fails

Example 2: Model errors dominate



$N(0, \sigma=1)$



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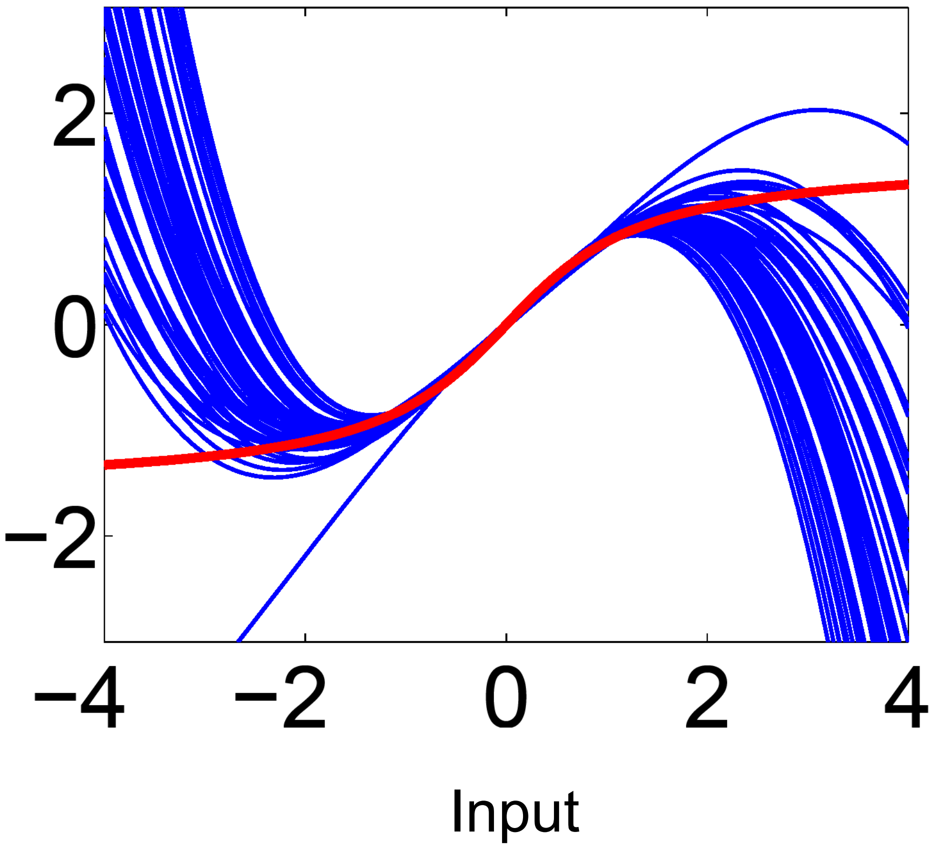
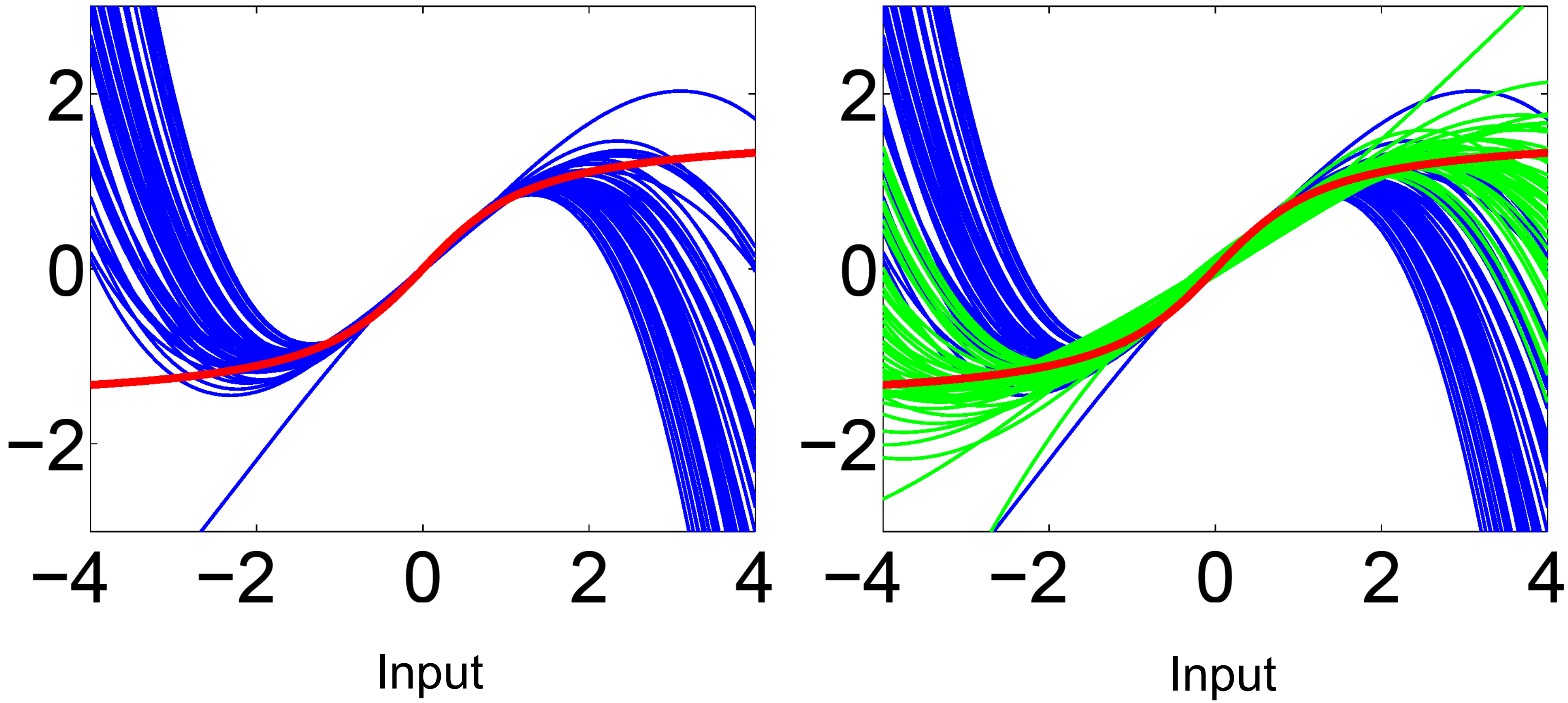


$N(0, \sigma=1)$

Example 2: Model errors dominate

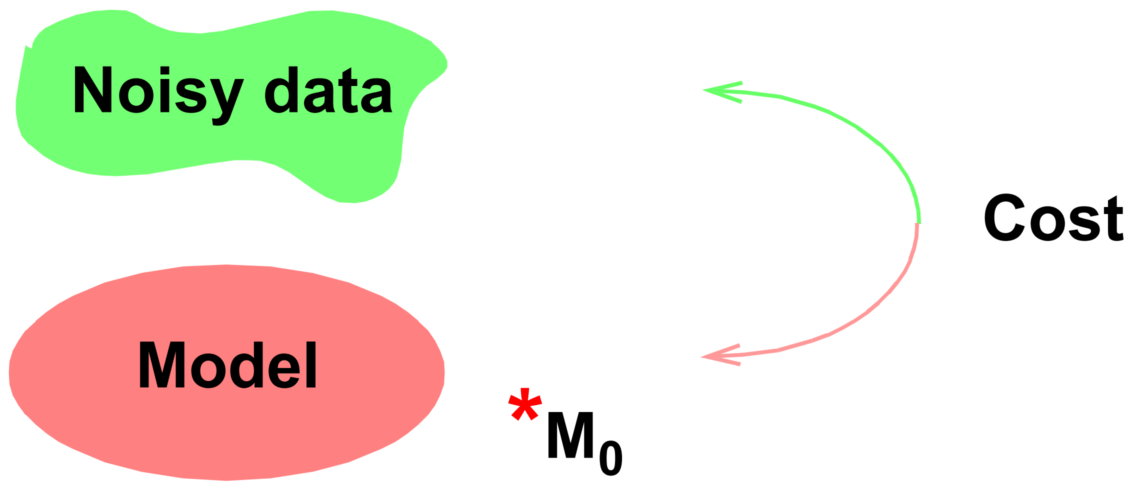


$N(0, \sigma^2=1)$ $N(0, \sigma^2=4)$

Identification of linear systems

in the presence of

nonlinear distortions




TC57
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Outline

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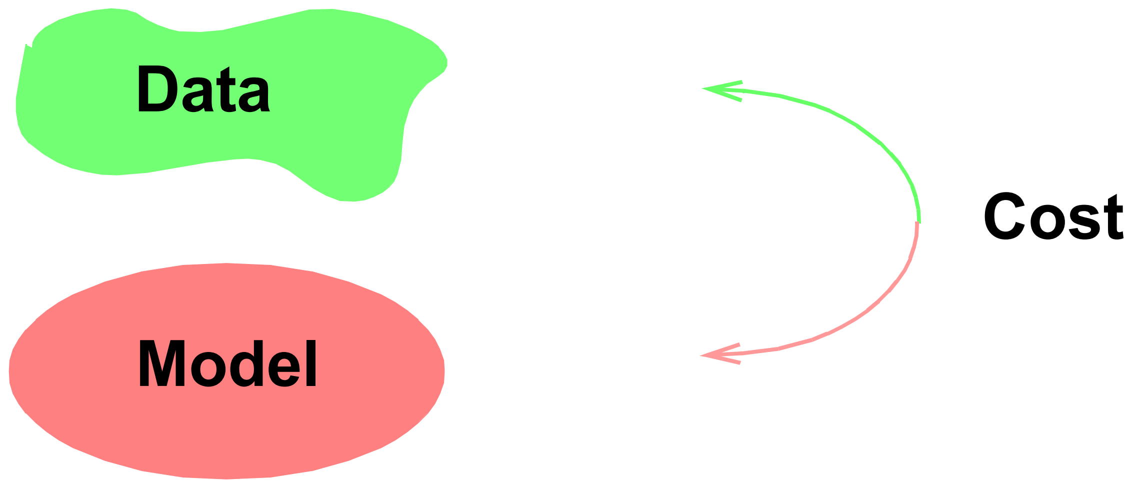
- Linear system identification

- Impact nonlinear distortions on the linear framework

Linear System Identification



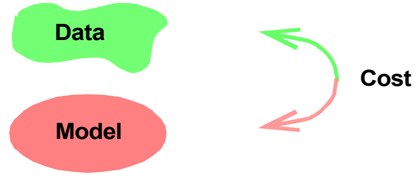
Data

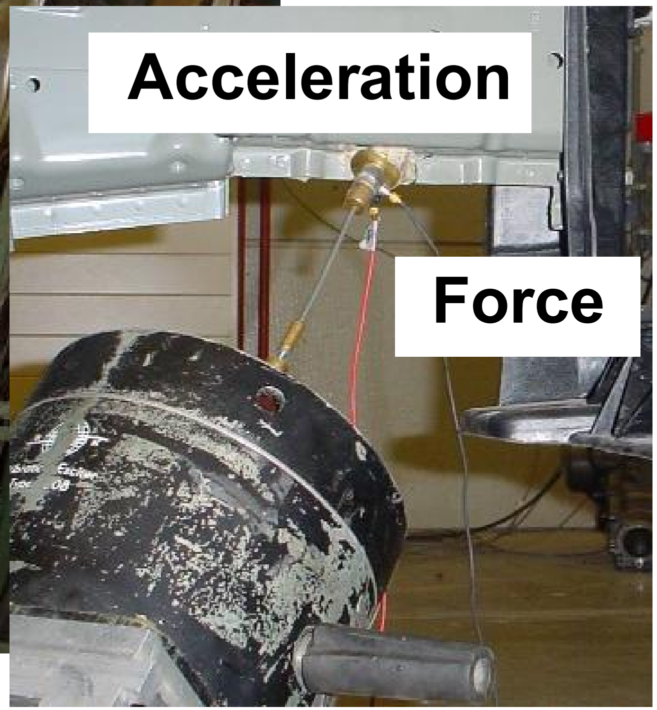
Data

Model

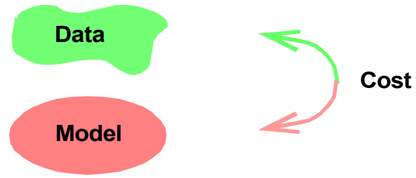
Cost

Data





Data

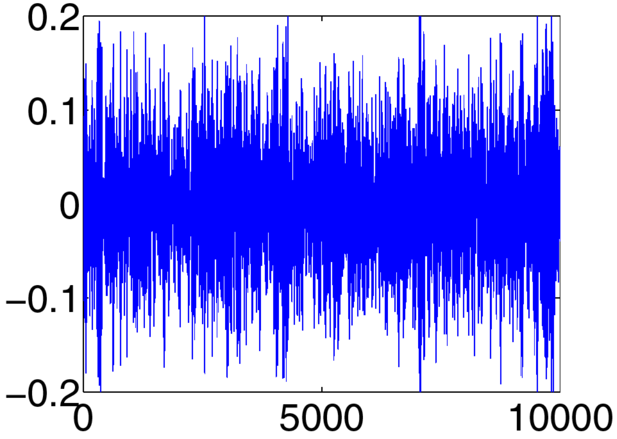


Force

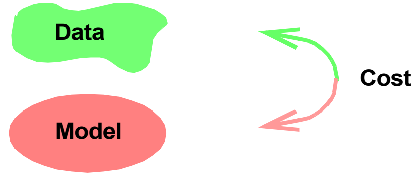
$$u(t) = u_0(t) + n_u(t)$$

Acceleration

$$y(t) = y_0(t) + n_y(t)$$



Data



Force

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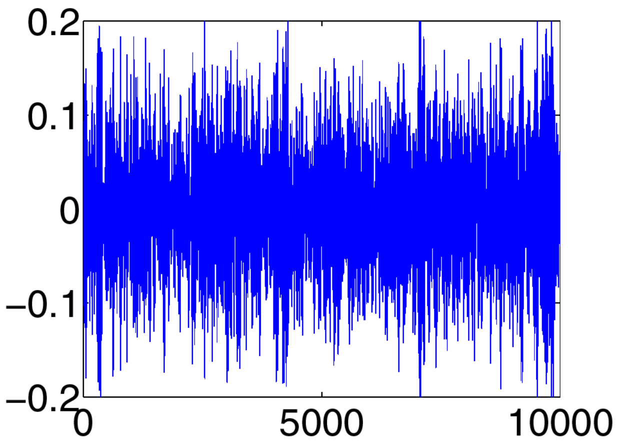
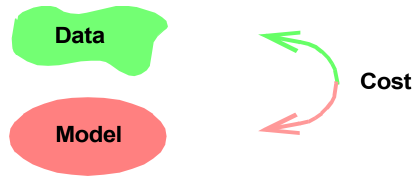


Data

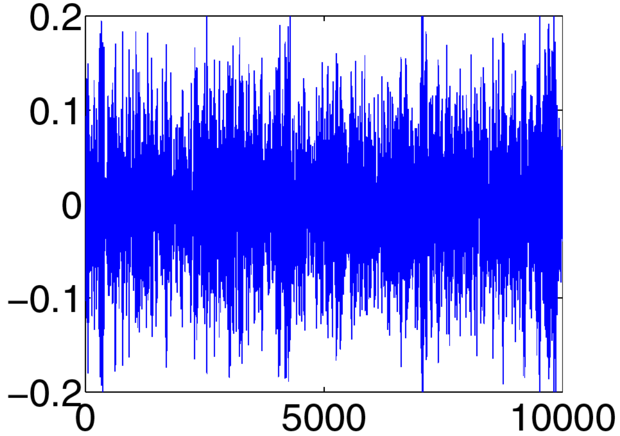


time-frequency transform

discrete Fourier transform

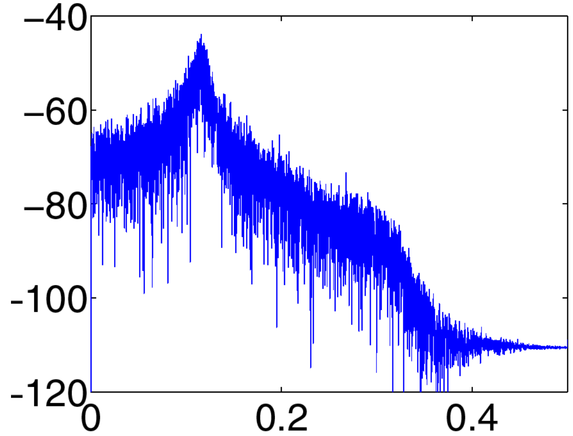
Force
$u(t) \;=\; u_0(t) + n_u(t)$

Acceleration
$y(t) \;=\; y_0(t) + n_y(t)$

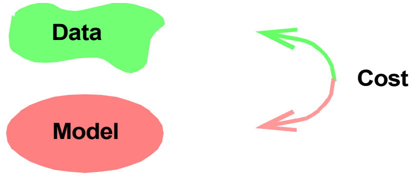


Force
$U(\Omega_k) \;=\; U_0(\Omega_k) + N_U(\Omega_k)$

Acceleration
$Y(\Omega_k) \;=\; Y_0(\Omega_k) + N_Y(\Omega_k)$



Model



time domain

continuous time

$$A(\frac{d}{dt})y_0(t) = B(\frac{d}{dt})u_0(t)$$

discrete time

$$A(q)y_0(t) = B(q)u_0(t)$$

Model

time domain

frequency domain

continuous time

$$A(\frac{d}{dt})y_0(t) = B(\frac{d}{dt})u_0(t)$$

discrete time

$$A(q)y_0(t) = B(q)u_0(t)$$

$$A(\Omega)Y_0(\Omega) = B(\Omega)U_0(\Omega)$$

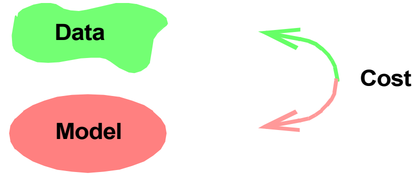
continuous time:

$$\Omega = j\omega$$

discrete time:

$$\Omega = e^{j2\pi\omega/\omega_s}$$

Cost

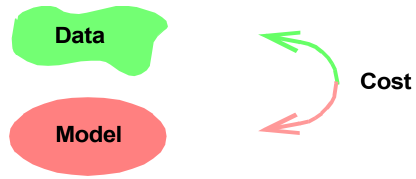


$$V_F(\theta, Z) = \frac{1}{F}\sum_{k=1}^{F}\begin{pmatrix} Y(k) - Y_{\mathrm{p}}(k) \\ U(k) - U_{\mathrm{p}}(k) \end{pmatrix}^H \begin{bmatrix} \sigma_Y^2(k) & \sigma_{YU}^2(k) \\ \sigma_{UY}^2(k) & \sigma_U^2(k) \end{bmatrix}^{-1} \begin{pmatrix} Y(k) - Y_{\mathrm{p}}(k) \\ U(k) - U_{\mathrm{p}}(k) \end{pmatrix}$$

under the constraint

$$A(\Omega_k, \theta) Y_{\mathrm{p}}(k) = B(\Omega_k, \theta) U_{\mathrm{p}}(k) \qquad k = 1, 2, \ldots, F$$

Cost



$$V_F(\theta, Z) = \frac{1}{F}\sum_{k=1}^{F} \begin{pmatrix} Y(k) - Y_p(k) \\ U(k) - U_p(k) \end{pmatrix}^H \begin{bmatrix} \sigma_Y^2(k) & \sigma_{YU}^2(k) \\ \sigma_{UY}^2(k) & \sigma_U^2(k) \end{bmatrix}^{-1} \begin{pmatrix} Y(k) - Y_p(k) \\ U(k) - U_p(k) \end{pmatrix}$$

under the constraint

$$A(\Omega_k, \theta) Y_p(k) = B(\Omega_k, \theta) U_p(k) \qquad k = 1, 2, \ldots, F$$

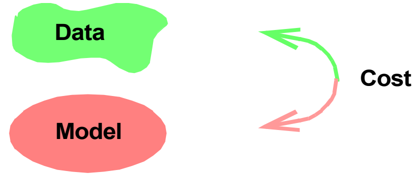
Remarks:

completely symmetric in U_p, Y_p

θ: $n_a + n_b + 1$ real parameters

U_p, Y_p: $2F$ complex parameters

Cost



$$V_F(\theta, Z) = \frac{1}{F}\sum_{k=1}^{F} \frac{|A(\Omega_k, \theta)Y(k) - B(\Omega_k, \theta)U(k)|^2}{\sigma_Y^2(k)|A(\Omega_k, \theta)|^2 + \sigma_U^2(k)|B(\Omega_k, \theta)|^2 - 2\mathrm{Re}(\sigma_{YU}^2(k)A(\Omega_k, \theta)\bar{B}(\Omega_k, \theta))}$$

Discussion points

- $\sigma_U^2(k), \sigma_Y^2(k), \sigma_{YU}^2(k)$?

