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**Abstract** Biological systems display complex dynamics emerging from intricate networks of interacting molecular components: cells use signalling pathways and regulatory control mechanisms to coordinate multiple processes, allowing them to respond and adapt to an ever-changing environment. Many structural and dynamical features of biological control systems can also be found in engineered control systems and, hence, feedback control theory can provide a useful approach for the analysis and design of complex biological systems. In this chapter we provide a control theoretic analysis of the osmoregulation system in *Saccharomyces cerevisiae* (see [8, 24, 26, 40]), where a complex biochemical signalling and regulatory network allows cells to maintain homeostasis in the face of osmotic shock.

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**Keywords (separated by '-')** Osmoregulation - Signalling pathway - Signaling pathway - Homeostasis - Osmosis - Osmoadaptation - Eukaryotic - Yeast - High osmolarity glycerol (HOG) - Integral feedback

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## Chapter 4

# Modelling and Analysis of Feedback Control Mechanisms Underlying Osmoregulation in Yeast

Francesco Montefusco, Ozgur E. Akman, Orkun S. Soyer  
and Declan G. Bates

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## 14 4.1 Introduction

15 Osmosis is the diffusion of water through a semipermeable membrane (permeable to  
 16 the solvent, but not the solute), from the compartment containing a low concentra-  
 17 tion (hypotonic) solution to the one at high concentration (hypertonic). The chemical  
 18 potential of water is central in this process and can be considered as a measure of  
 19 the effective water concentration in a given area. The water potential is influenced  
 20 by two factors, [17]: the osmotic potential and the pressure potential. The first is  
 21 approximately proportional to the concentration of dissolved molecules of solutes:  
 22 when the concentration of solute molecules increases, the water potential decreases.  
 23 The second takes into account the hydrostatic pressure, the pressure exerted by a fluid  
 24 at equilibrium due to the force of gravity. For two regions of water with different  
 25 potentials and separated from each other by a semipermeable membrane, there is a  
 26 water flow to the region of lower potential by osmosis: the movement of the fluid  
 27 from the hypotonic to the hypertonic solution, while decreasing the concentration  
 28 difference, increases the pressure of the hypertonic solution with respect to the hypo-  
 29 tonic, thus producing a force that counteracts the osmosis. When these two effects  
 30 balance each other, the osmotic equilibrium is reached: there is no net movement of  
 31 solvent and the pressure required to maintain an equilibrium is defined as the osmotic  
 32 pressure.

33 Osmosis is particularly important for cells, since many biological membranes  
 34 are permeable to small molecules like water, but impermeable to larger molecules  
 35 and ions. Osmosis provides the primary means by which water is transported into  
 36 and out of cells. Typically, a cell has a higher intra cellular osmotic pressure ( $P_i$ )  
 37 than extra cellular osmotic pressure ( $P_e$ ). The main reason for this difference is  
 38 that highly charged macromolecules and metabolites attract many small inorganic  
 39 ions to the cell interior (the Donnan effect, see [1]). Due to this difference, water  
 40 will flow into the cell, leading to swelling and potentially to cell rupture. The yeast  
 41 *Saccharomyces cerevisiae* prevents the fundamental problem of water inflow and cell  
 42 swelling by its cell wall, which is less elastic than the plasma membrane. The cell wall  
 43 resists the expansion of the cell and creates an inward pressure on the cell contents,  
 44 Gervais and Beney [9]. This pressure is called the turgor pressure  $P_t$ , defined as the  
 45 difference in the hydrostatic pressure between the inside and the outside of the cell.  
 46 At equilibrium (equil.), the water potential is equal inside and outside of the cell and  
 47 the turgor pressure balances the difference in osmotic pressures, as in [33],

$$48 \quad P_i = P_e + P_t \quad (\text{equil.}). \quad (4.1)$$

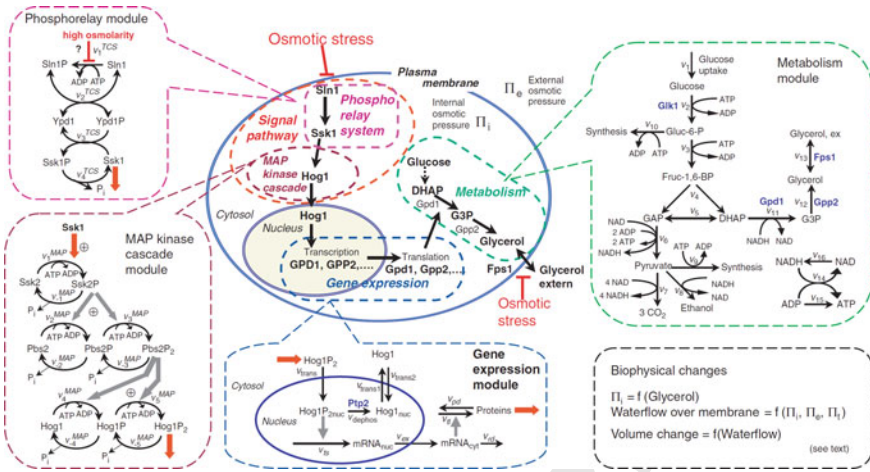
49 Osmotic shocks arise due to a sudden rise (for example the addition of salt to the  
 50 cell medium) or fall in the concentration of a solute in the cell's environment, resulting  
 51 in rapid movements of water through the cell's membrane. These movements can  
 52 produce dramatic consequences for the cell, since loss of water inhibits the transport  
 53 of substrates and cofactors into the cell, while the uptake of large quantities of water  
 54 can lead to swelling, rupture of the cell membrane or apoptosis. Due to their more

55 direct contact with their environment, single-celled organisms are generally more  
56 vulnerable to osmotic shock. However, cells in large animals such as mammals also  
57 suffer similar stresses under certain conditions, Ho [12].

58 Osmoadaptation is the mechanism by which cells sense and respond to various  
59 changes in their environmental conditions to avoid the aforementioned dramatic con-  
60 sequences. Organisms have evolved a variety of mechanisms to respond to osmotic  
61 shock. Typically, cells recognise changes in the osmolarity of their surroundings  
62 by using surface sensors which generate signals by activating signal transduction  
63 networks. These pathway are found in all eukaryotic organisms and are important  
64 in coordinating the response from the cell membrane into the cell, Rep et al. [30].  
65 Recent experimental research indicates that most eukaryotic cells use the mitogen  
66 activated protein (MAP) kinase pathways for this purpose, Klitz and Burg [16].