- selection of the model order

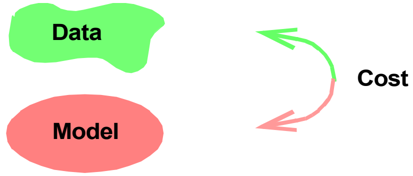
- generation of initial estimates

- numerical optimization

- numerical stability

- impact initial conditions >< leakage

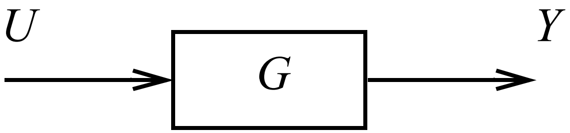
Cost

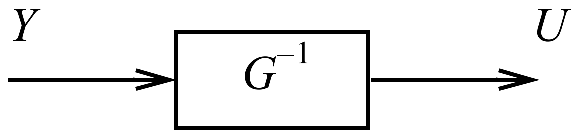


$$V_F = \frac{1}{F}\sum \frac{|AY - BU|^2}{\sigma_Y^2|A|^2 + \sigma_U^2|B|^2 - 2\mathrm{Re}(\sigma_{YU}^2 A\bar{B})}$$

$$\frac{1}{F}\sum \frac{|Y - GU|^2}{\sigma_Y^2 + \sigma_U^2|G|^2 - 2\mathrm{Re}(\sigma_{YU}^2 \bar{G})}$$

$$\frac{1}{F}\sum \frac{|G^{-1}Y - U|^2}{|G^{-1}|^2\sigma_Y^2 + \sigma_U^2 - 2\mathrm{Re}(\sigma_{YU}^2 \bar{G}^{-1})}$$





Time domain - frequency domain identification

$$\frac{1}{F}\sum\frac{|Y-GU|^2}{\sigma_Y^2+\sigma_U^2|G|^2-2\mathrm{Re}(\sigma_{YU}^2\overline{G})}$$

Input exactly known: $\sigma_U^2 = 0$

Frequency domain

$$\frac{1}{F}\sum\frac{|Y-GU|^2}{\sigma_Y^2}$$

plant model: parametric
noise model: nonparametric

Time domain - frequency domain identification

$$\frac{1}{F}\sum \frac{|Y - GU|^2}{\sigma_Y^2 + \sigma_U^2 |G|^2 - 2\mathrm{Re}(\sigma_{YU}^2 \overline{G})}$$

Input exactly known: $\sigma_U^2 = 0$

Frequency domain

$$\frac{1}{F}\sum \frac{|Y - GU|^2}{\sigma_Y^2}$$

plant model: parametric
noise model: nonparametric

Time domain

$$\frac{1}{N}\sum |H^{-1}(y - Gu)|^2$$

plant model: parametric
noise model: parametric
σ_Y^2 --> $|H|^2$

Time domain - frequency domain identification

$$\frac{1}{F}\sum\frac{|Y-GU|^2}{\sigma_Y^2+\sigma_U^2|G|^2-2\mathrm{Re}(\sigma_{YU}^2\overline{G})}$$

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$$\frac{1}{F}\sum\frac{|Y-GU|^2}{\sigma_Y^2}$$

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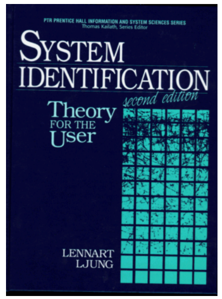
noise model: nonparametric

Time domain

$$\frac{1}{N}\sum|H^{-1}(y-Gu)|^2$$

plant model: parametric
noise model: parametric
σ_Y^2 --> $|H|^2$



Time domain - frequency domain identification

$$\frac{1}{F}\sum\frac{|Y-GU|^2}{\sigma_Y^2+\sigma_U^2|G|^2-2\mathrm{Re}(\sigma_{YU}^2\overline{G})}$$

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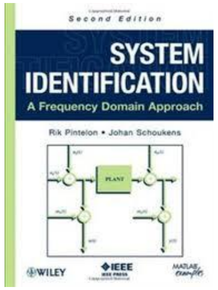
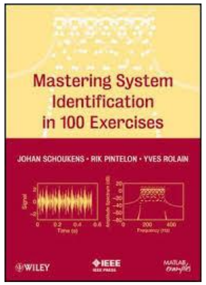
noise model: nonparametric



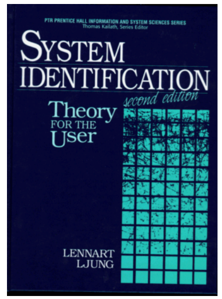


Time domain

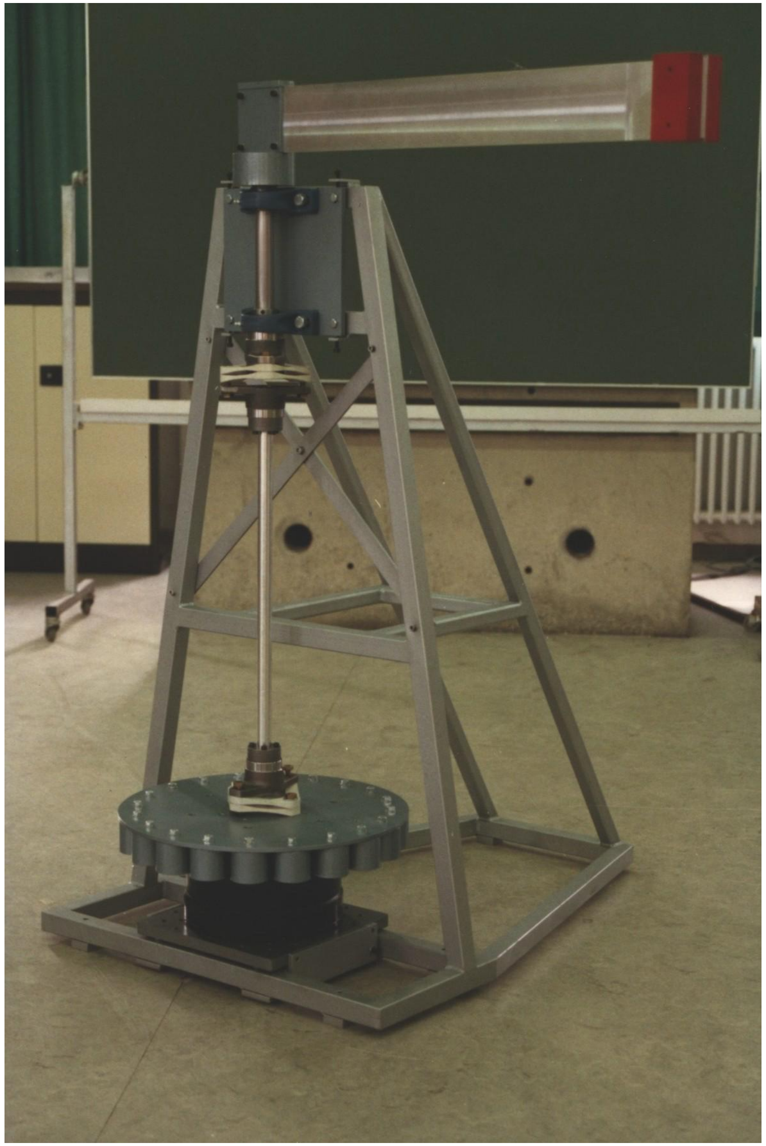
$$\frac{1}{N}\sum|H^{-1}(y-Gu)|^2$$

plant model: parametric
noise model: parametric

σ_Y^2 --> $|H|^2$

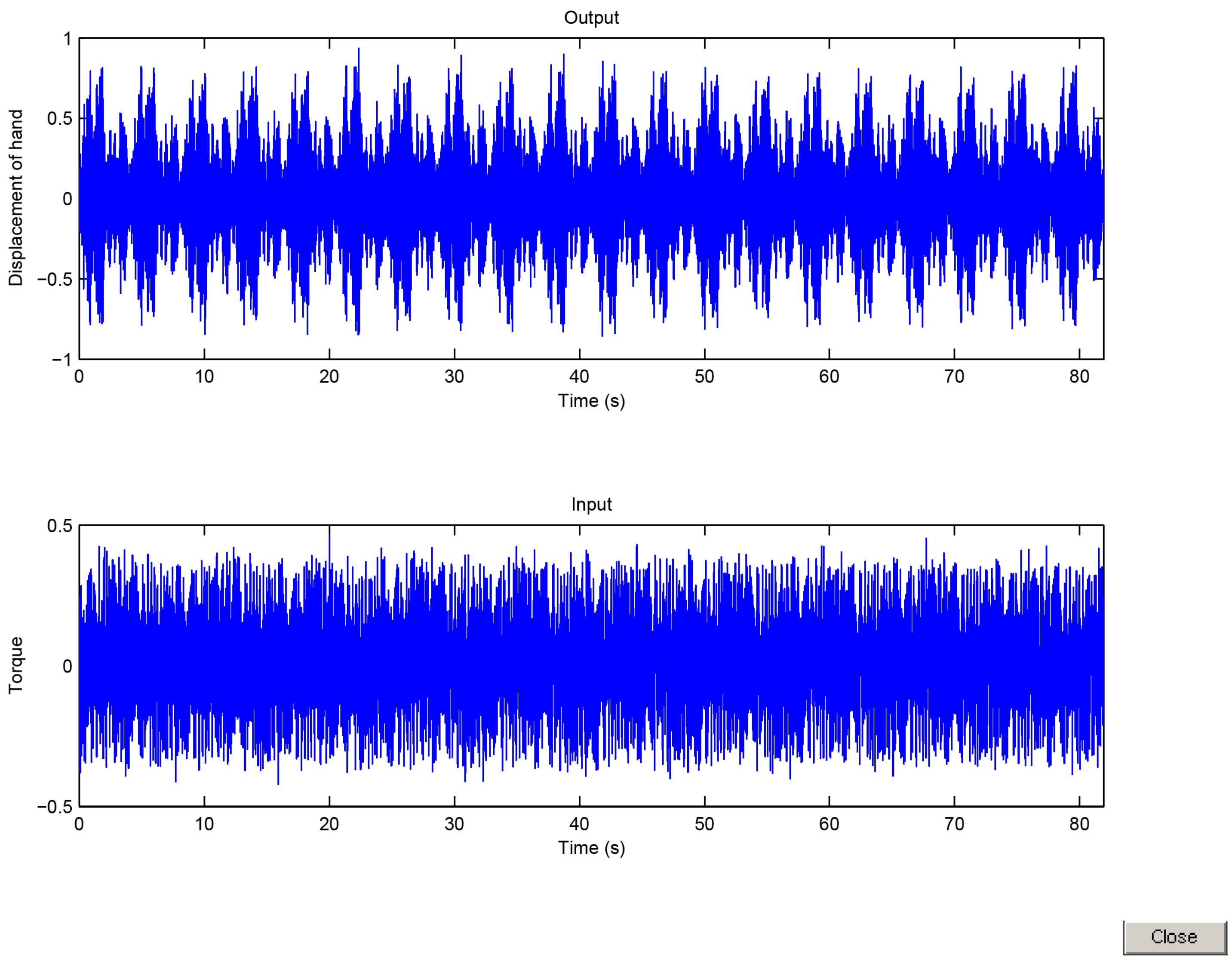


Application
The flexible robot arm



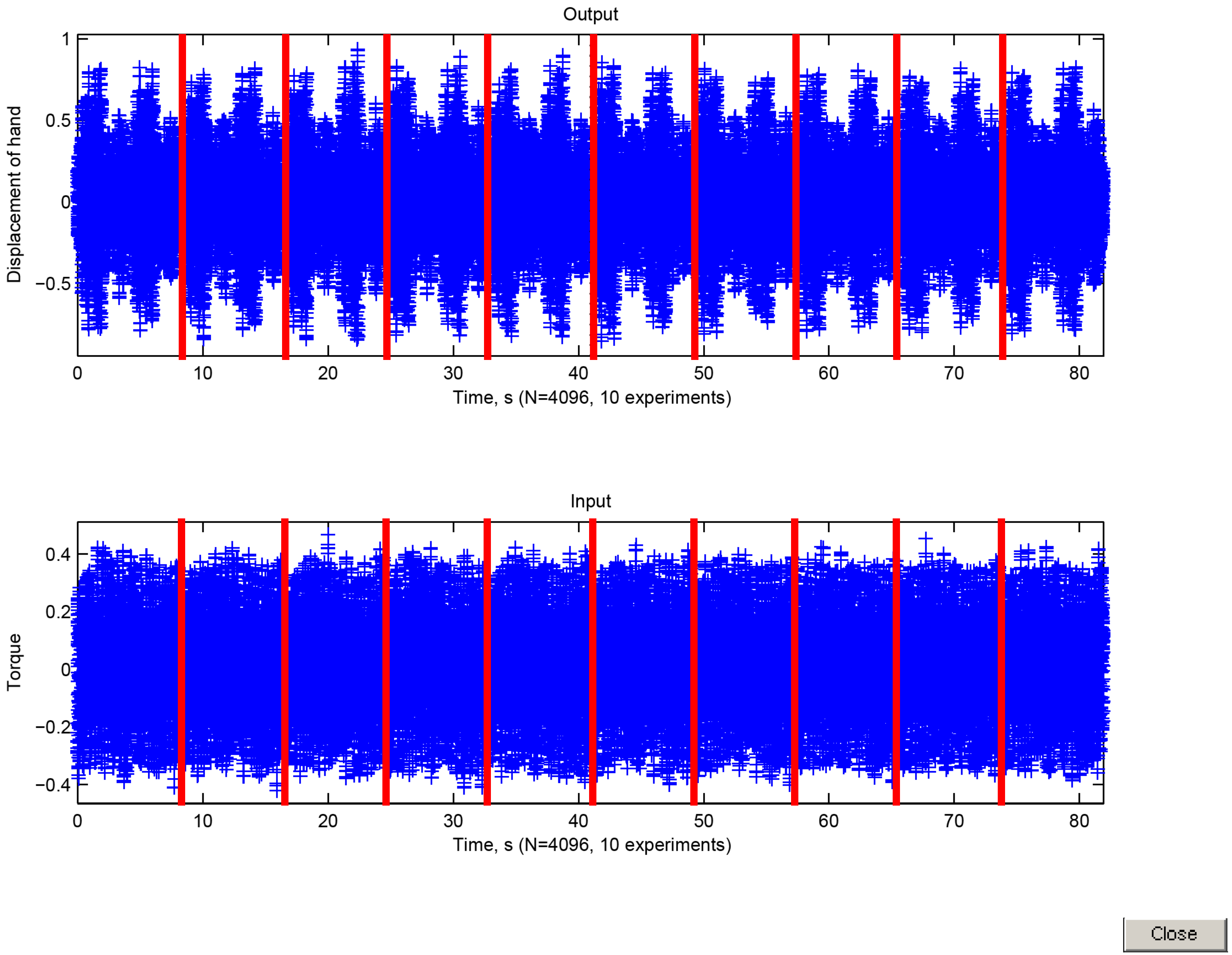
Data from Jan Swevers, KULeuven, PMA

Raw data

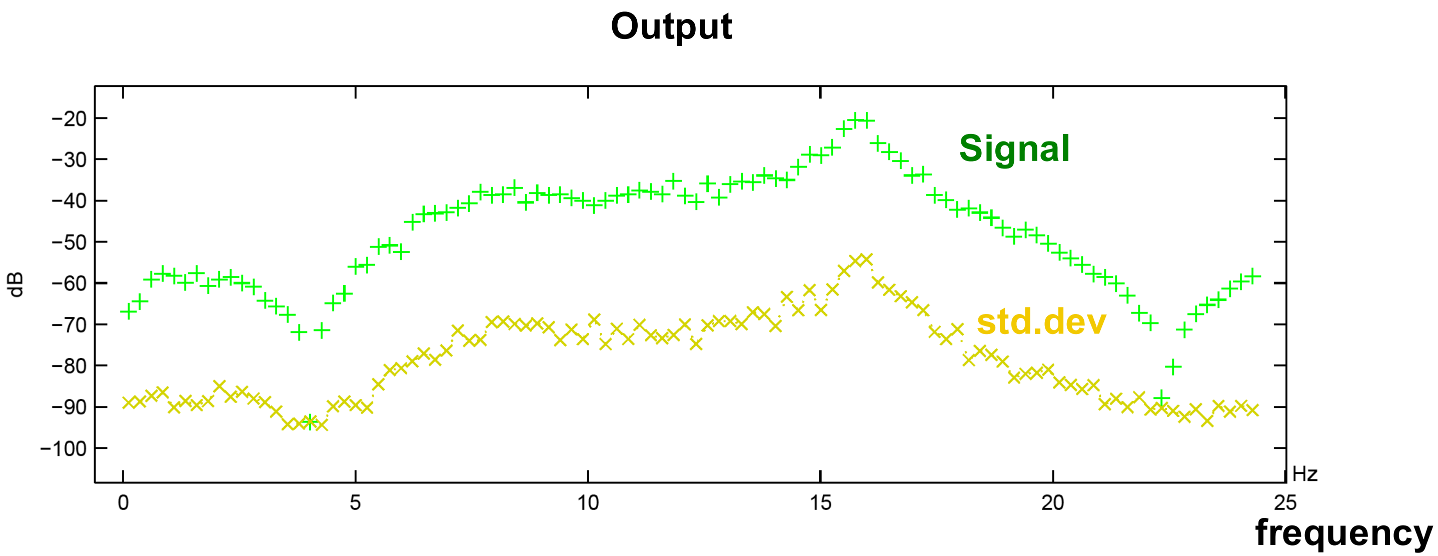


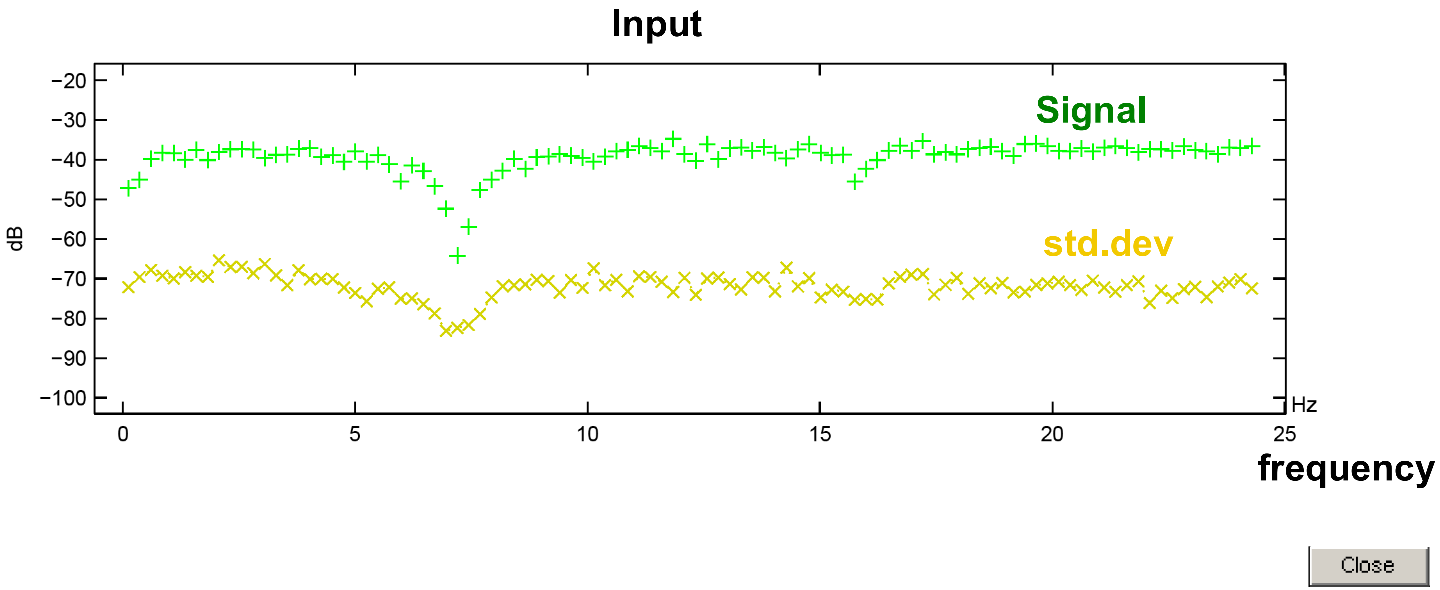
Output
Displacement of hand
1
0.5
0
−0.5
−1
0
10
20
30
40
50
60
70
80
Time (s)
Input
Torque
0.5
0
−0.5
0
10
20
30
40
50
60
70
80
Time (s)
Close

Segment the record
10 periods

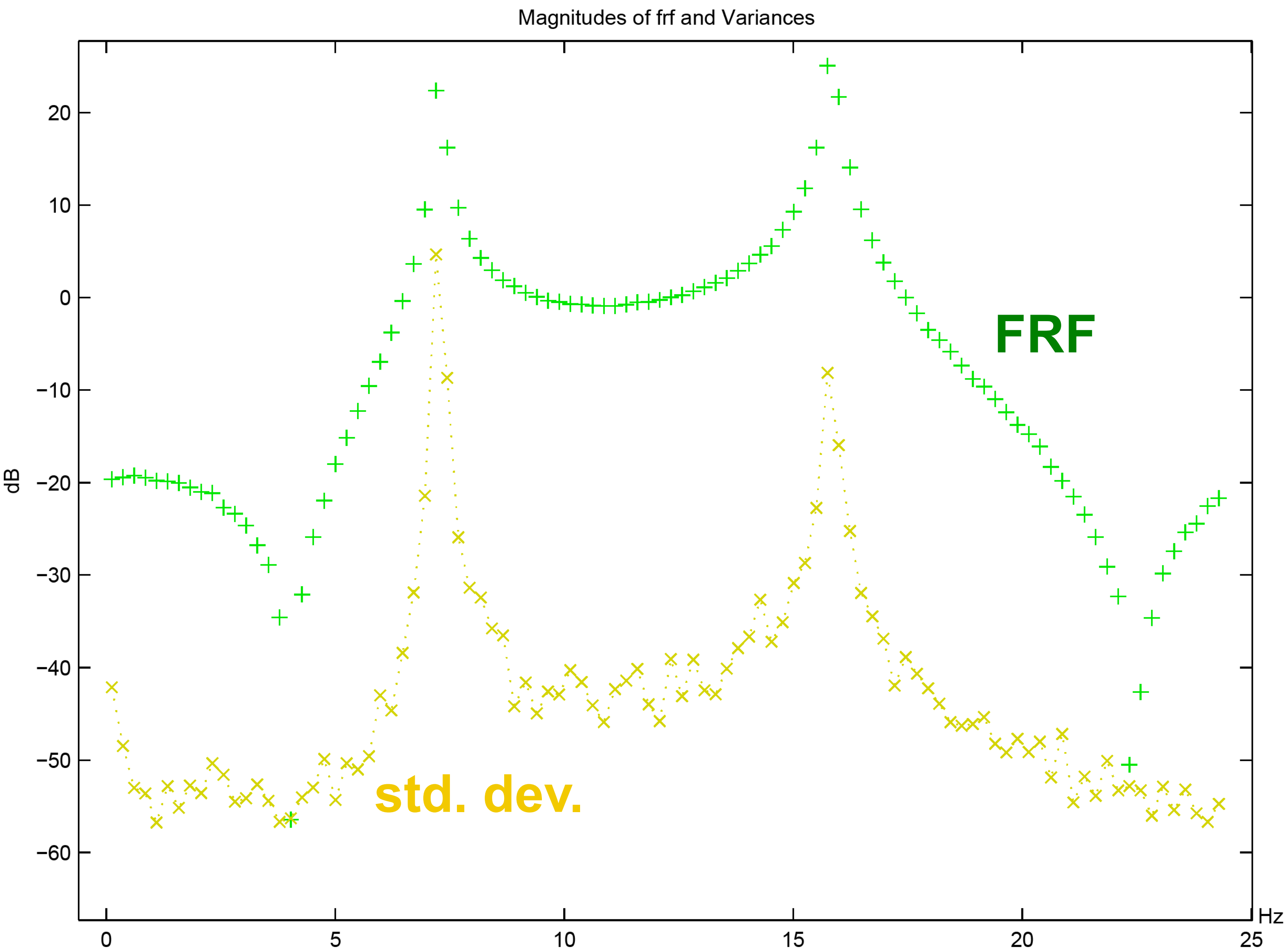


Variance analysis

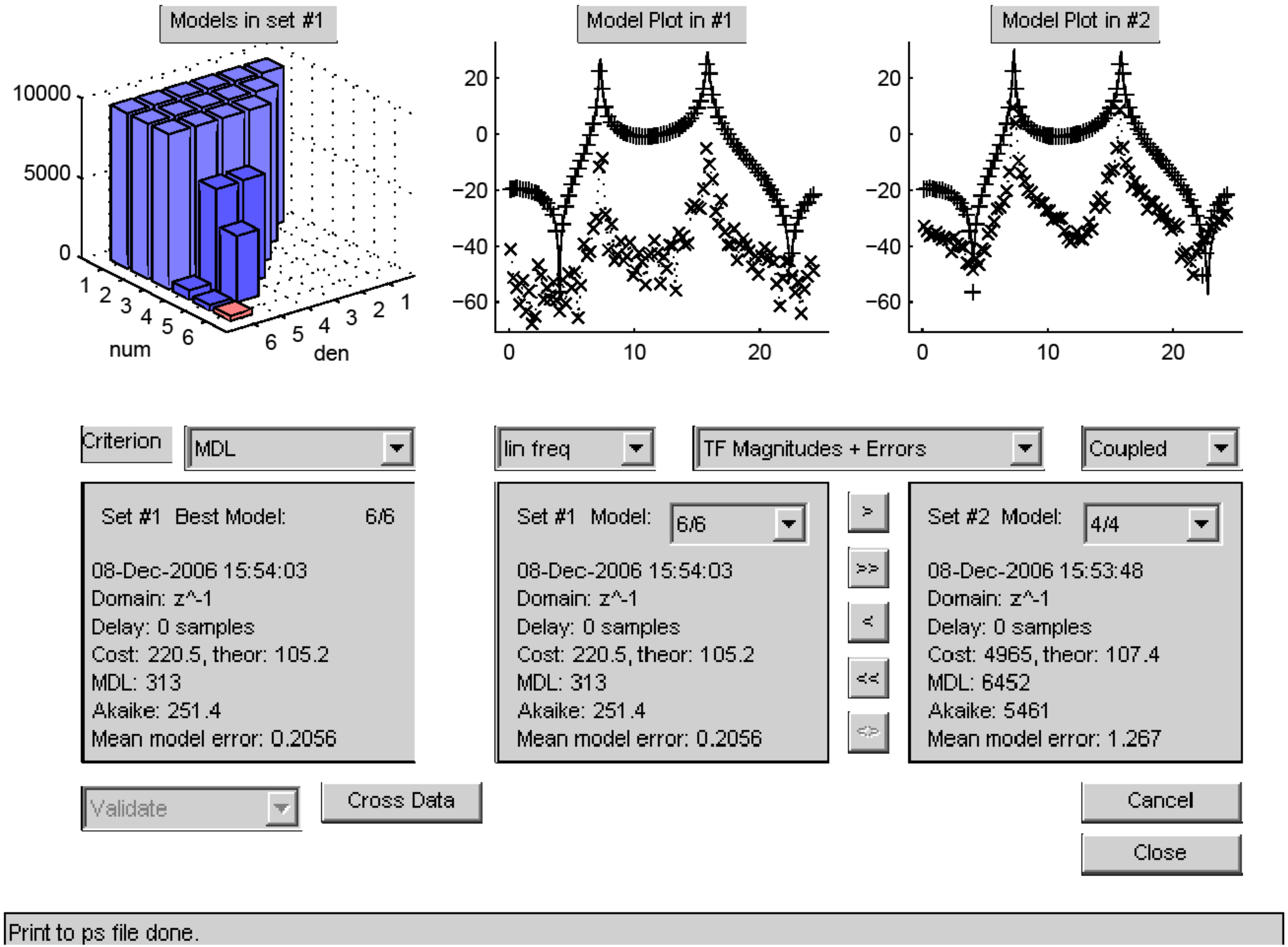




Variance analysis
FRF

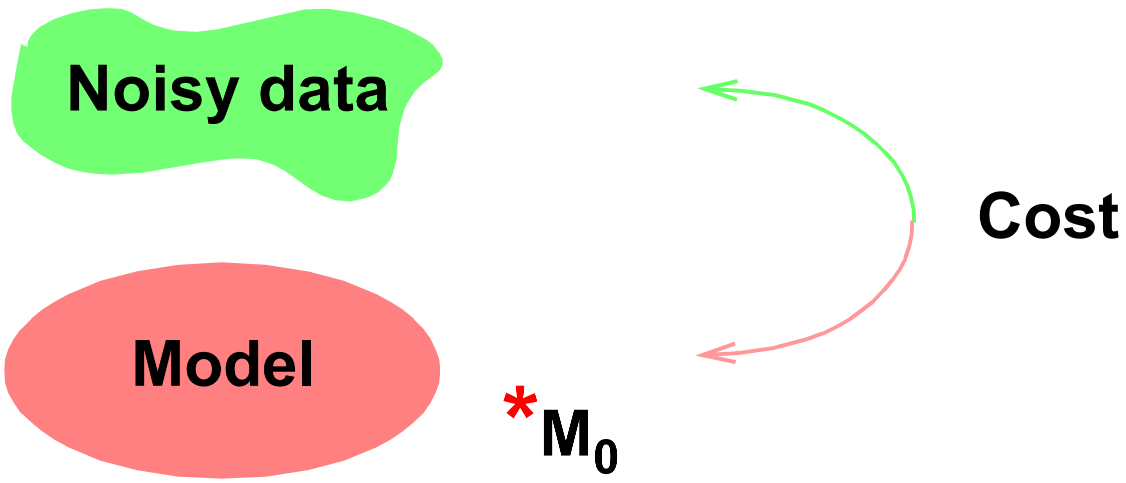


Estimated model



Outline

- Introduction
- Intuitive solutions?
- Linear system identification
- Impact nonlinear distortions on the linear framework



System Identification in a real world

Linear

Nonlinear

Time-Varying

Linear SI versus Nonlinear SI

Linear SI
- mature field
- well developed tools
- inexpensive

Nonlinear SI
- hot research topic
- expensive

Linear SI versus Nonlinear SI

Linear SI
- mature field
- well developed tools
- inexpensive

Nonlinear SI
- hot research topic
- expensive

Questions

- Do we face a nonlinear identification problem?
- Safe to use a linear system identification approach?
- How much to gain with a nonlinear model?

System Identification in a real world

Detection, qualification, quantification NL

Linear identification in the presence of nonlinear distortions

Nonlinear system identification

Detection, qualification, quantification NL

Goal

- characterize nonlinear behaviour
- no increase of the measurement time
- little user interaction

Detection, qualification, quantification NL

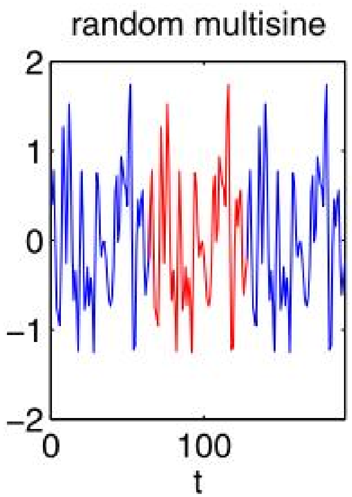
Goal

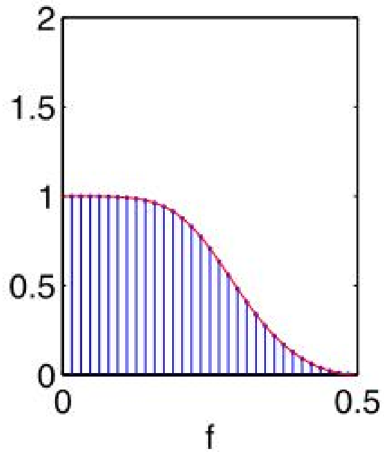
characterize nonlinear behaviour
no increase of the measurement time
little user interaction

Tool

periodic excitations

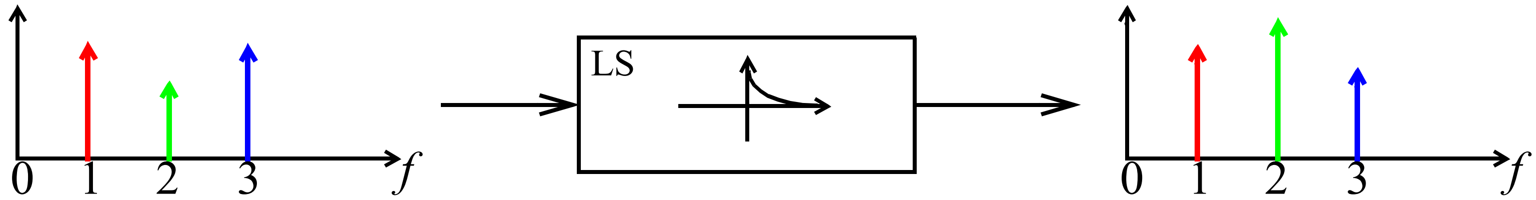
$$u(t) = \frac{1}{F}\sum_{k=1}^{F} A_k \cos(2\pi k f_0 t + \varphi_k)$$





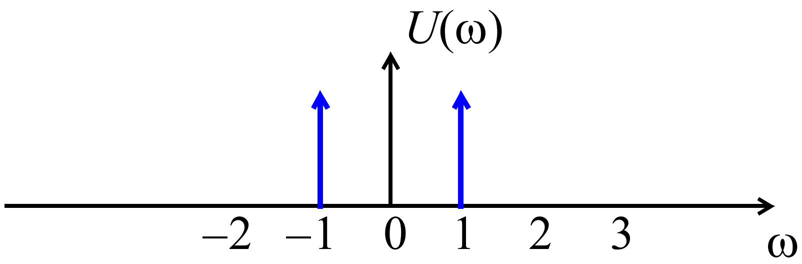
Behaviour of a nonlinear system

A linear system



Behaviour of a nonlinear system
Basic Idea

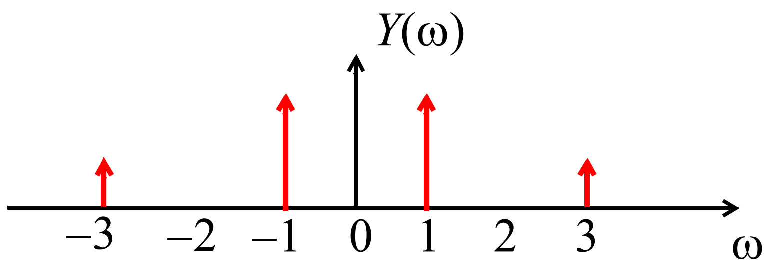
$$u(t) = 2\cos\omega t = e^{j\omega t} - e^{-j\omega t}, \quad \omega = 1$$



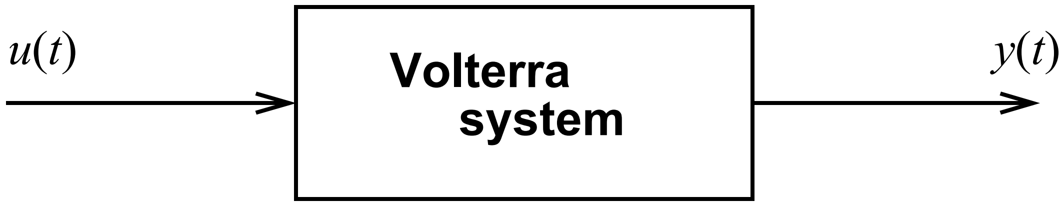
$$u^3 = (e^{j\omega t} - e^{-j\omega t})(e^{j\omega t} - e^{-j\omega t})(e^{j\omega t} - e^{-j\omega t})$$

Output: all possible combinations, 3 by 3, of the frequencies -1 and 1

1	1	1	3
1	1	-1	1
1	-1	1	1
1	-1	-1	-1
-1	1	1	1
-1	1	-1	-1
-1	-1	1	-1
-1	-1	-1	-3



Volterra theory in a nut shell
time domain



$$y(t) = \sum_{k=1}^{\infty} y^{[k]}(t)$$

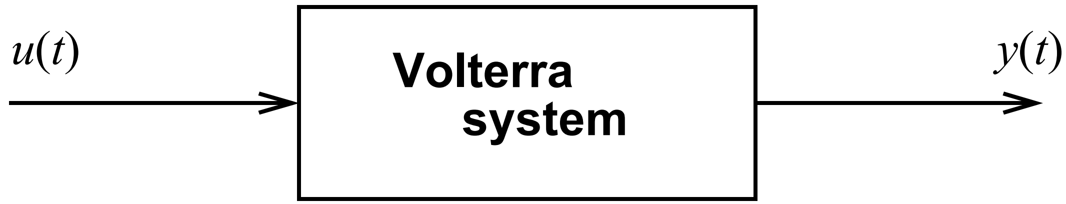
with

$$y^{[1]}(t) = \int_{-\infty}^{\infty} u(\sigma_1) h_1(t - \sigma_1) d\sigma_1$$

$$y^{[2]}(t) = \iint_{-\infty}^{\infty} u(\sigma_1) u(\sigma_2) h_2(t - \sigma_1, t - \sigma_2) d\sigma_1 d\sigma_2$$

...