## 67 4.2 Osmoregulation Process in Yeast

68 In recent years, the osmoregulatory response in yeast has emerged as an important  
69 model system for studying adaptive, homeostatic responses to environmental distur-  
70 bances (see [8, 15]). The underlying molecular control system is well characterized  
71 in *Saccharomyces cerevisiae* (see [26, 40]), where it comprises three separate mech-  
72 anisms that act to adjust the glycerol production in order to keep the cell's turgor  
73 pressure and volume constant in the face of environmental changes: (1) the regu-  
74 lation of the membrane protein Fps1 determining the glycerol export rate; (2) the  
75 transcription of several genes, whose proteins are involved in glycerol production,  
76 by the activation of the high osmolarity glycerol (HOG) mitogen-activated protein  
77 kinase (MAPK) signaling pathway and (3) the HOG kinase dependent regulation of  
78 the glycerol via non-transcriptional mechanisms. Despite its biochemical complex-  
79 ity (see Fig. 4.1), the osmoregulation system in yeast can be naturally abstracted  
80 as a feedback control system comprised of distinct branches as described above.  
81 This approach was taken in recent studies, which aimed to use standard engineering  
82 control models to capture the experimentally observed responses of yeast to osmotic  
83 shock and to further predict its structural and dynamic features (see [8, 24, 26]). Gen-  
84 nemark et al. [8] combined proportional controllers to model the above-described  
85 biochemical branches. Mettetal et al. [24] developed a concise model by using linear  
86 systems theory, and then revised this model arguing for the necessity of at least one  
87 branch of the system to implement integral control to achieve the experimentally  
88 observed adaptive responses in the system, Muzzey et al. [26]. The role of integral  
89 feedback in perfectly adaptive systems is by now well-studied in the Systems Biology  
90 literature (see [26, 27]), and it is highly likely that the osmoregulation system in yeast  
91 does indeed include a biochemical implementation of integral feedback, as seen in  
92 other systems (see [6, 27, 39]). It is still unclear, however, exactly how biological  
93 control systems such as osmoregulation might have evolved to use integral feedback  
94 control, and whether other alternative mechanisms might produce similar (or better)  
95 performance properties.



**Fig. 4.1** Overview of the response of yeast to osmotic shock, the figure has been taken from [15] and reproduced with permission of Nature Publishing Group

96 As a first step towards answering this question, we recently extended the propor-  
 97 tional controller model devised in [8] with the implementation of an ultrasensitive  
 98 controller, Montefusco et al. [25]. Ultrasensitivity describes a particular form of  
 99 sensitivity in biological systems, where the system does not respond to incoming  
 100 signals outside of a certain regime, but responds in a highly sensitive manner within  
 101 this regime. Such an input-output relationship (i.e. ultrasensitivity) can be described  
 102 by a specific nonlinear function, is shown to be a ubiquitous feature in several biolog-  
 103 ical systems, and can be biochemically implemented through a variety of mechanisms  
 104 such as phosphorylation cycles and cooperative binding (see [4, 11]). The MAPK  
 105 systems, which are also found in osmoregulation, are theoretically shown to be capa-  
 106 ble of embedding ultrasensitivity (see [3, 13]), and bistability [21]. Starting from the  
 107 proportional control model developed by [8], we explore the consequences of such  
 108 potential ultrasensitivity and show that it significantly increases system performance  
 109 in achieving homeostasis to osmotic perturbations.

110 In the following sections we present the model devised in [8], then we focus  
 111 our attention on the results presented in [24, 26]. Finally, we provide an updated  
 112 description of the recent results first presented in [25].

### 113 4.3 A Proportional Control Based Model 114 of the Osmoregulation in Yeast

115 In this section we describe the model presented in [8], where the authors devised a  
 116 simple ordinary differential equation (ODE) model of the adaptive response to an  
 117 osmotic shock in *S. cerevisiae*. They abstracted several elements to yield a reduced

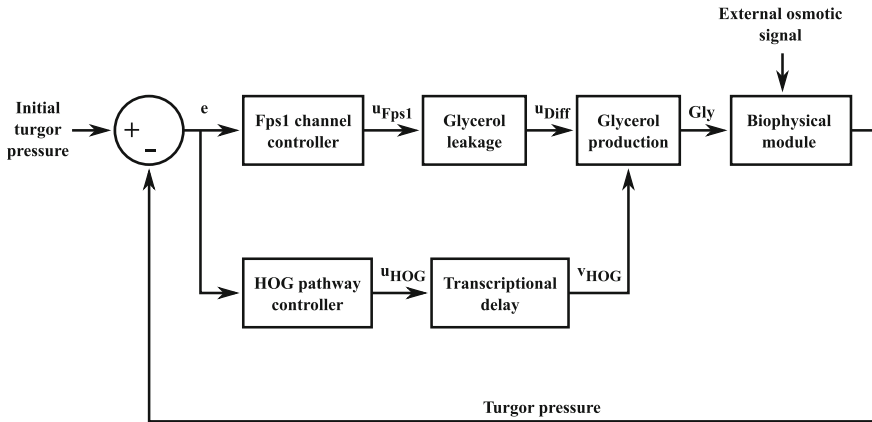


Fig. 4.2 Mathematical model of the osmoregulation process. This figure has been adapted from [8]

118 representation of the system, starting from the detailed model developed in [15] (see  
 119 the diagram in Fig. 4.2). The model, in particular, consists of two main components.  
 120 First, a biophysical model describing how the cell volume and the turgor pressure are  
 121 affected by varying extra-cellular osmolarity. Second, the two parallel mechanisms  
 122 for controlling the biophysical system in order to keep turgor pressure and volume  
 123 constant: one by controlling the production of glycerol via the HOG pathway and  
 124 the other by controlling the outflow of glycerol via the Fps1 channel. The complete  
 125 model consists of 4 ODE's, 3 algebraic equations and 10 parameters, that have been  
 126 estimated using experimental data on glycerol. The authors have validated the model  
 127 by predicting the behaviour of modified strains and input functions.