Volterra theory in a nut shell
multi dimensional frequency domain



$$y(t) = \sum_{k=1}^{\infty} y^{[k]}$$

Define

$$y^{[2]}(t_1, t_2) = \iint_{-\infty}^{\infty} u(\sigma_1)u(\sigma_2)h_2(t_1 - \sigma_1, t_2 - \sigma_2)d\sigma_1 d\sigma_2$$

Then

$$Y^{[2]}(\omega_1, \omega_2) = \iint_{-\infty}^{\infty} y^{[2]}(t_1, t_2)e^{-j\omega_1 t_1}e^{-j\omega_2 t_2}dt_1 dt_2$$

Volterra theory in a nut shell
frequency domain relations

$$Y^{[n]}(\omega_1, \omega_2, \ldots, \omega_n) = H^{[n]}(\omega_1, \omega_2, \ldots, \omega_n) U(\omega_1) \ldots U(\omega_n)$$

with

$$H^{[n]}(\omega_1, \omega_2, \ldots, \omega_n) = \int \ldots \int_{-\infty}^{\infty} h_n(t_1, t_2, \ldots, t_n) e^{-j\omega_1 t_1} \ldots e^{-j\omega_n t_{nn}} dt_1 dt_2 \ldots dt_n$$

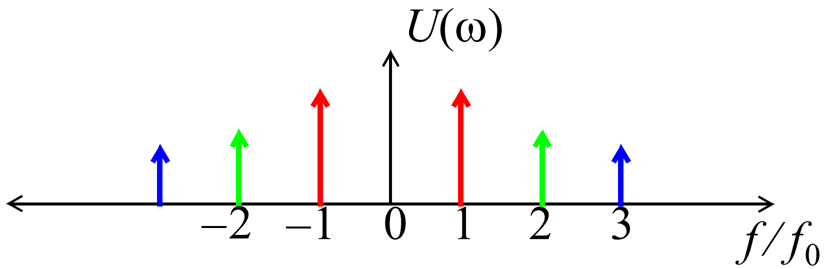
Corresponding one-dimensional frequency representation

$$Y^{[n]}(\omega_1, \omega_2, \ldots, \omega_n) \to Y(\omega_1 + \omega_2 + \ldots + \omega_n)$$

$\omega_1 + \omega_2 + \ldots + \omega_n$ indicates that contribution results from n^{th} degree NL

Volterra theory in a nut shell
frequency domain relations for periodic signals

$$Y^{[n]}(\omega_1, \omega_2, \ldots, \omega_n) = H(\omega_1, \omega_2, \ldots, \omega_n)U(\omega_1)\ldots U(\omega_n)$$



with

$$Y^{[2]}(\omega) = \int_{-\infty}^{\infty} y^{[2]}(\omega, \omega - \omega_1)d\omega_1 \rightarrow Y^{[2]}(k) = \sum_l y^{[2]}(l, k-l) = \sum_l H^{[2]}(l, k-l)U(l)U(k-l)$$

similar

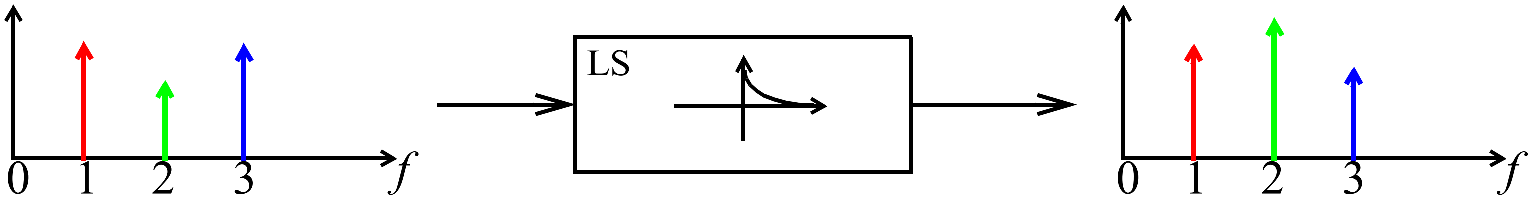
$$Y^{[3]}(k) = \sum_{l_1}\sum_{l_2} \ldots U(l_1)U(l_2)U(k-l_1-l_2)$$

Conclusion

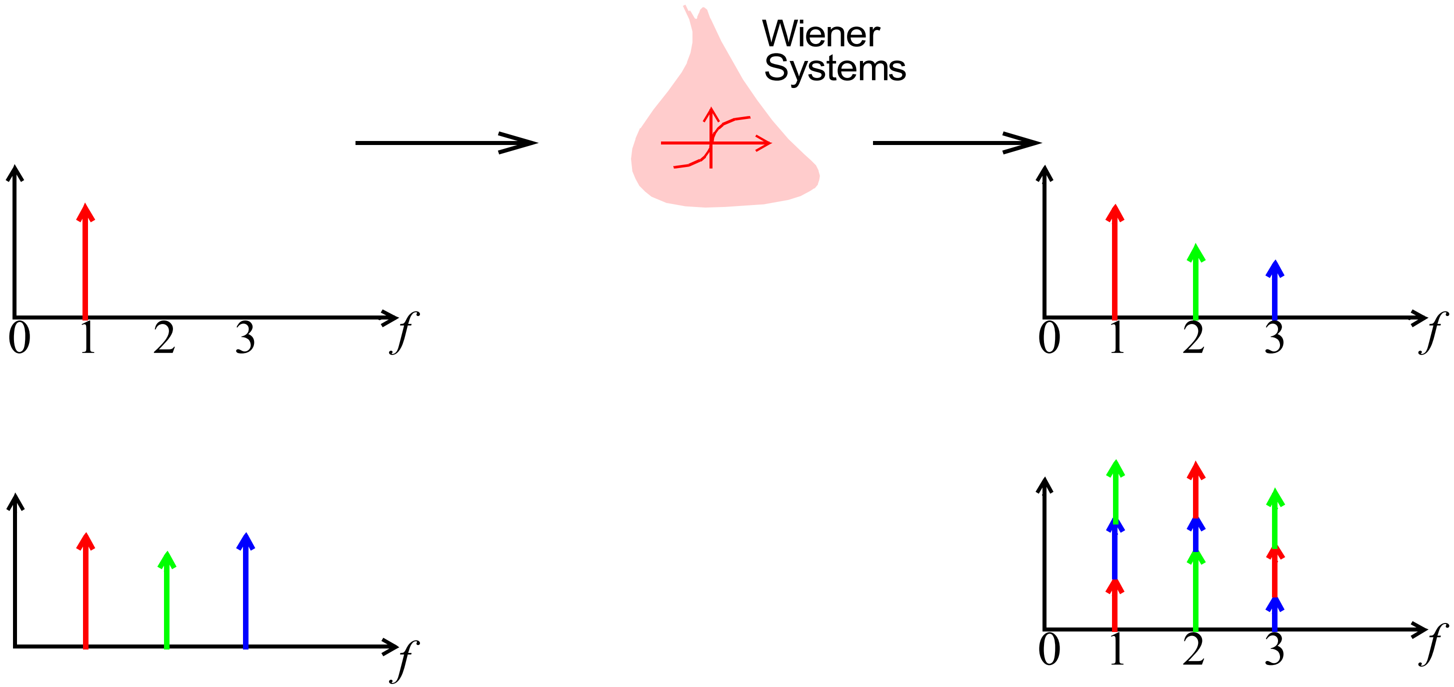
$Y^{[3]}(k)$ sum over all combination $U(l_1)U(l_2)U(l_3)$ such that $l_1 + l_2 + l_3 = k$

Behaviour of a nonlinear system

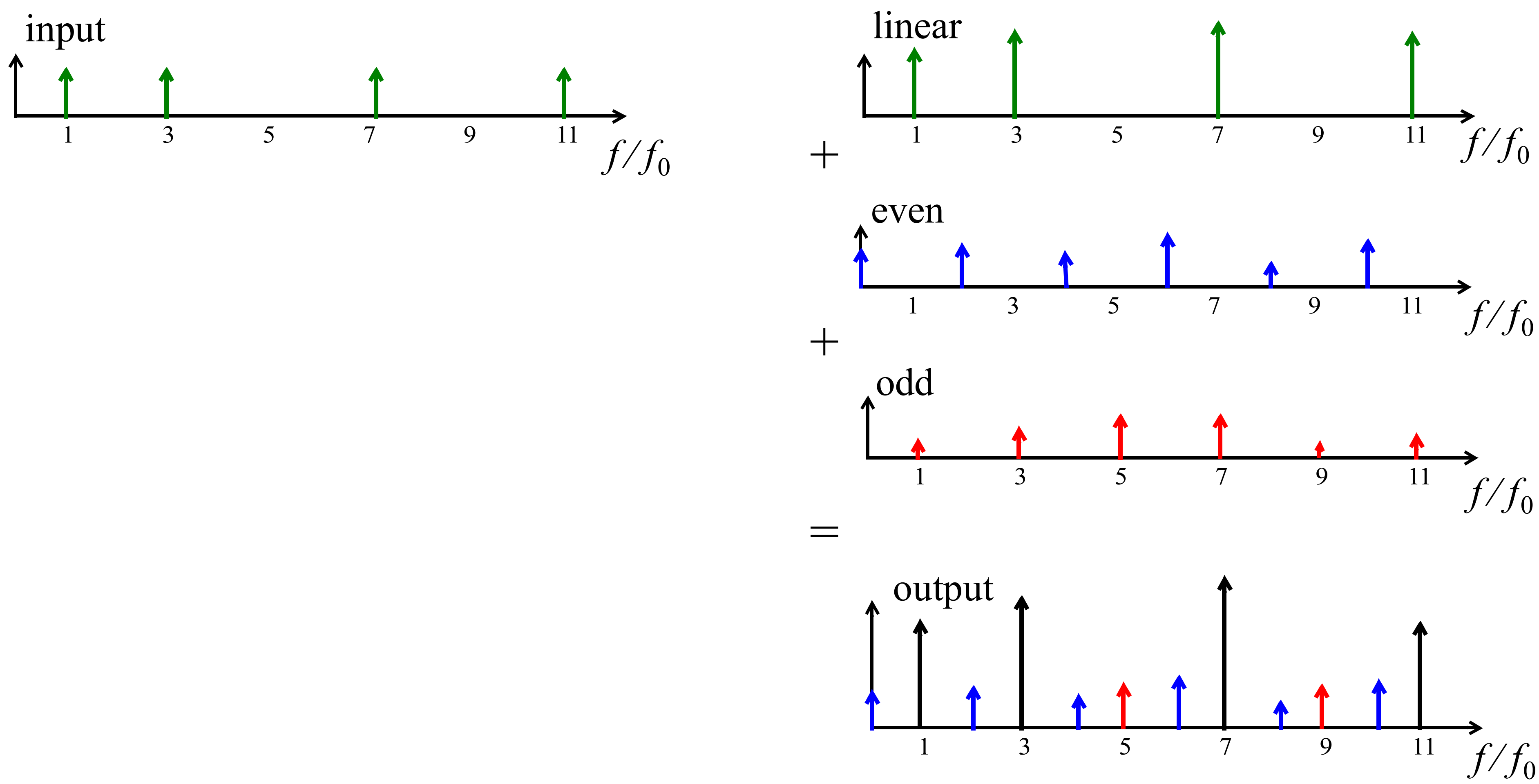
Linear system



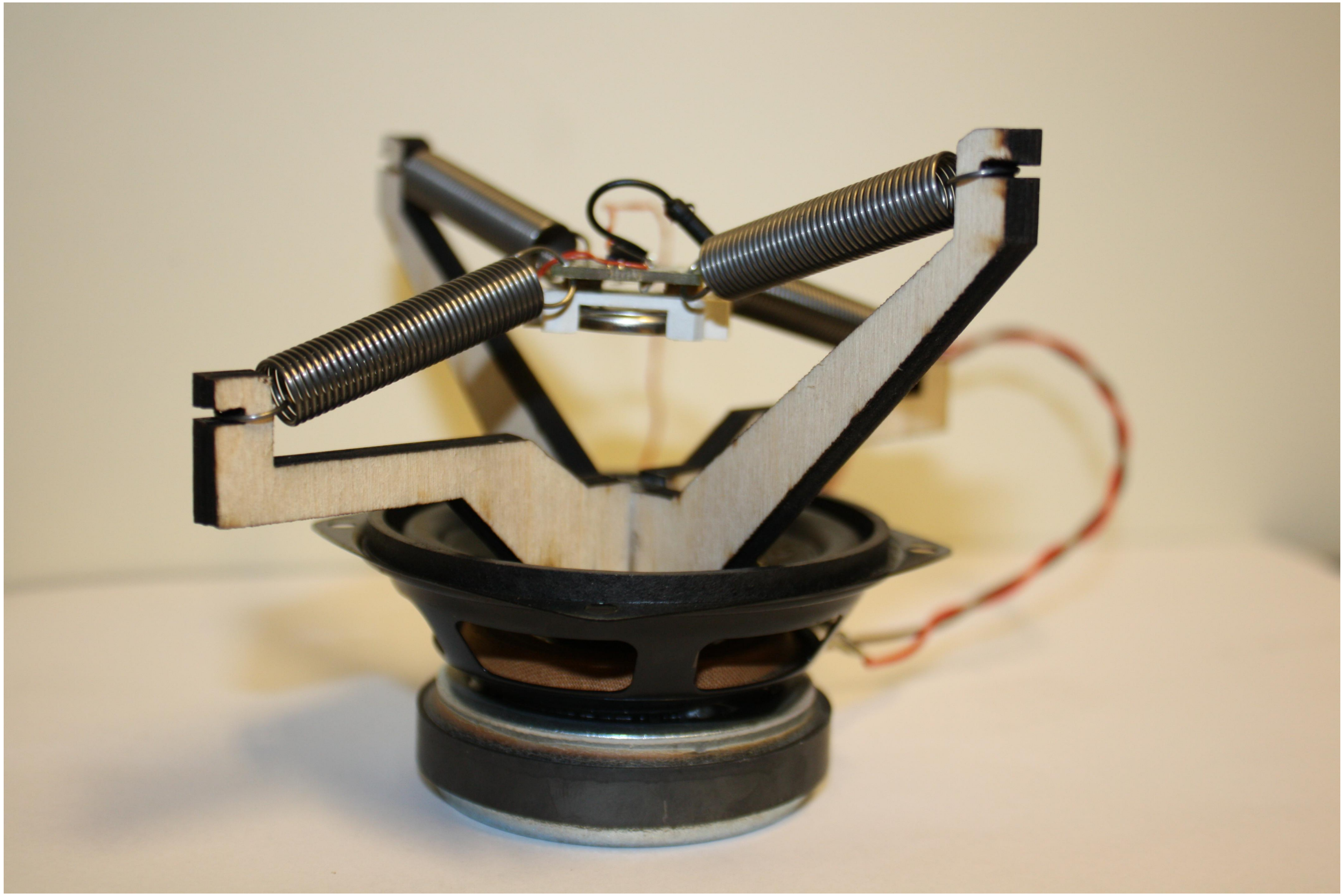
A Nonlinear system



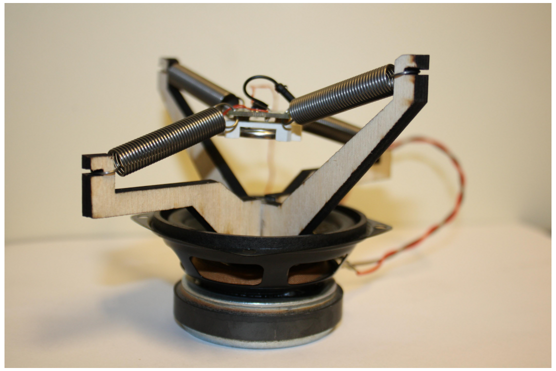
Detection, Qualification, Quantification NL
Basic Idea