### 128 4.3.1 The Mathematical Model

129 The mathematical model presented in [8] is described in the following paragraphs.

#### 130 4.3.1.1 The Biophysical Module

131 The biophysical system is modelled by considering the dependencies between cell  
 132 volume  $V$ , the turgor pressure  $P_t$ , the intra-cellular osmotic pressure  $P_i$  and the  
 133 extra-cellular osmotic pressure  $P_e$ . At any given time  $t$ ,  $P_i(t)$ ,  $P_e(t)$  and  $P_t(t)$  are  
 134 determining the flow of water across the cell membrane, which is proportional to  
 135  $(P_i(t) - P_e(t) - P_t(t))$ . Assuming that the cell volume is only affected by the inflow  
 136 and outflow of water, then the change in volume can be expressed as

$$137 \quad \frac{dV}{dt} = k_{p1}(P_i(t) - P_e(t) - P_t(t)), \quad (4.2)$$

with  $k_{p1}$  denoting a hydraulic water permeability constant. At equilibrium (equil.), i.e. constant volume and no net flow of water over the membrane, the Eq. (4.2) reduces to (4.1).

The only osmolyte considered explicitly in the model is glycerol (*Gly*) and, hence, ions and other small molecules, changing upon osmotic shock, Sunder et al. [35], are not considered. This assumption is motivated by experimental results from [29], where the authors found that glycerol counter-balances approximately 80% of applied NaCl in *S. cerevisiae*. Therefore, the intra-cellular osmotic pressure, according to van't Hoff's law, is expressed as

$$P_i(t) = \frac{s + Gly(t)}{V(t) - V_b}, \quad (4.3)$$

with  $s$  being the concentration of the sum of osmolytes (assumed constant) other than glycerol present in the cell, and  $V_b$  being the non-osmotic volume of the cell, subsuming non-polar cellular components, such as membranes. According to Eq. (4.3), the intra-cellular osmotic pressure increases with the glycerol, which can be used to control the turgor pressure of the cell. The extra-cellular osmotic pressure is only modified by the input signal, for example applied salt stress, and is then independent of changes in other variables. The turgor pressure is linearly dependent on the volume according to [17], in the following manner:

$$P_t(t) = \varepsilon \left( \frac{V(t)}{V(0)} - 1 \right) + P_t(0), \quad (4.4)$$

where  $V(0)$  is the initial volume,  $P_t(0)$  is the initial turgor pressure, and  $\varepsilon$  is the volumetric elastic modulus. By expressing the volume at which  $P_t = 0$  with the notation  $V^{P_t=0}$ , (4.4) can be rewritten as

$$P_t(t) = \begin{cases} P_t(0) \frac{V(t) - V^{P_t=0}}{V(0) - V^{P_t=0}}, & V(t) > V^{P_t=0} \\ 0, & \text{otherwise.} \end{cases}$$

#### 4.3.1.2 The Controller Modules

There are two branches of control in the model: the first represents the closure of Fps1 glycerol transporter channels as a reaction to osmotic shock, and the second the activation of the HOG pathway, leading to glycerol production after a time delay. The input signal  $e$  arriving at the controllers is expressed as

$$e(t) = P_t(0) - P_t(t), \quad (4.5)$$

which is the difference in turgor pressure. The output of the Fps1 branch, which corresponds to the response of the transporter channels, is given by



$$u_{Fps1}(t) = \begin{cases} k_{p2} \frac{P_t(0) - e(t)}{P_t(0)}, & e(t) > 0 \\ k_{p2}, & \text{otherwise.} \end{cases} \quad (4.6)$$

The function  $u_{Fps1}$  returns real values in the interval  $[0, k_{p2}]$ , where 0 corresponds to completely closed and where  $k_{p2}$  is the glycerol permeability coefficient in a completely open Fps1 channel.

The output of the HOG branch, which corresponds to the HOG pathway dependent glycerol production, is expressed as

$$u_{HOG}(t) = \begin{cases} k_{HOG} \cdot e, & e(t) > 0 \\ 0, & \text{otherwise,} \end{cases} \quad (4.7)$$

where  $k_{HOG}$  is the gain of this branch.

The time delay accounting for transcription and translation in the HOG pathway is approximated by

$$\frac{d\tilde{u}_{HOG}}{dt} = \frac{1}{T_d}(u_{HOG}(t) - \tilde{u}_{HOG}(t)),$$

with  $\tilde{u}_{HOG}(t)$  being the time delayed variable and  $T_d$  being the amount of time delay considered. As reported in [8], very simple proportional controllers have been used in order to reduce the complexity of the model, even though it is known that, for example, MAPK signalling pathways often exhibit a switch-like behaviour, Huang and Ferrell [13]. In the last section of this chapter we compare the dynamics of this model with those obtained by using a HOG controller implementing ultrasensitivity, Montefusco et al. [25] (Table 4.1).

AQ1

### 4.3.1.3 The Glycerol Module

The exchange of internal and external glycerol,  $u_{Diff}$  over the Fps1 channel is modelled by using Fick's first law of diffusion as

$$u_{Diff}(t) = u_{Fps1}(t) \left( \frac{Gly(t)}{V(t) - V_b} - \frac{Gly_e(t)}{V_e} \right),$$

with  $V_e$  being the extra-cellular volume and  $Gly_e$  being the glycerol concentration in the extra-cellular compartment. Intra-cellular glycerol  $Gly$  production is expressed, combining the output of the two controllers described above, as

$$\frac{dGly}{dt} = \tilde{u}_{HOG}(t) - u_{Diff}(t)$$

**Table 4.1** Model parameters: all volumes are scaled such that the initial volume of the cell is 1

Parameters		Bounds
$k_{p1}$	Water perm. coeff.	[0.0052, 160] Osm <sup>-1</sup>
$k_{p2}$	Fps1 control const.	[0, ∞]
$T_d$	Time delay	[5, 30] min
$k_{HOG}$	HOG control const.	[0, ∞] Osm <sup>-1</sup>
$Gly(0)$	Initial $Gly$	[1.1 5] × 10 <sup>-4</sup>
$P_i(0)$	Initial $P_i$	[0.6 0.7] Osm
$P_e(0)$	Initial $P_e$	[0.24 0.25] Osm
$V_b$	Non osmotic volume	[0.31 0.46]
$V^{P_i=0}$	$V$ when $P_i = 0$	[0.5 0.99]
$V_e$	External volume	[0.5 5] × 10 <sup>3</sup>
Dependent parameters		Value
$V(0)$	Initial $V$ - relative volume	1
$Gly_e(0)$	Initial $Gly_e$	$\frac{V_e Gly(0)}{(V(0) - V_b)}$
$P_i(0)$	Initial $P_i$	$P_i(0) - P_e(0)$
$s$	No. of osmolytes other than $Gly$	$P_i(0)(V(0) - V_b)$ $- Gly(0)$