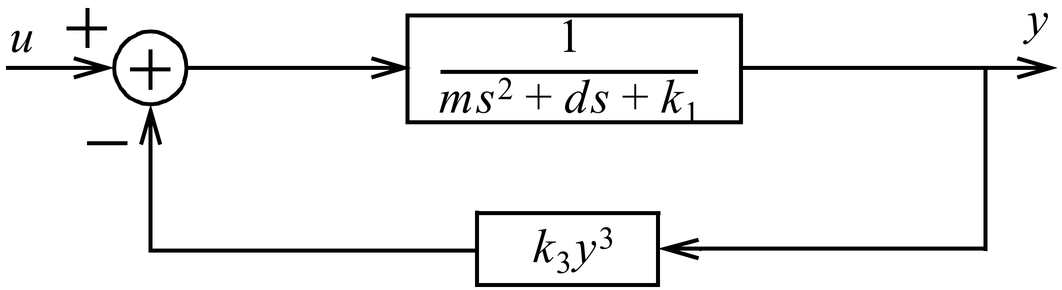


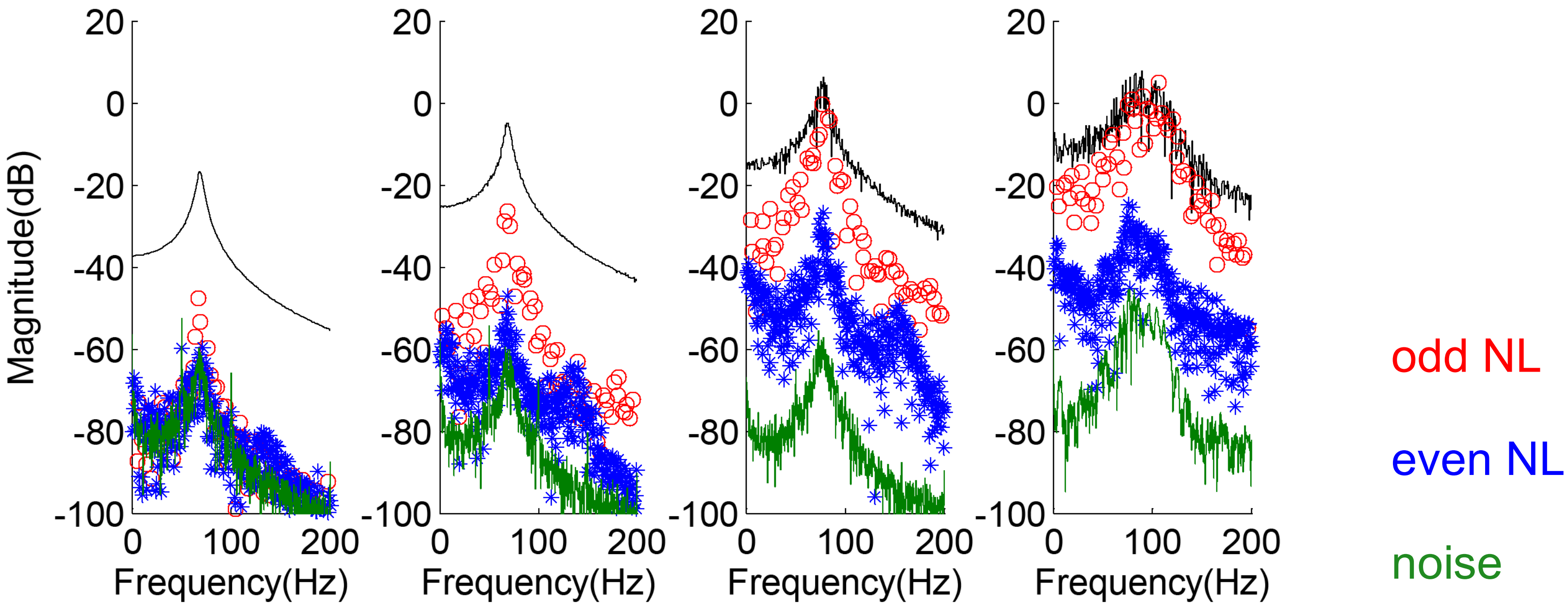
Detection, qualification, quantification of nonlinear distortions



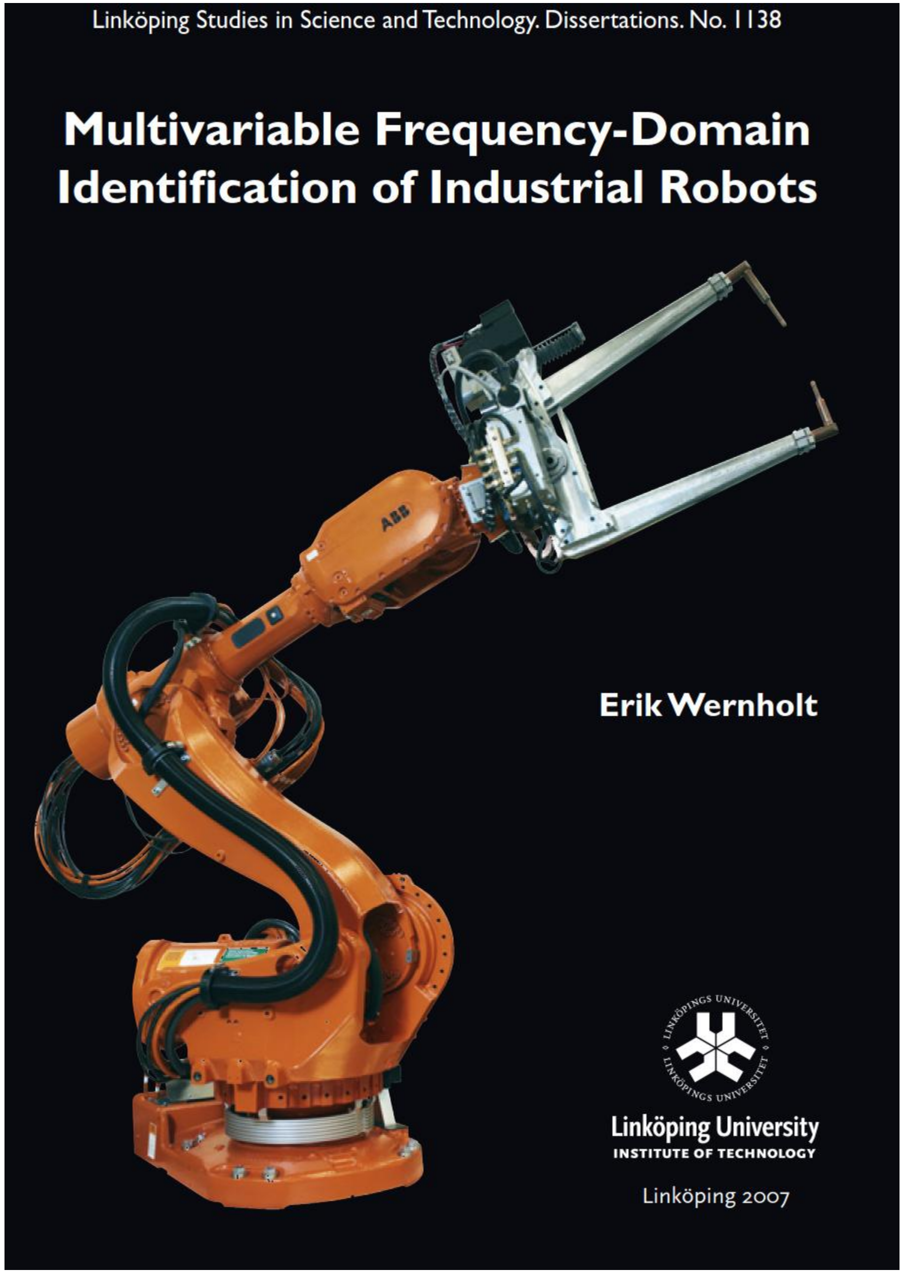
Detection, qualification, quantification of nonlinear distortions

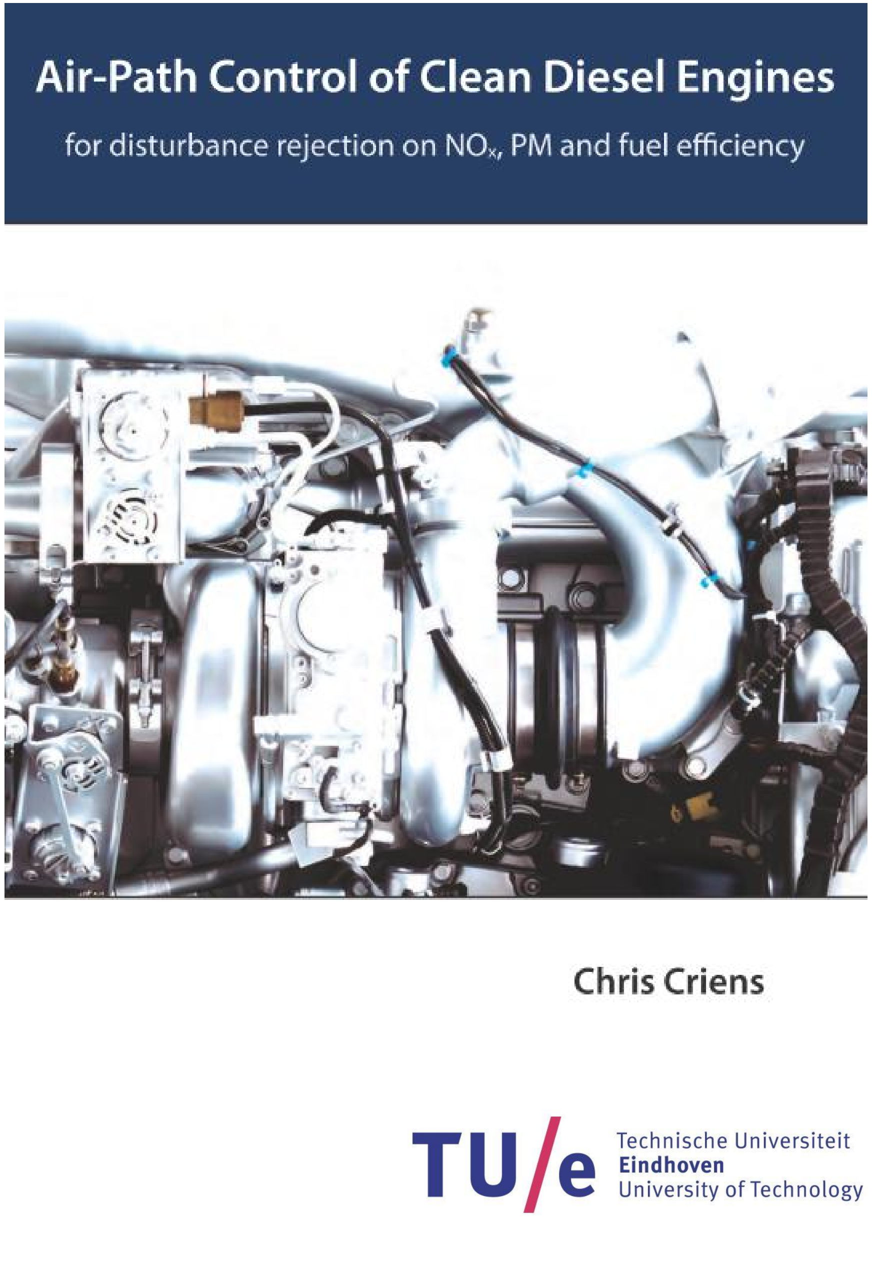






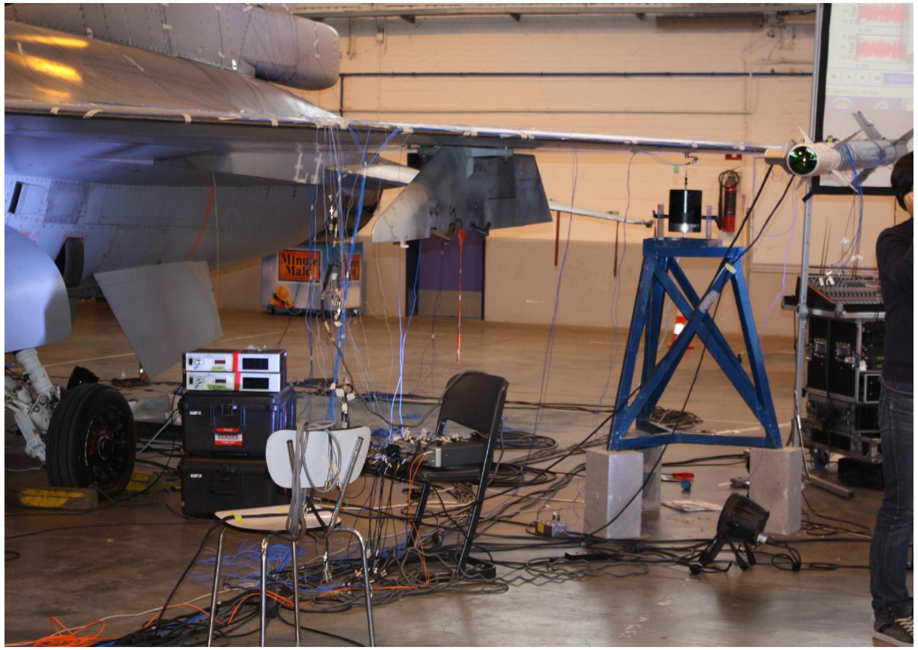
Industrial Examples

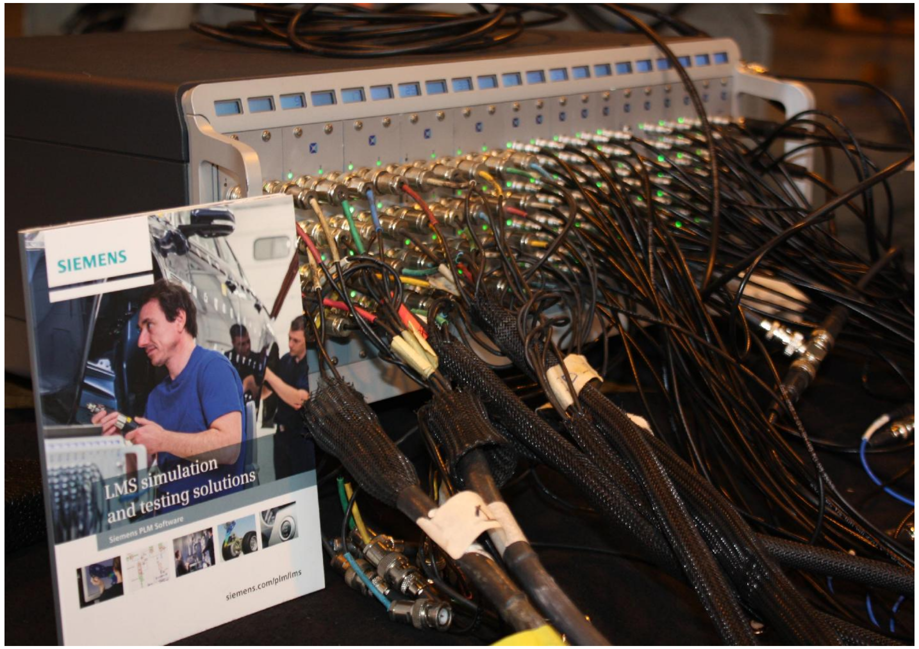


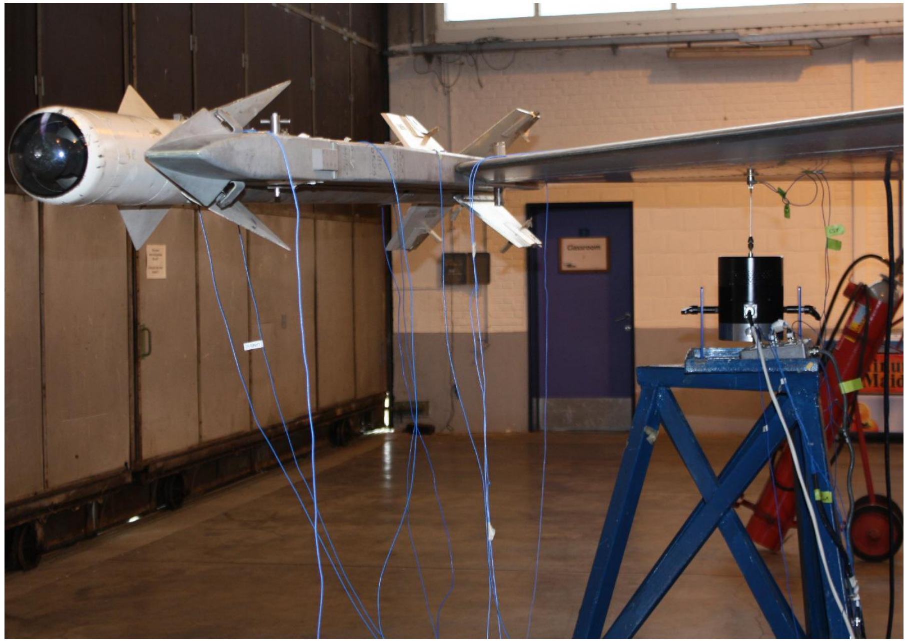


Ground vibration test on an F16-fighter

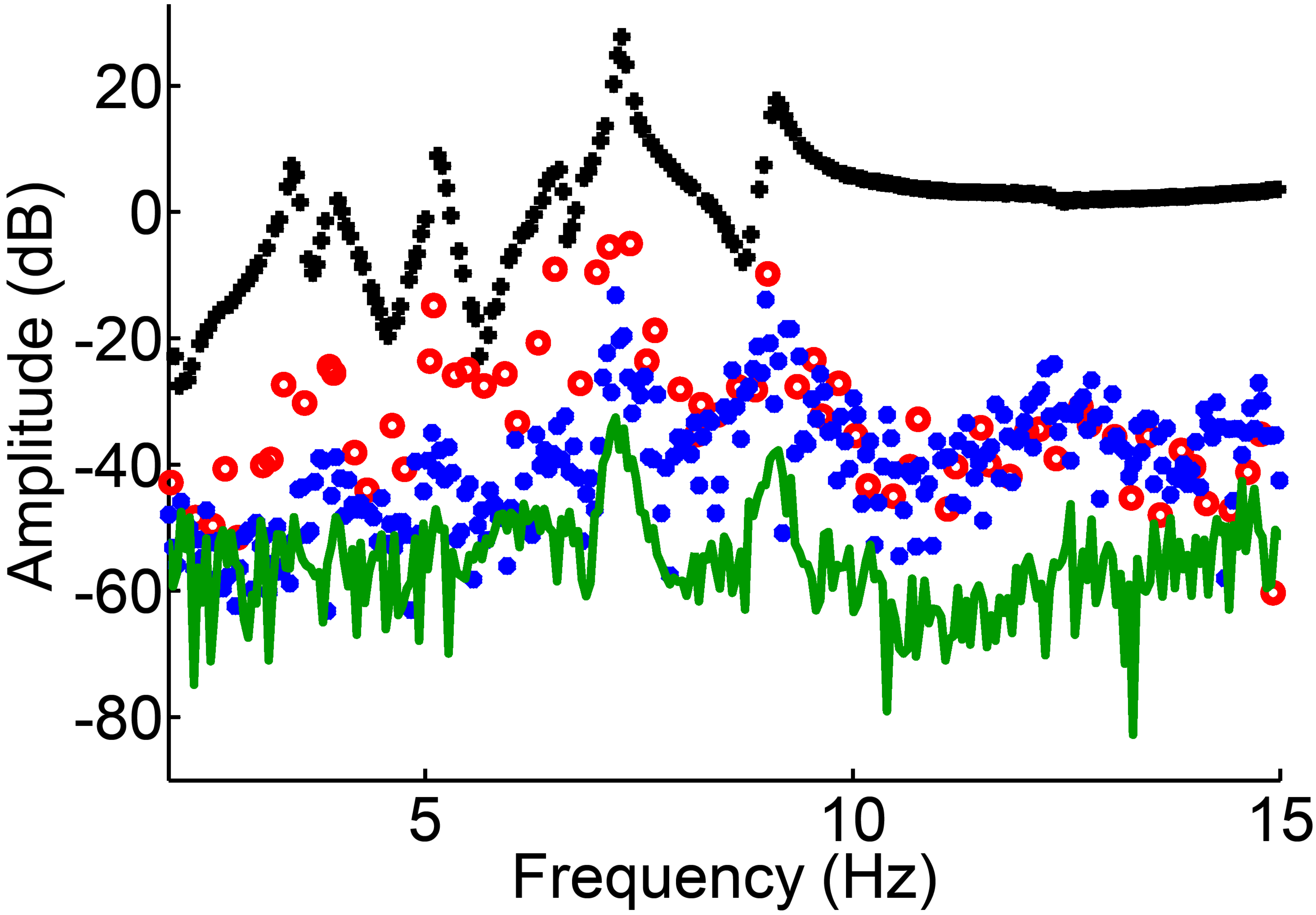
Ground vibration test on an F16-fighter







Ground vibration test on an F16-fighter



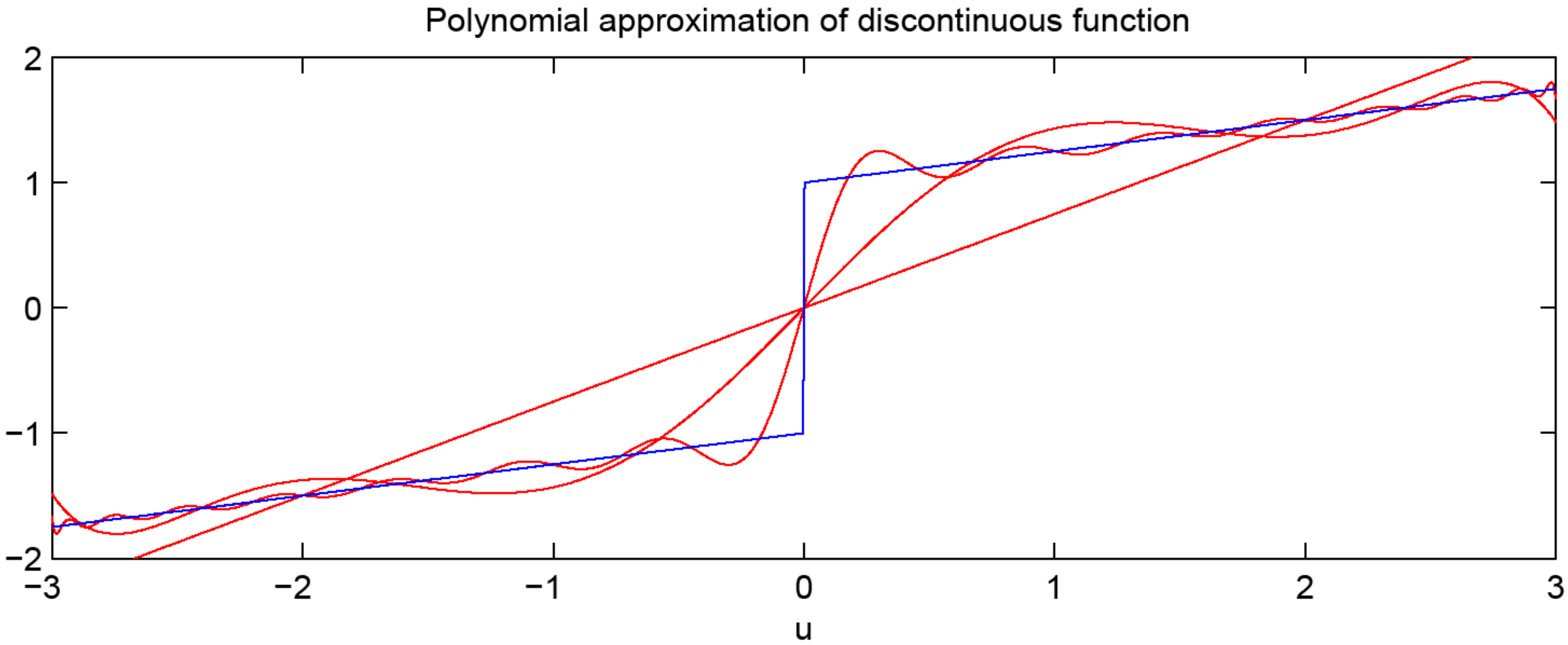
Approximation of nonlinear systems

User choices

- convergence criterion
- approximation method
- excitation

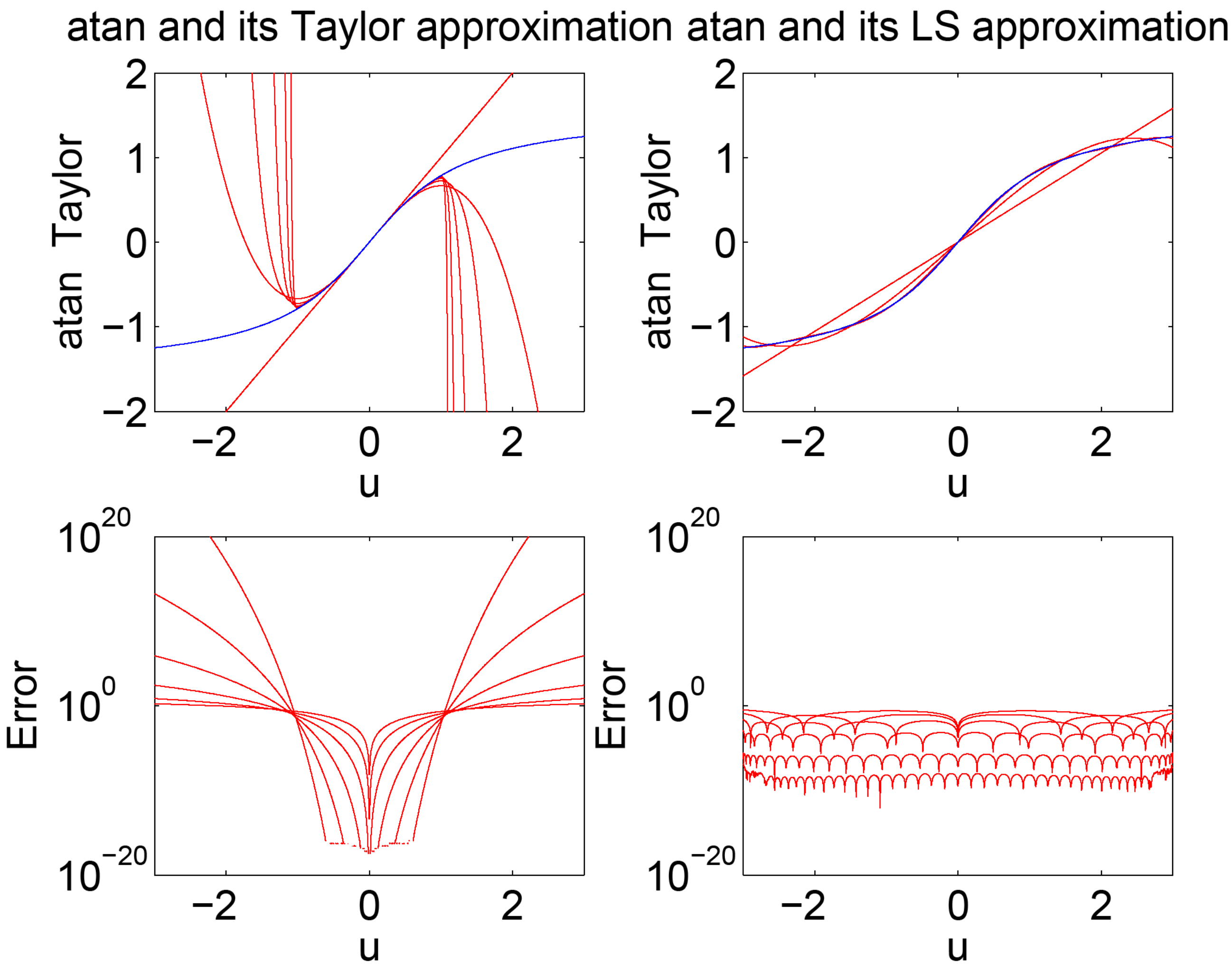
Approximation of nonlinear systems

convergence criterion



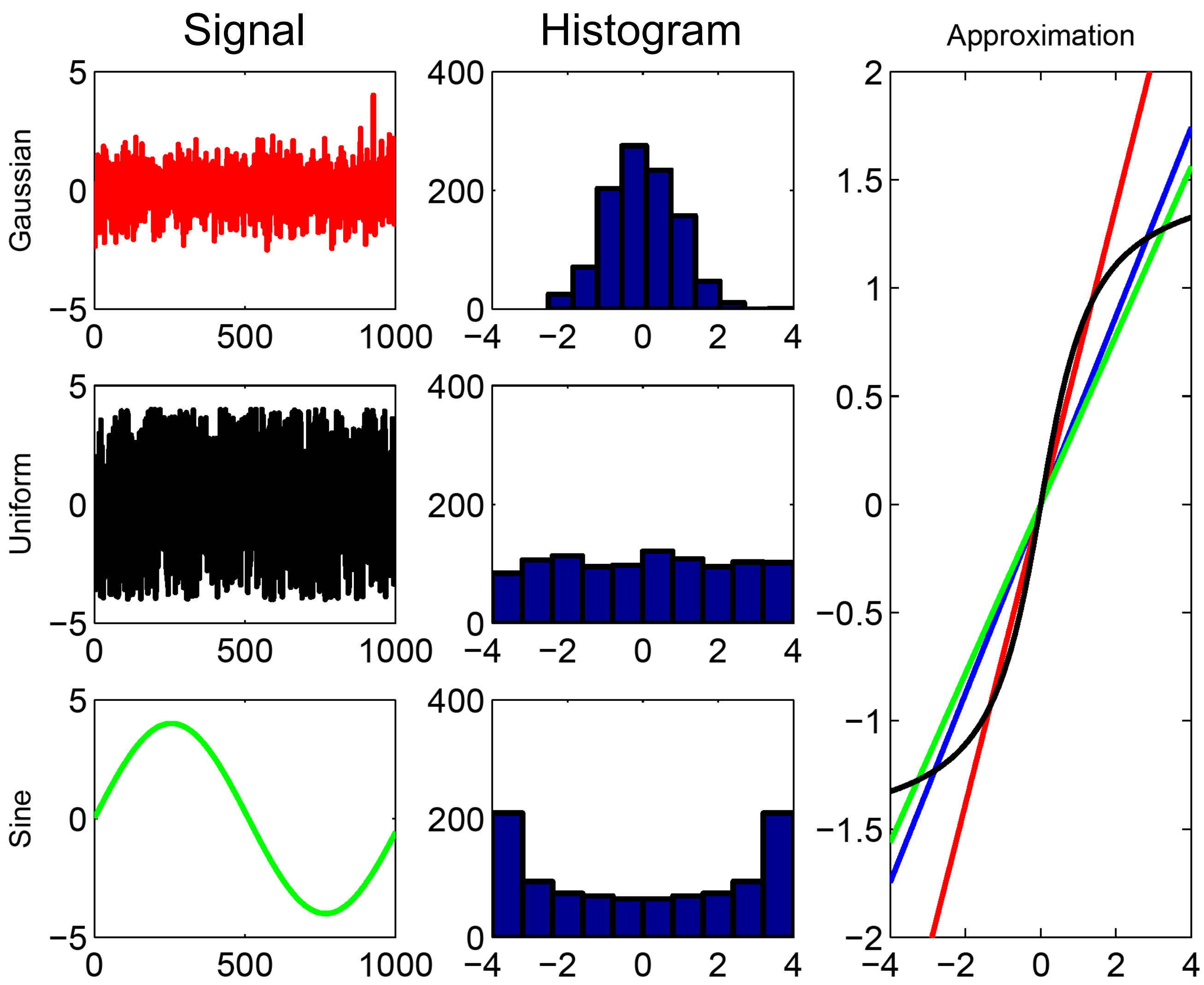
uniform convergence >< point wise convergence