Both  $Gly$  and  $Gly_e$  represent number of molecules (mol scaled by  $V(0)$ )

195 and extra-cellular glycerol, depending only on the diffusion over the Fps1 channel,  
196 is described by

$$197 \quad \frac{dGly_e}{dt} = u_{Diff}(t).$$

### 198 4.3.2 Parameter Estimation and Results

199 The model contains 14 parameters, 4 of which are dependent, as given in Table 4.5.  
200 In [8], the other parameters are estimated by simulating the model and minimising  
201 the error defined as the sum of the squares of the difference between simulated,  $X(t)$ ,  
202 and experimental time series data,  $\hat{X}(t_i)$ , for intra-cellular and total glycerol. The  
203 error for one time series is calculated as

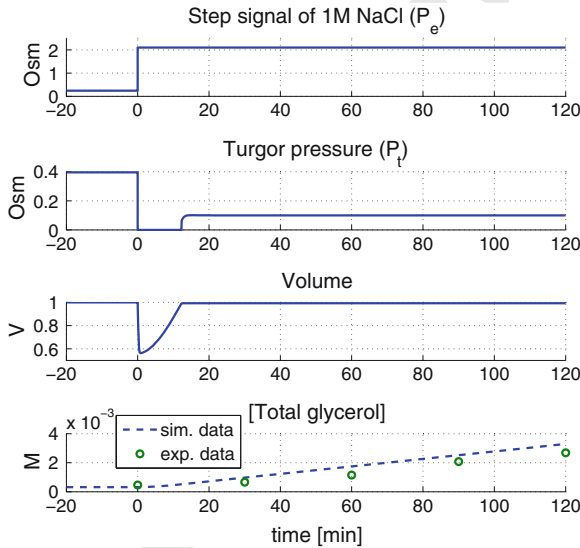
$$204 \quad \text{error} = \sum_i (X(t_i) - \hat{X}(t_i))^2. \quad (4.8)$$

205 The best parameters found are given in Table 4.2. To find a possible global mini-  
206 mum point of the error function, the authors in [8] evaluated several randomly chosen  
207 starting points in the feasible region of the parameter space. The research was contin-  
208 ued for the sets of parameter with sufficient low error by using the function *fmincon*  
209 from the MATLAB Optimization Toolbox, MATLAB [22]. Figures 4.3 and 4.4 show  
210 the simulated data using the simple model devised in [8] and the parameter set given

**Table 4.2** Optimized parameters by using the relation (4.8): the values of  $P_e$  and  $k_{p1}$  were fixed while the remaining 8 parameters were estimated from time series data

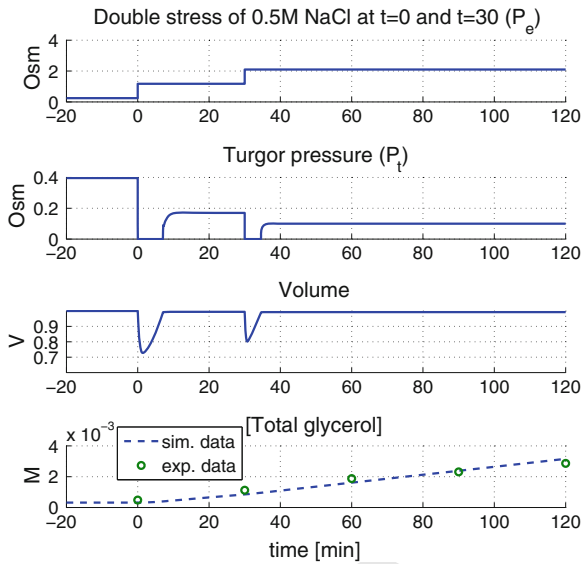
Parameters		Value
$k_{p1}$	Water perm. coeff.	$1 \text{ Osm}^{-1}$
$k_{p2}$	Fps1 control const.	0.316
$T_d$	Time delay	8.61 min
$k_{HOG}$	HOG control const.	$0.416 \text{ Osm}^{-1}$
$Gly(0)$	Initial $Gly$	$2 \times 10^{-4}$
$P_i(0)$	Initial $P_t$	0.636 Osm
$P_e(0)$	Initial $P_e$	0.240 Osm
$V_b$	Non osmotic volume	0.368
$V^{P_i=0}$	$V$ when $P_t = 0$	0.99
$V_e$	External volume	$4.79 \times 10^3$

$Gly$  represents number of molecules (mol scaled by  $V(0)$ )



**Fig. 4.3** Simulation of a step signal of 1M NaCl at  $t = 0$ . *Upper plot* external osmotic signal. *Second plot* the turgor pressure. *Third plot* the volume response. *Lower plot* the total glycerol concentration for the simulated (sim.) and experimental (exp.) data taken from [8]

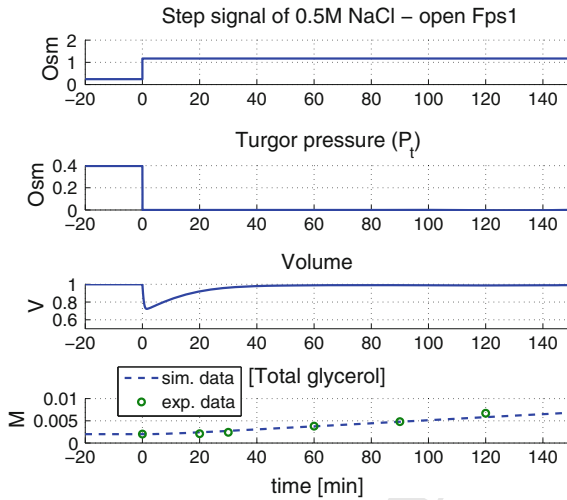
211 in Table 4.2 and a comparison with the time-series experimental data. Figure 4.3  
 212 shows the simulation of the model by applying an osmotic stress of 1M NaCl at time  
 213  $t = 0$ , corresponding to an increase in the extra-cellular osmotic pressure by 1.86  
 214 Osm, while Fig. 4.4 shows the response to a double stress of 0.5M NaCl at  $t = 0$  and  
 215  $t = 30$ . The simulated data show how the turgor pressure and volume drop immedi-  
 216 ately upon the osmotic stress. While the volume returns to approximately the same  
 217 value as before the stress, the turgor pressure, the controlled variable, doesn't reach  
 218 its previous value. The main reason for incomplete recovery is that the model para-