Approximation of nonlinear systems
Approximation method

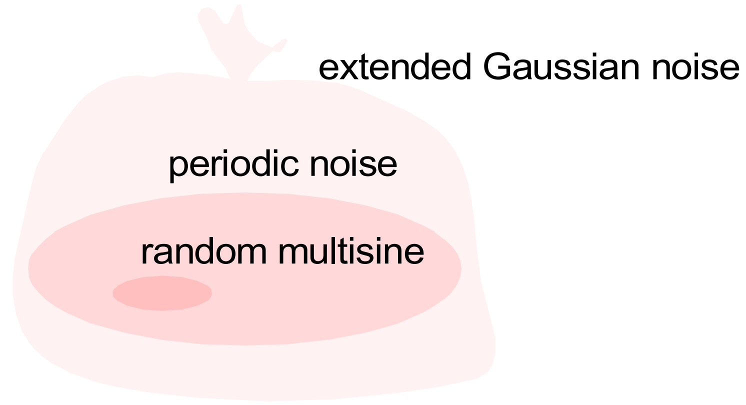


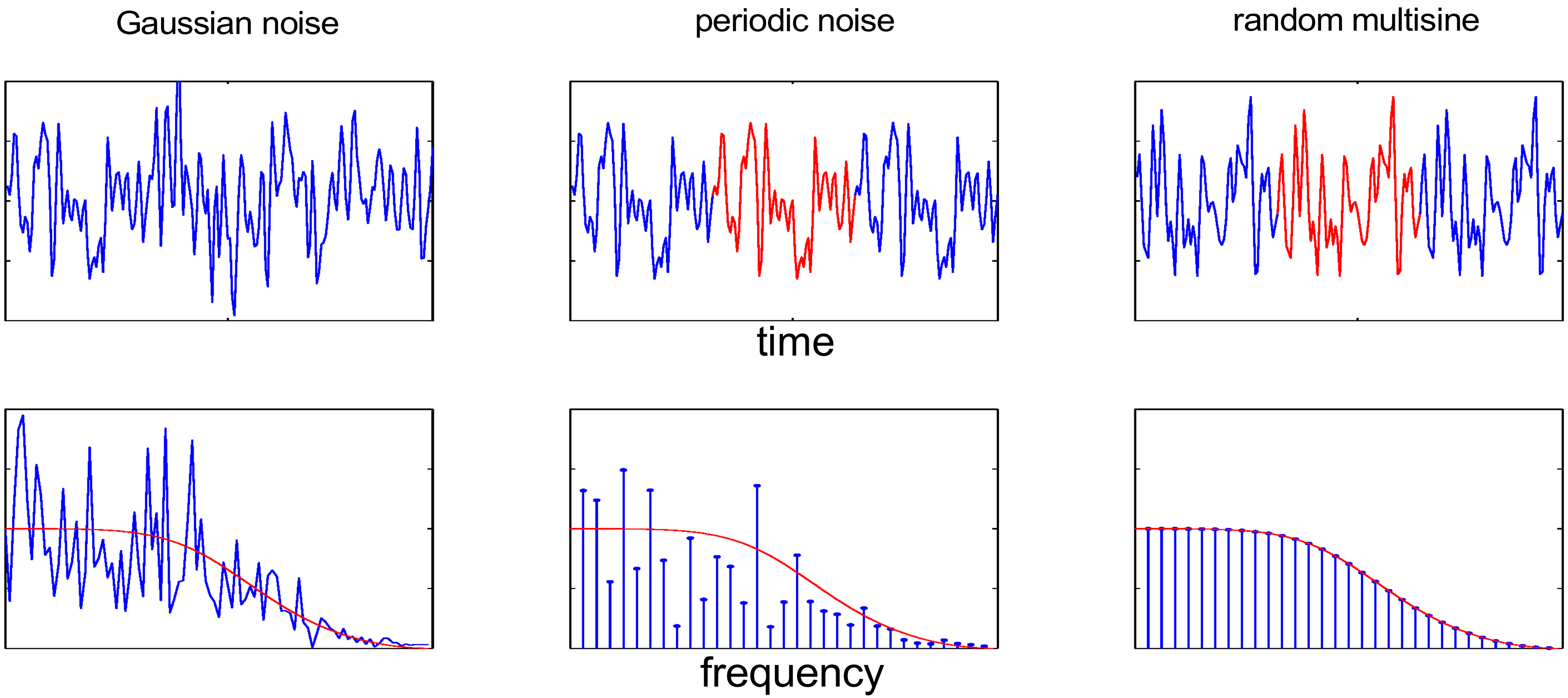
Taylor >< Least Squares

Approximation of nonlinear systems
Excitation



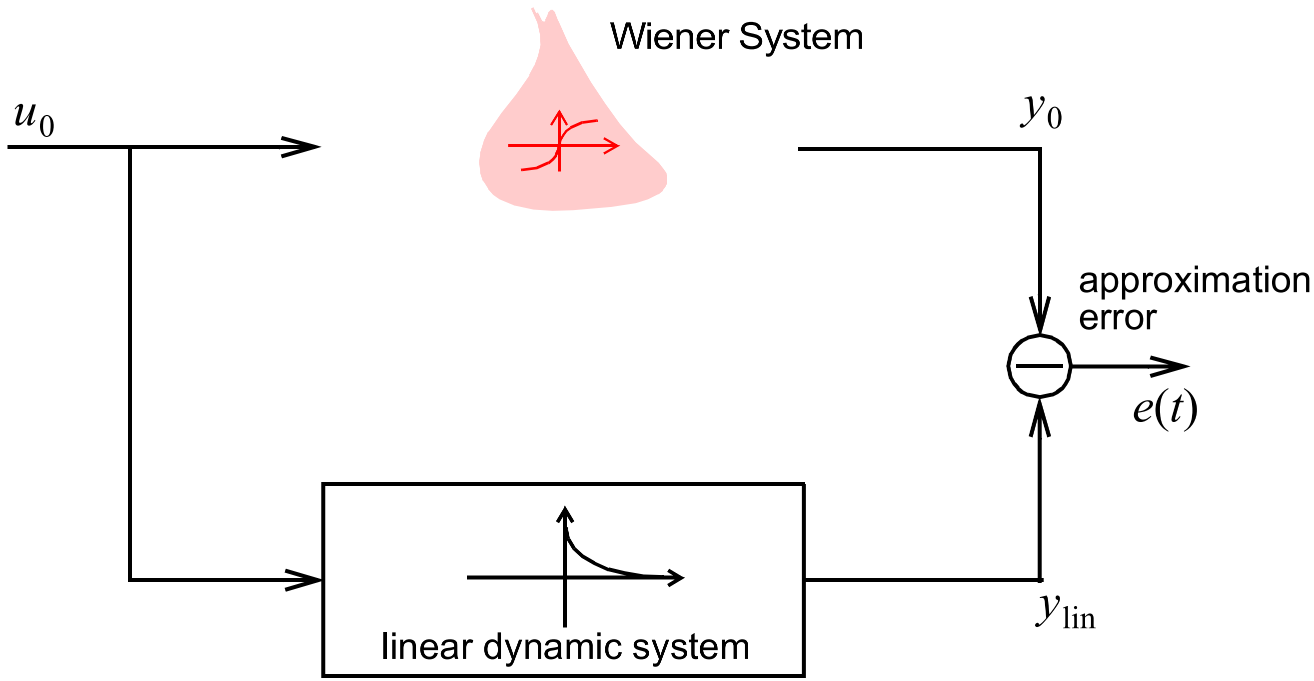
Class of excitation signals





$$u(t) = \frac{1}{F}\sum_{k=1}^{F} A_k \cos(2\pi k f_0 t + \varphi_k)$$

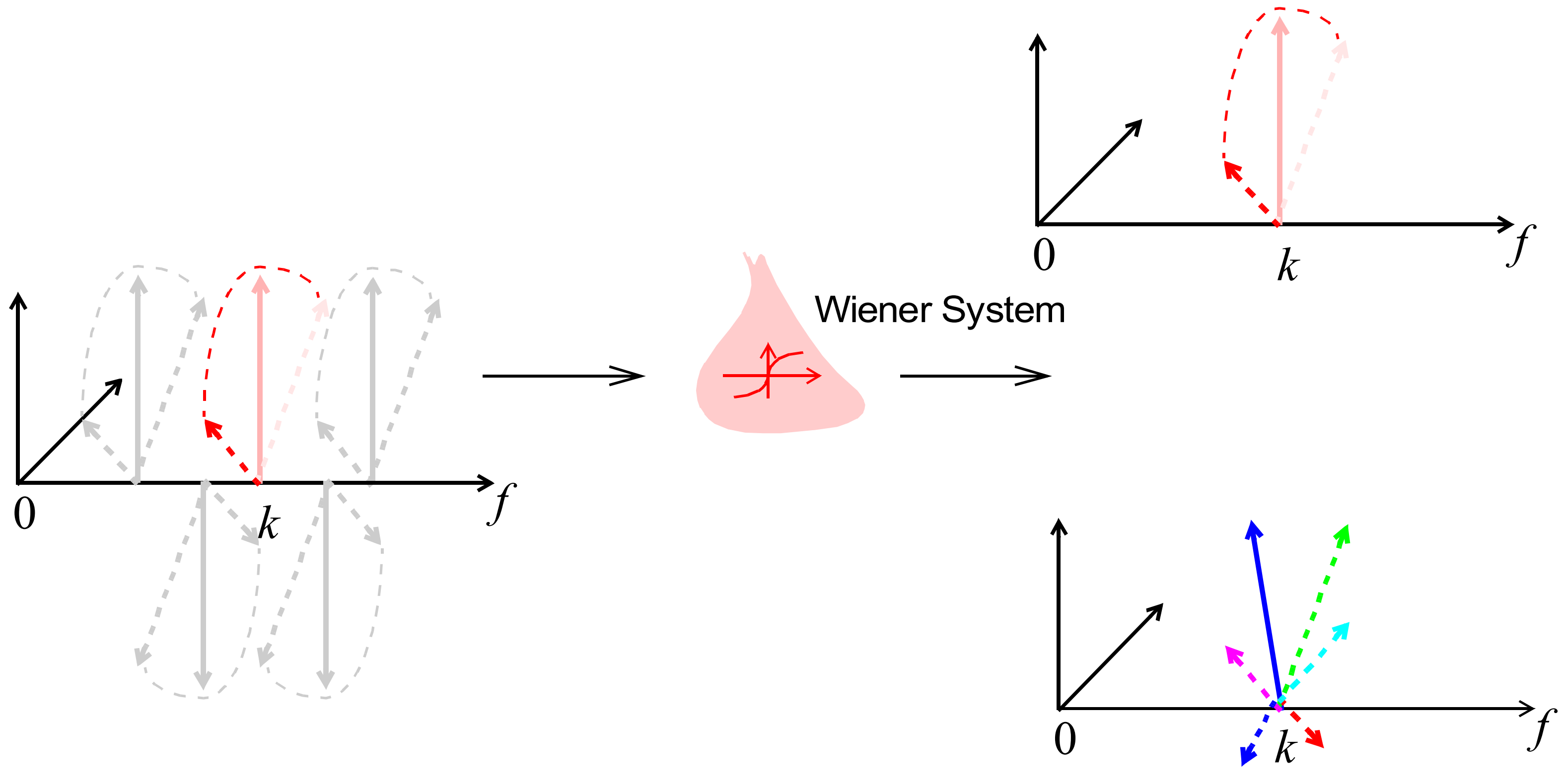
Approximation of a nonlinear system by a linear system



$$G_{BLA} = \arg\min_{G} E_U\{|Y - GU|^2\}$$

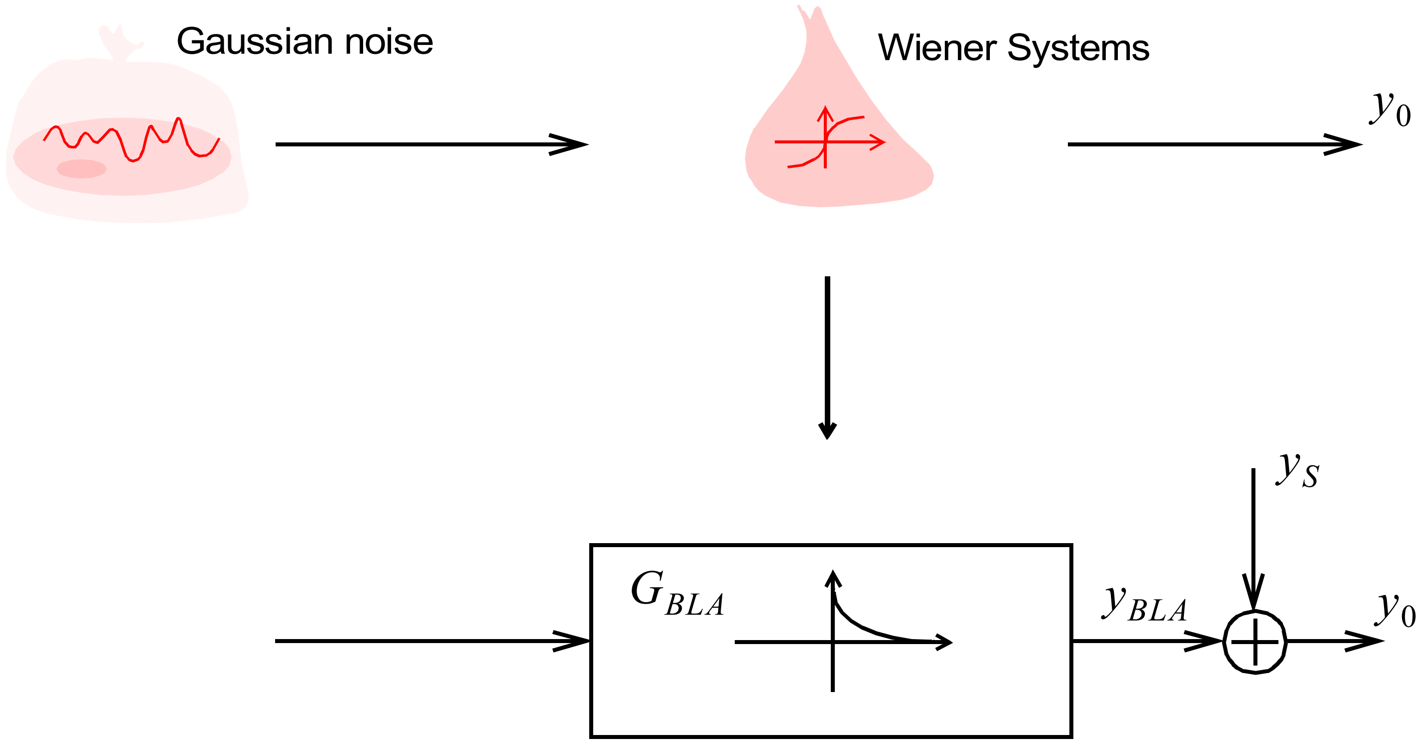
Behaviour of a nonlinear system

coherent $\angle Y(k) = \angle U(k) + \varphi_0(k)$



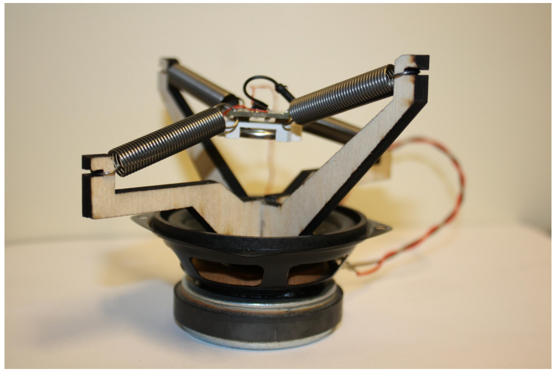
non coherent

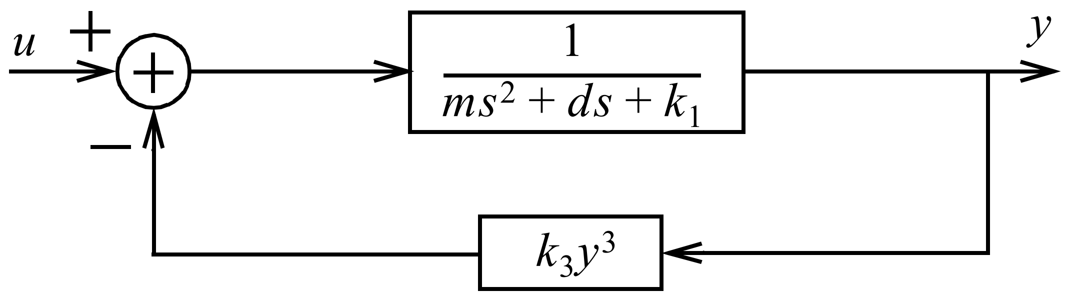
A new paradigm

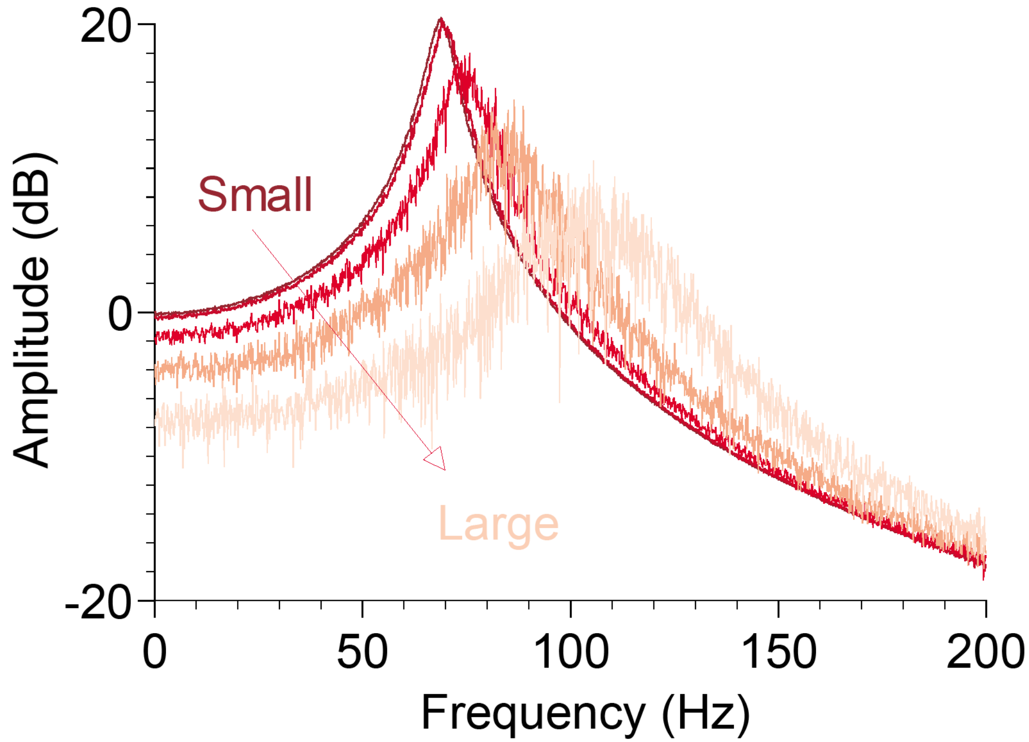


$$Y(k) = G_{BLA}(j\omega_k)U(k) + Y_S(k)$$

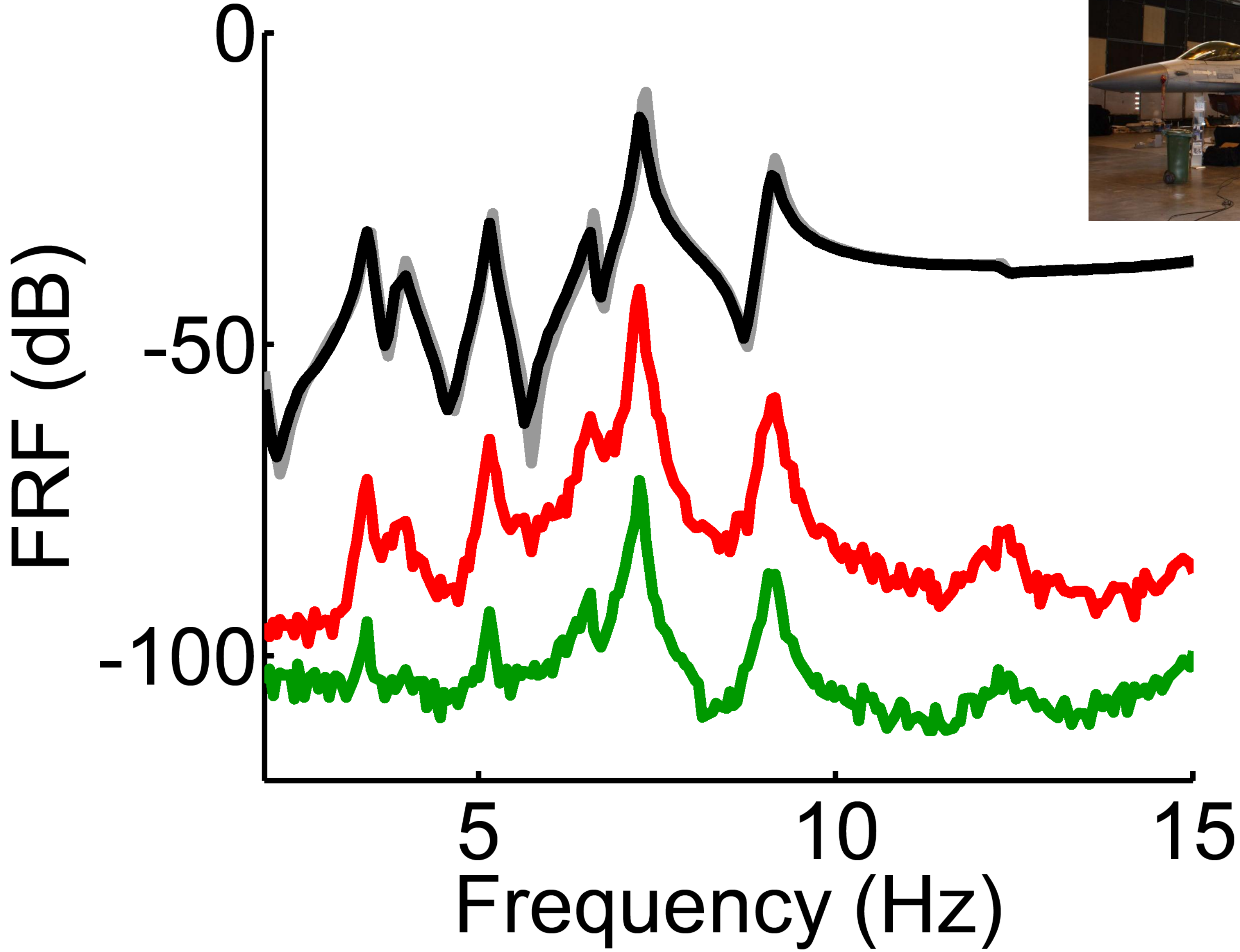
Example : hardening spring



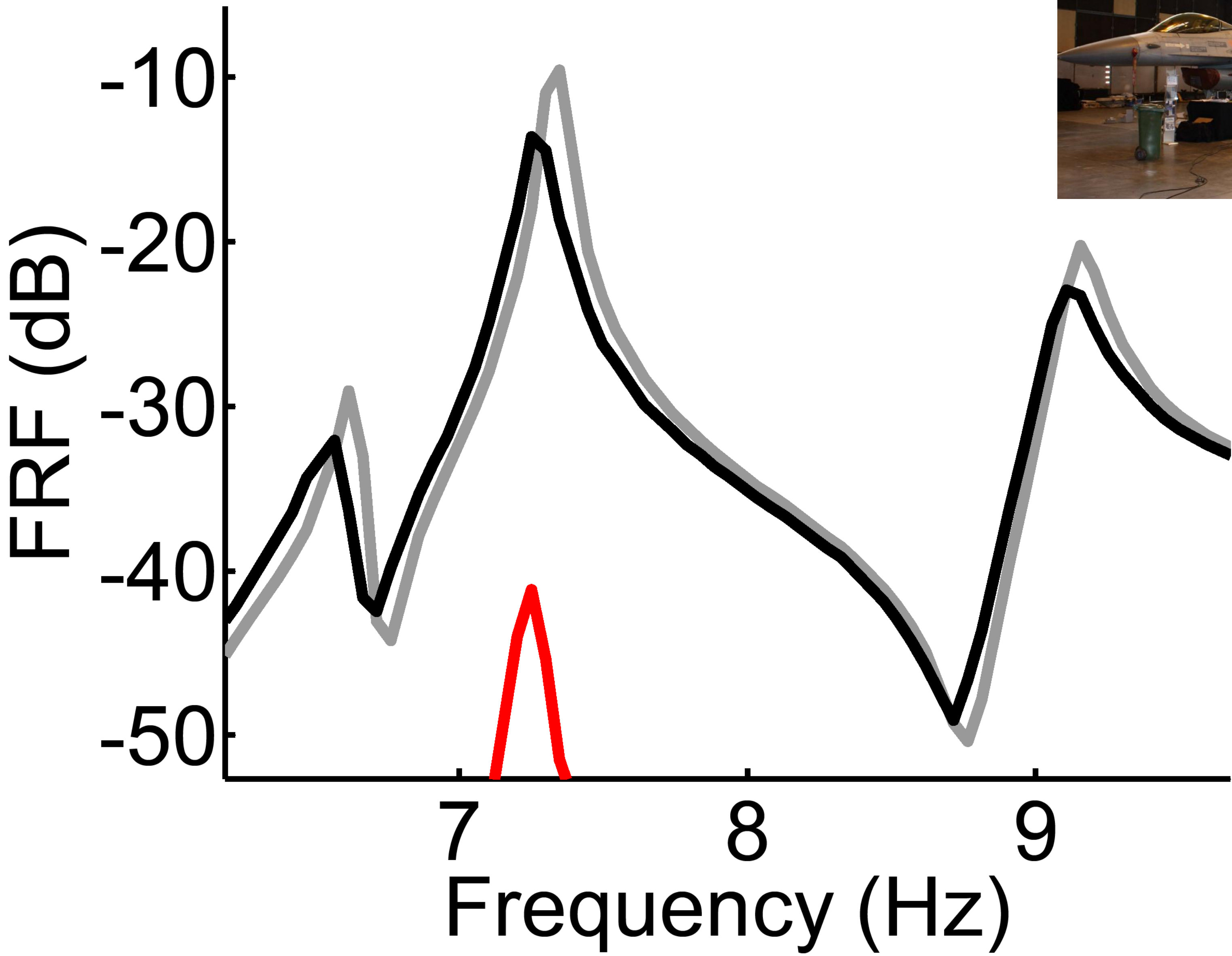




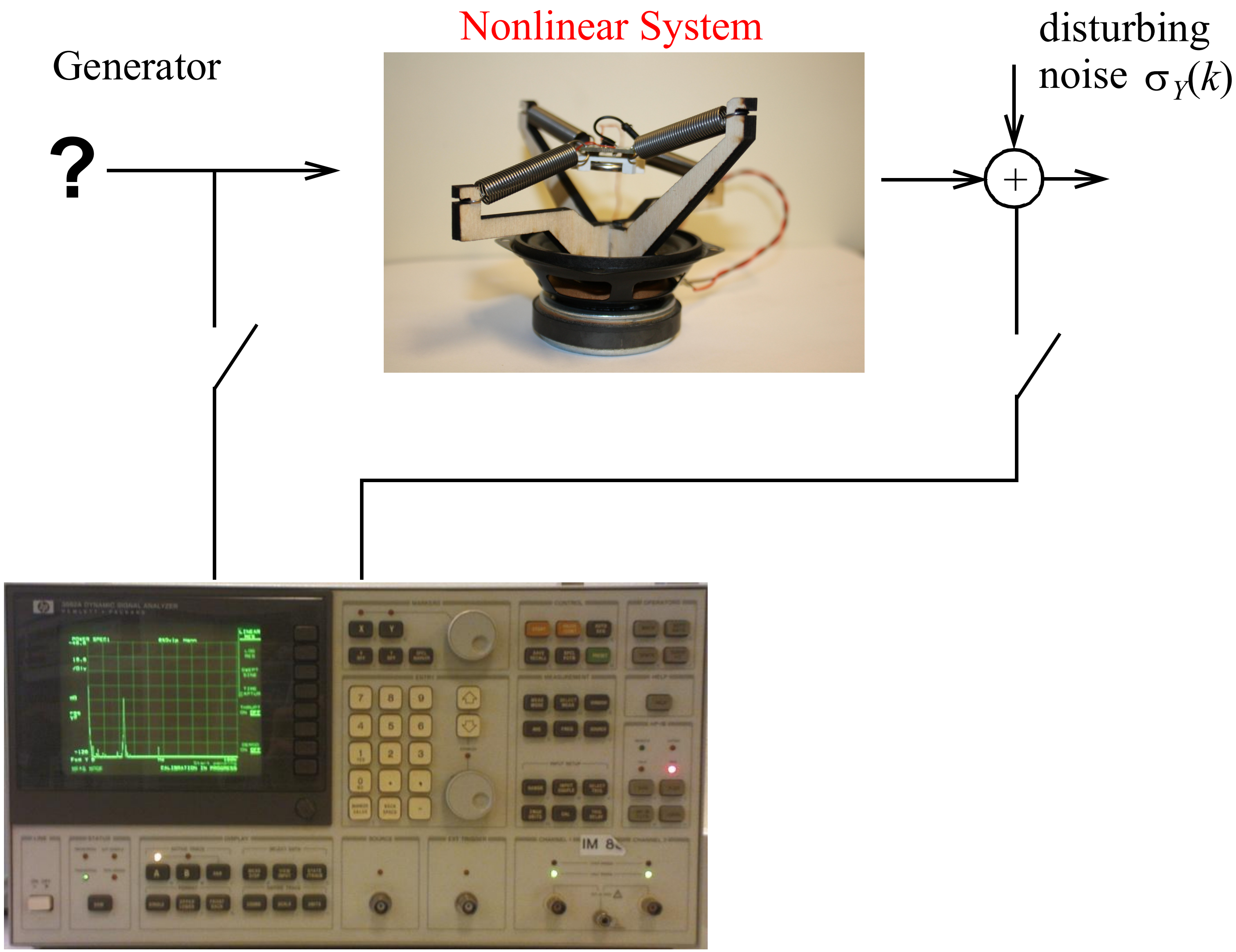
Example: F16-fighter measurements



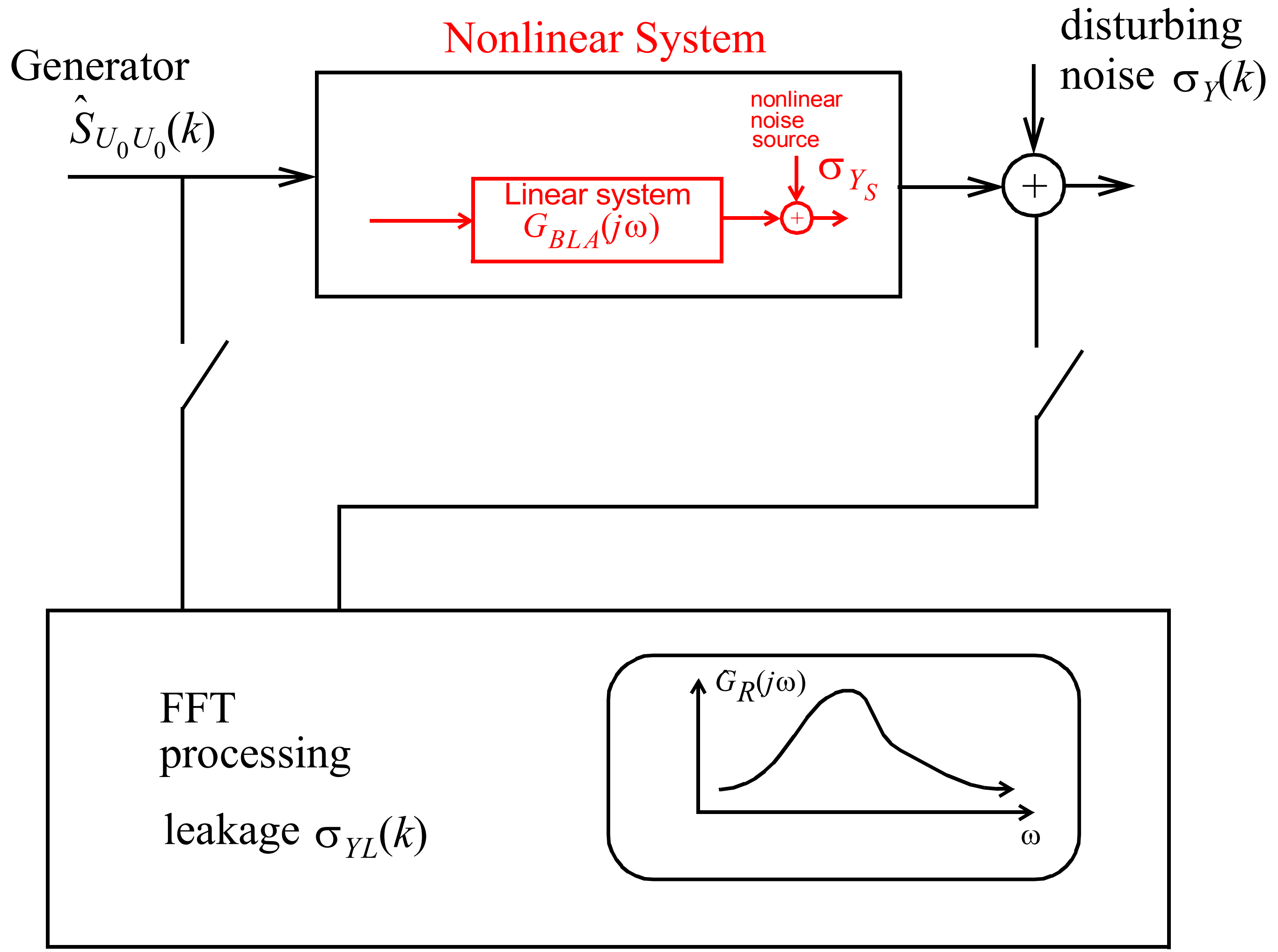
Example: zoom F16-fighter measurements



FRF-measurements in the presence of NL-distortions



FRF-measurements in the presence of NL-distortions

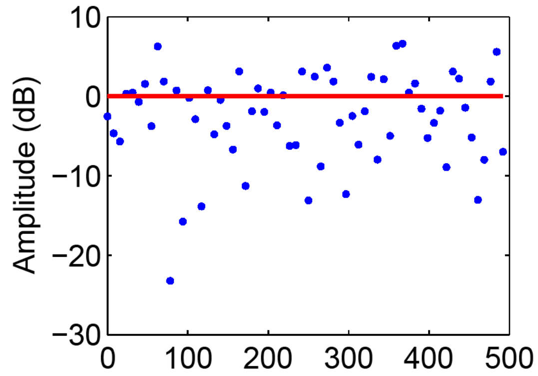


$$\sigma^2_{G_{BLA}}(k) = \frac{\sigma^2_{YL}(k) + \sigma^2_{Y_S}(k) + \sigma^2_{Y}(k)}{\hat{S}_{U_0U_0}(k)}$$

FRF-measurements in the presence of NL-distortions (Cont'd)

$$\sigma^2_{G_{BLA}}(k) = \frac{\sigma^2_{YL}(k) + \sigma^2_{Y_S}(k) + \sigma^2_{Y}(k)}{\hat{S}_{U_0U_0}(k)}$$

Avoid dips in $\hat{S}_{U_0U_0}(k)$



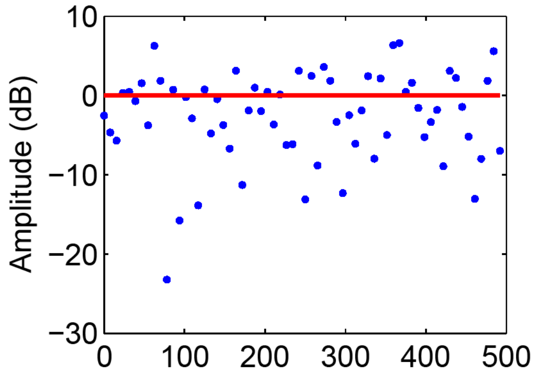
deterministic signals >< noise

FRF-measurements in the presence of NL-distortions (Cont'd)

$$\sigma^2_{G_{BLA}}(k) = \frac{\sigma^2_{\hat{Y}L}(k) + \sigma^2_{\hat{Y}_S}(k) + \sigma^2_{\hat{Y}}(k)}{\hat{S}_{U_0U_0}(k)}$$

Avoid dips in $\hat{S}_{U_0U_0}(k)$

deterministic signals >< noise



Reduction of the leakage errors $\sigma^2_{\hat{Y}L}$

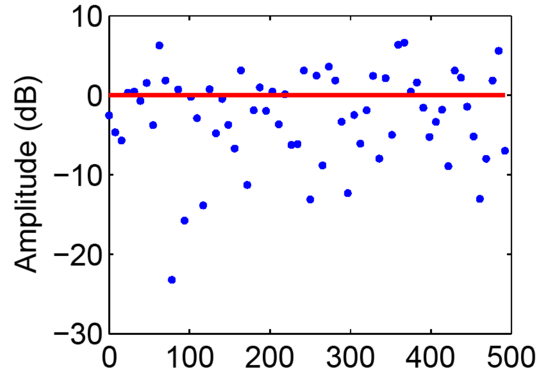
periodic signals

FRF-measurements in the presence of NL-distortions (Cont'd)

$$\sigma^2_{G_{BLA}}(k) = \frac{\sigma^2_{YL}(k) + \sigma^2_{Y_S}(k) + \sigma^2_{Y}(k)}{\hat{S}_{U_0U_0}(k)}$$

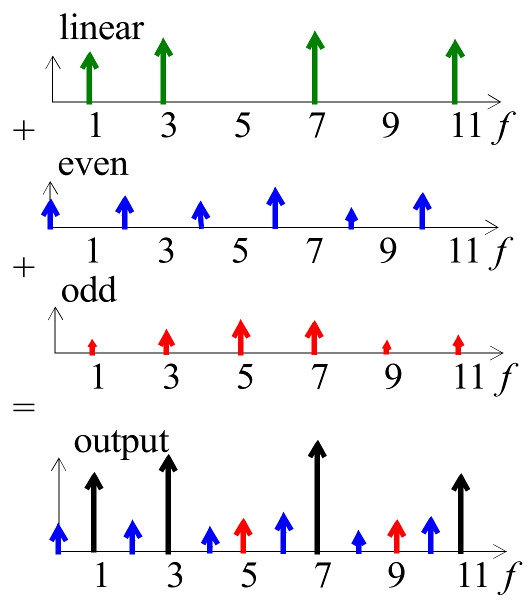
Avoid dips in $\hat{S}_{U_0U_0}(k)$

deterministic signals >< noise



Reduction of the leakage errors σ^2_{YL}

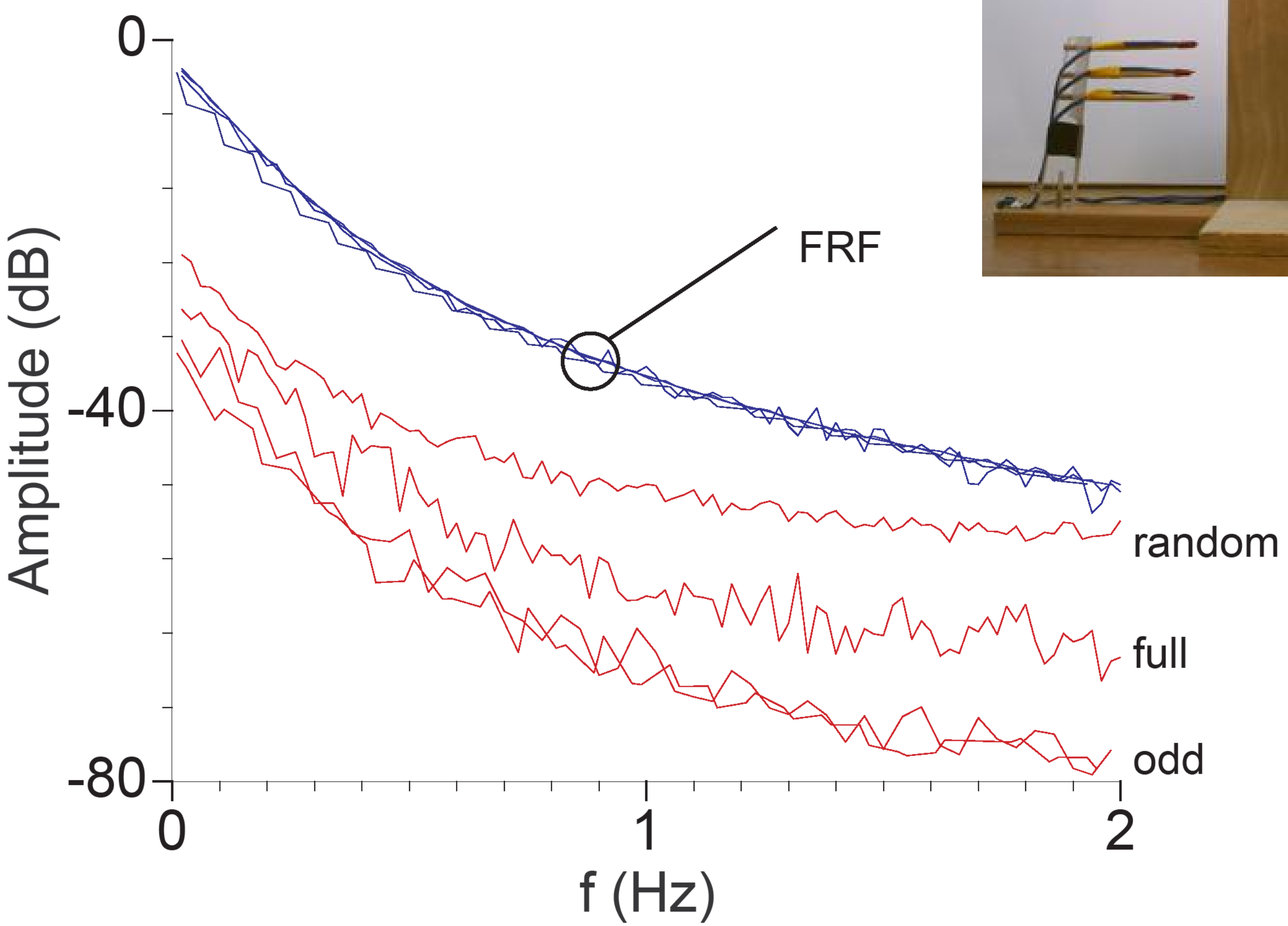
periodic signals

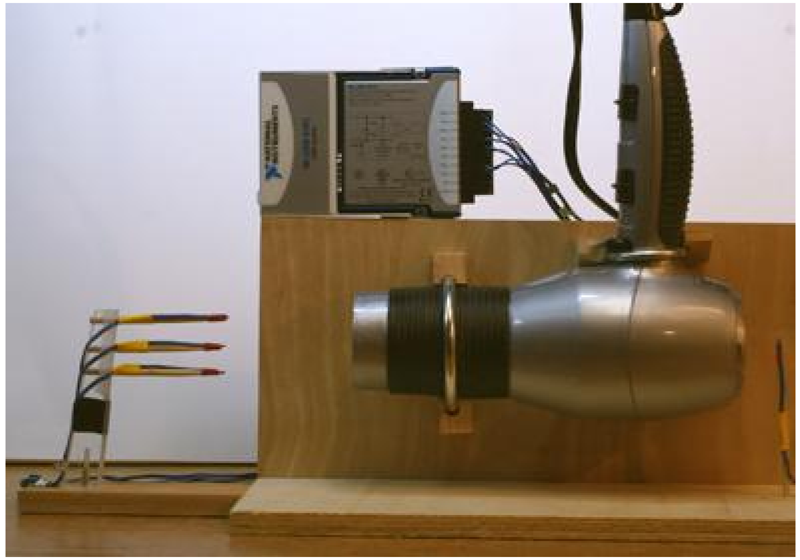


Reduction of the impact of nonlinear distortions $\sigma^2_{Y_S}$

Odd excitations

Hair dryer experiment





Best Linear Approximation : Parametric modelling

$$G_{BLA}(j\omega, \theta)$$

Linear identification framework

Consistent estimate

True model retrieved for large data sets

Uncertainty bounds are wrong

Nonlinear induced variance underestimated by factor 7 or more

Conclusions

- Intuitive solutions?

- Linear system identification

- Impact nonlinear distortions on the linear framework

Conclusions

- Intuitive solutions? --> Dangerous

- Linear system identification

- Impact nonlinear distortions on the linear framework

Conclusions

- Intuitive solutions? --> Dangerous

- Linear system identification --> A versatile tool

- Impact nonlinear distortions on the linear framework

Conclusions

- Intuitive solutions? --> Dangerous

- Linear system identification --> A versatile tool

- Impact nonlinear distortions on the linear framework

  --> Bring SI to the real world