**Fig. 4.4** Simulation of a double stress of 0.5M NaCl at  $t = 0$  and  $t = 30$ . *Upper plot* external osmotic signal. *Second plot* the turgor pressure. *Third plot* the volume response. *Lower plot* the total glycerol concentration for the simulated (sim.) and experimental (exp.) data taken from [8]

219 meters are estimated by using the measured glycerol data, which are not sufficient  
 220 for complete recovery of both volume and turgor pressure. The reason why volume  
 221 and not turgor pressure is recovered is due to the high value of the estimated  $V^{P_t=0}$ ,  
 222 indicating a low elasticity of the cell wall. Therefore, turgor pressure is not recovered  
 223 until the volume is almost completely recovered. For a lower value of  $V^{P_t=0}$ , the  
 224 turgor pressure would be recovered faster and the volume slower. Figure 4.4 shows  
 225 that the model can reproduce the regulatory behaviour of the system to a series of  
 226 osmotic shocks. Moreover, it is able to predict the behaviour of modified strains. For  
 227 example, Fig. 4.5 shows the simulation to an osmotic shock in a modified strain with  
 228 constitutively open Fps1 (i.e. only one control mechanism via the HOG pathway).  
 229 This test was experimentally demonstrated in [15]. To simulate this experiment we  
 230 set  $u_{Fps1} = k_{p2}$  (see Eq. (4.6)) and adjust the value of  $Glyc(0)$  to obtain a realistic  
 231 initial value of total glycerol. Figure 4.5 shows that the model correctly predicts  
 232 the levels of total glycerol. Note, in particular, an over-production of the glycerol as  
 233 experimentally measured (double production compared to wild type experiment) and  
 234 a prolonged activation time of the HOG pathway (see Fig. 4d in [15]), that can not  
 235 be explicitly observed using this model, since Hog1 is not a variable of the model,  
 236 but implicitly deduced from the delay of volume recovery.

237 This model can therefore give us significant insight into the functioning of the sys-  
 238 tem, and the results indicate that even such a simple model can predict the behaviour  
 239 of different strains and the response to different input functions. It is also easier to  
 240 understand and analyse than the detailed model developed in [15] (compare Fig. 4.1



**Fig. 4.5** Simulation of one osmotic stress of 0.5M NaCl at  $t = 0$  in modified system (open Fps1). *Upper plot* external osmotic signal. *Second plot* the turgor pressure. *Third plot* the volume response. *Lower plot* the total glycerol concentration for the simulated (sim.) and experimental (exp.) data taken from [8]

241 with Fig. 4.2). However, detailed models are often important to completely under-  
 242 stand a particular phenomena. For instance, in [15] the authors extracted novel infor-  
 243 mation on the features of the system: the switch-like behaviour of the phosphorelay  
 244 module consisting of three protein (Sln1, Ypd1 and Ssk1) that become more pro-  
 245 nounced for higher number of components (see Fig. 2a in [15], where a comparison  
 246 of the steady-state characteristics is performed for phosphorelay systems consisting  
 247 of one, two and three proteins); and the main role of the phosphatases, that is to  
 248 constantly counteract HOG pathway activation to set thresholds and reduce noise  
 249 instead of providing a direct downregulation of the pathway.

## 250 4.4 Systems-Engineering Approaches

251 In this section we introduce some methods based on systems-engineering tools to  
 252 better understand the dynamics of the osmo-adaptation response. In this area, impor-  
 253 tant contributions have been produced by the group of van Oudenaarden. In a first  
 254 work (see [24]), the authors analysed the dynamics of the system in the frequency  
 255 domain, a feasible approach which allows the derivation of a concise model of the  
 256 basic mechanisms of the osmoregulation, that emerge from an intricate network  
 257 of interactions acting at very different time-scales, e.g. ligand binding or unbind-  
 258 ing, phosphorylation, diffusion between compartments and transcription of genes.  
 259 In [26], the authors later found that Hog1-dependent glycerol accumulation is crucial  
 260 for the perfect adaptation of yeast to simple step increases of osmotic change,

261 suggesting that Hog1 may implement integral–feedback via an as yet–unknown role  
 262 for protein–protein interactions that increase the internal osmolyte concentration.

#### 263 4.4.1 A LTI System Identification

264 In this section we apply frequency domain analysis to derive a concise model of the  
 265 HOG MAPK cascade in the budding yeast *S. cerevisiae*. Our treatment is mainly  
 266 based on the results presented in [24]. After a hyper osmotic shock, membrane  
 267 proteins trigger a signal transduction cascade that culminates in the activation of  
 268 the MAPK Hog1. This activated protein, which is primarily cytoplasmic before the  
 269 shock, is then imported into the nucleus, where it activates several transcription  
 270 responses to osmotic stress. When the osmotic balance is restored, Hog1 is deacti-  
 271 vated through dephosphorylation, thus allowing its export back in the cytoplasm. In  
 272 order to identify the model, the input and the output of the system to be predicted have  
 273 to be defined: in this case the input is the extra–cellular osmolyte concentration and the  
 274 output is the concentration of active Hog1 protein. In [24] the input is manipulated by  
 275 varying the salt concentration of the medium surrounding the cells, whereas the out-  
 276 put is measured by estimating the localisation of Hog1 in the nucleus,  $R(t)$ , through  
 277 fluorescence image analysis: the cellular localisation of Hog1–YFP, a yellow fluores-  
 278 cent protein fused to Hog1, and Nrd1–RFP, a red fluorescent protein fused to a strictly  
 279 nuclear protein, are simultaneously monitored and  $R(t)$  is measured as the nuclear  
 280 to total Hog1 ratio in the cell ( $R(t) = (\langle YFP \rangle_{nucleus} / \langle YFP \rangle_{cell})_{population}$ ,  
 281 averaged over the 50–300 cells observed in the microscope’s field of view).

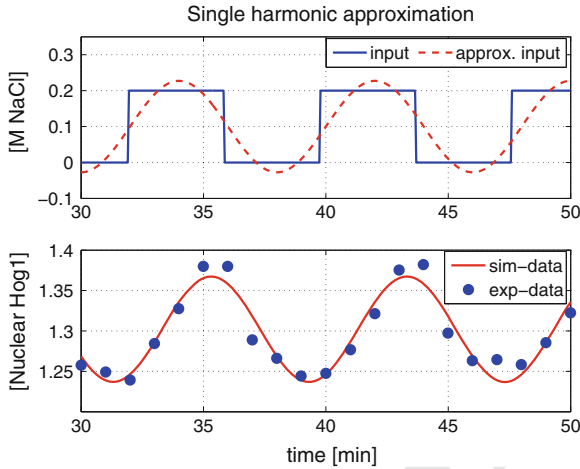
282 The experiments are performed by applying pulse wave signals to the cells with  
 283 different values of the period  $T_0$ , ranging from 2 to 128 min and they show that  
 284 the steady–state response is approximately sinusoidal, with period  $T_0$  (see Fig. 4.6).  
 285 Using Fourier analysis, both the input and the output can be approximated as sine  
 286 waves oscillating with a period  $T_0 = 2\pi/\omega_0$ . In particular, the experimental input,  
 287 using a first harmonic approximation (see [5] pp. 26–30), can be written as

$$288 \quad u(t) \approx 0.2 \left( \frac{1}{2} + \frac{2}{\pi} \sin(\omega_0 t) \right) \quad (4.9)$$

289 and the steady–state response  $R_\infty(t)$  as

$$290 \quad R_\infty(t) = R_0 + A(\omega_0) \sin(\omega_0 t + \phi(\omega_0)), \quad (4.10)$$

291 where  $R_0$  is the offset term and  $A$  and  $\phi$  are two parameters that characterise the  
 292 oscillations.  $A$  and  $\phi$  are represented through the absolute value and phase of the  
 293 complex number  $\tilde{R}(\omega_0)$ , respectively. This complex number is calculated from the  
 294 Fourier coefficient of the experimental data,  $R_\infty(t)$ , taken for stimuli with period  $T_0$   
 295 using the following relation:



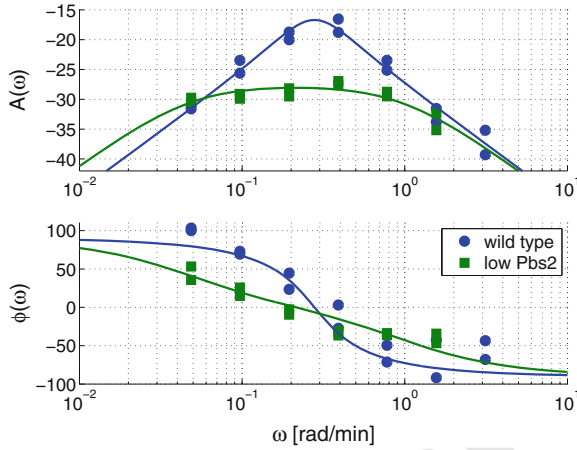
**Fig. 4.6** Upper plot Pulse signal of 0.2 M NaCl with  $T_0 = 8$  min and its approximation using the first harmonic. Lower plot the function  $R_\infty(t)$  (sim.) defined by (4.11) and fitted to the experimental (exp.) measurements of nuclear Hog1 enrichment taken from [24]

$$\tilde{R}(\omega_0) = 2 \int_{nT_0}^{(n+m)T_0} \frac{\exp^{-i\omega_0 t} R_\infty(t)}{mT_0} dt, \quad (4.11)$$

The amplitude of the signal, defined as  $A(\omega_0) = |\tilde{R}(\omega_0)|$ , represents half the distance from the peak to the trough of the output sine wave. The phase parameter,  $\phi(\omega_0)$ , can be written implicitly as  $\frac{\tilde{R}(\omega_0)}{|\tilde{R}(\omega_0)|} \exp^{i(\phi(\omega_0) - \pi/2)}$ . The parameter  $n$  is chosen so that the system is allowed to approach steady state before computing  $\tilde{R}(\omega_0)$ . The parameter  $m$ , which represents the number of periods over which the Fourier transform is computed, is set to be at least two for periods less than 64 min. For periods greater than or equal to 64 min, it is found that the first period is a good representation of the steady state oscillations and thus  $\tilde{R}(\omega_0)$  is computed over this period alone. However, the values  $A(\omega_0)$  and  $\phi(\omega_0)$  can be computed for different values of  $\omega_0$  by fitting the parameters of the Eq. (4.11) to the experimental time response as shown in the lower plot of Fig. 4.6 for  $\omega_0 = 2\pi/8$  rad/min. The resulting frequency response is shown on the Bode plots in Fig. 4.7.

A predictive model can be identified from the available experimental data by using linear systems theory: a linear input–output relationship in Fourier space is defined by

$$\tilde{Y}(\omega) = A_0 \frac{\prod_{i=1}^n (z_n + i\omega)}{\prod_{i=1}^n (p_n + i\omega)} \tilde{U}(\omega), \quad (4.12)$$



**Fig. 4.7** The experimental data of the Fourier amplitude  $A(\omega)$  and phase  $\phi(\omega)$  (two measurements at each frequency), for wild type (*circles*) and underexpressed Pbs2 mutant (*squares*) strains, with the fitting models (*solid lines*). Experimental data taken from [24]

**Table 4.3** Best-fitting parameters for the Eq. (4.13)

	$p_1$ [ $\text{min}^{-1}$ ]	$p_2$ [ $\text{min}^{-1}$ ]	$A_0$ [ $\text{min}^{-1}$ ]
Wild type	$-0.1434 + 0.239i$	$-0.1434 - 0.239i$	0.3292
Low Pbs2	$-0.0466$	$-0.9755$	0.3169

313 where  $\tilde{Y}(\omega)$  and  $\tilde{U}(\omega)$  are the output and input Fourier spectra, respectively,  $z_n$  are  
 314 the  $n$  roots of the numerator of Eq. (4.12), also called *zeros*, and  $p_n$  are the  $n$  roots  
 315 of the denominator, also called *poles*. The simplest such model from this class, that  
 316 describes the experimental points in Fig. 4.7, exhibits a zero at the origin ( $z_1 = 0$ )  
 317 and a pair of poles  $p_1$  and  $p_2$  yielding

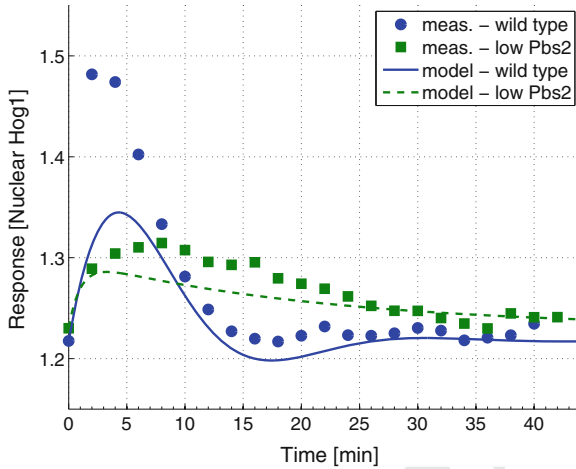
$$318 \quad (p_1 + i\omega)(p_2 + i\omega)\tilde{Y}(\omega) = (i\omega)A_0\tilde{U}(\omega). \quad (4.13)$$

319 The best-fit parameters for the wild type and for the mutant (Pbs2 underexpression)  
 320 strains are shown in Table 4.3. Applying the inverse Fourier transform (note a mul-  
 321 tiplication by  $i\omega$  in Fourier domain corresponds to the derivative operator in time  
 322 domain) the following relationship in the time-domain is given:

$$323 \quad \ddot{y}(t) + (p_1 + p_2)\dot{y}(t) + (p_1 p_2)y(t) = A_0\dot{u}(t). \quad (4.14)$$

324 The identified second-order LTI models, defined by the Eq. (4.13), are used to  
 325 predict the response of the two strains to a step input of 0.2 M NaCl. Figure 4.8 shows  
 326 the predicted responses of the two models and a comparison with the experimental  
 327 measurements: the responses of the linear systems are offset by a constant value  
 328 (1.23 M NaCl), which is the experimentally measured basal activity level of Hog1.  
 329 The two models show a good qualitative match to the different sets of data for the two





**Fig. 4.8** Time domain response of the system to a step increase of 0.2 M NaCl: comparison of the responses predicted by the two linear models developed in the frequency domain vs the experimental measurements taken from [24]

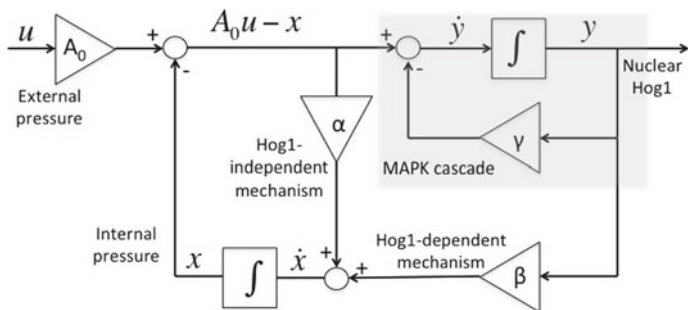
330 yeast strains (the match is not perfect, since these are linear models of a process that  
 331 will clearly also involve some nonlinear dynamics). Note that the wild type model  
 332 exhibits a pair of complex conjugated poles and therefore the response is oscillatory,  
 333 with a larger overshoot and a faster response than the low Pbs2 model, as expected  
 334 from the experimental data. Indeed, the latter has two real poles, and thus exhibits a  
 335 limited initial overshoot, a fast initial rise (due to the pole with small time constant)  
 336 and a slow decay (caused by the large time constant associated with the other real  
 337 pole). The identified LTI model can be written as a pair of differential equations, that  
 338 is more readily interpreted in terms of biological process:

$$339 \begin{pmatrix} \dot{z}(t) \\ \dot{y}(t) \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} z(t) \\ y(t) \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix} u(t) \quad (4.15)$$

341 with rate constants  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$  and  $f$ . The variable  $y(t)$  is assumed to represent the  
 342 observable output of the system (the level of Hog1 activity), whereas the variable  
 343  $z(t)$  represents the hidden state and  $u(t)$  the osmotic stimulus. When these equations  
 344 are simplified to remove the hidden variable  $z(t)$ , a single second order differential  
 345 equation in  $y(t)$  is obtained:

$$346 \ddot{y}(t) = (a + d)\dot{y}(t) + (bc - ad)y(t) + (ec - af)u(t) + f\dot{u}(t) \quad (4.16)$$

347 This equation is equivalent to the one in (4.14), if  $f = A_0$ ,  $ce = aA_0$  and  $c \neq 0$ .  
 348 Substituting these relations in (4.15), we obtain the following system:



**Fig. 4.9** Block diagram representation of the system (4.18): two negative feedback loops (Hog1-independent and Hog1-dependent mechanisms) act to reduce the difference between the stimulus,  $A_0u(t)$ , and the internal-state variable,  $x(t)$ , representing the internal pressure

$$\begin{pmatrix} \dot{z}(t) \\ \dot{y}(t) \end{pmatrix} = \begin{pmatrix} \frac{ce}{A_0} & b \\ c & d \end{pmatrix} \begin{pmatrix} z(t) \\ y(t) \end{pmatrix} + \begin{pmatrix} e \\ A_0 \end{pmatrix} u(t) = \begin{pmatrix} \frac{e}{A_0} & b \\ 1 & d \end{pmatrix} \begin{pmatrix} A_0u(t) + cz(t) \\ y(t) \end{pmatrix}. \quad (4.17)$$

Defining  $x(t) = -cz(t)$ ,  $\alpha = -\frac{ec}{A_0}$ ,  $\beta = -bc$  and  $\gamma = -d$  the system (4.17) is written as

$$\begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ 1 & -\gamma \end{pmatrix} \begin{pmatrix} A_0u(t) - x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} \alpha(A_0u(t) - x(t)) + \beta y \\ A_0u(t) - x(t) - \gamma y \end{pmatrix}. \quad (4.18)$$

Comparing this relation with the LTI model, we can equate coefficients to obtain the relations:

$$\alpha + \gamma = p_1 + p_2, \quad p_1 = \frac{1}{2}((\alpha + \gamma) + \sqrt{(\alpha - \gamma)^2 - 4\beta}),$$

and

$$\alpha\gamma + \beta = p_1p_2, \quad p_2 = \frac{1}{2}((\alpha + \gamma) - \sqrt{(\alpha - \gamma)^2 - 4\beta}).$$

The identified model, described by the relation (4.18), contains two negative feedback loops, which act to reduce the difference,  $(A_0u(t) - x(t))$ , between the stimulus,  $A_0u(t)$ , and the internal-state variable  $x(t)$  (see Fig. 4.9). This enables us to assign a physical meaning also to the variable,  $x(t)$ : since the input is the external pressure,  $x$  represents the internal pressure. Moreover the model tells us that one feedback mechanism is mediated by the Hog1 MAPK pathway ( $\beta y$  changes  $x$  through the activity of the observable output  $y$ ), whereas a second one is mediated by a pathway which is independent of Hog1. Since Hog1 is activated by Pbs2, we can derive useful insight by comparing the responses of the wild type strain with the mutant strain, in which Pbs2 is underexpressed (see Fig. 4.8). This comparison suggests that the feedback action provided by the Hog1 pathway is stronger,

374 producing a faster response. As discussed in Sect. 4.3, the hyperosmotic-shock  
375 response in yeast is regulated by two parallel mechanisms: 1) the Hog1-independent  
376 pathway activating the membrane protein Fps1 that quickly (<2 min) responds by  
377 decreasing the glycerol-export rate (see [19, 36]); 2) the Hog1-dependent pathway  
378 increasing the expression of Gpd1 and Gpd2 which accelerate the production of the  
379 glycerol over a longer time scale (>30min—see [2]). Although the topology of the  
380 model identified corresponds closely to that of the known biological system (see  
381 Fig. 4.9) the dynamic differences suggest that the MAPK Hog1 plays a role not only  
382 in the transcriptional regulation of glycerol producing proteins, but also in the control  
383 of the rapid accumulation of glycerol, consistent with previous studies (see [19, 36,  
384 37]): from Fig. 4.8 the peak times of the responses of both wild type and mutant  
385 strains are less than 10 min and in both cases the response is much faster than the  
386 characteristic dynamics of gene expression. From this analysis the authors, in [24],  
387 have hypothesized that gene expression may be more important as a longer-time  
388 scale feedback in the hyperosmotic-shock response. To test this hypothesis, they  
389 stimulated cells with periodic pulses of NaCl (see Fig. S5 in [24]). The cells were  
390 shocked either in the absence or presence of cycloheximide, a small molecule that  
391 inhibits protein synthesis. They showed that cells respond very similarly to an initial  
392 pulse of osmolyte both in the absence or presence of cycloheximide. On the other  
393 hand, to adapt to subsequent pulses, cells need less time in the absence of cyclo-  
394 heximide and more in its presence. These results suggest that non transcriptional  
395 feedback mediates short-time scale osmolyte accumulation (see [8, 15, 28, 37]),  
396 whereas gene expression plays a role in osmolyte production only over longer time  
397 scales and for more intense shocks.

## 398 4.5 Perfect Adaptation in Yeast Osmoregulation

399 As shown in the last section the concise model developed by [24] is able to predict  
400 the Hog1 response by using only two differential equations. However, a detailed  
401 comparison of the LTI model's predictions with the experimental data sets shows  
402 that this model (only containing two negative feedback loops that control the rapid  
403 accumulation of glycerol) is too simple to fully reproduce the quantitative dynamics  
404 of the Hog1 nuclear enrichment when the cell are stimulated multiple times with  
405 periodic pulses of NaCl (see Figs. S8 and S9 in [40]). In particular, the experimental  
406 data sets and the model presented in [40] suggest that yeast can remember the first  
407 pulse of high osmolarity and needs less time to adapt to subsequent pulses of simu-  
408 lation. The LTI model developed by [24] fails to capture this dynamical property  
409 and, in [26], the same group proposed a revised concise LTI model by implementing  
410 an integral feedback mechanism which requires Hog1 kinase activity. They started  
411 with a minimalist model represented by the network diagram of Fig. 4.10, which  
412 aims to predict the dynamics of the osmoregulation system with only a few key  
413 parameters, starting from input-output data, and, using biological measurements and  
414 engineering principles, to better understand the relation of its dynamics with the



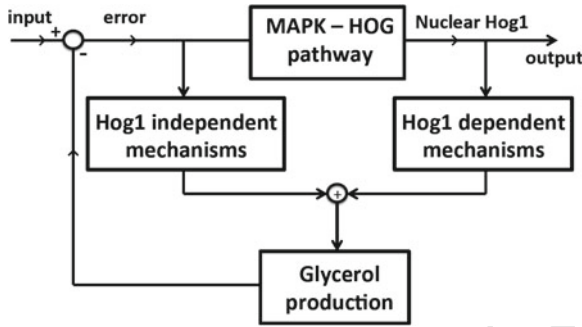
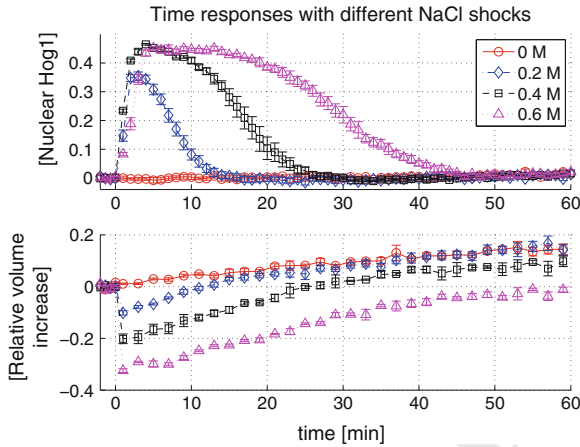


Fig. 4.10 Network diagram of the osmoregulation system presented in [26]

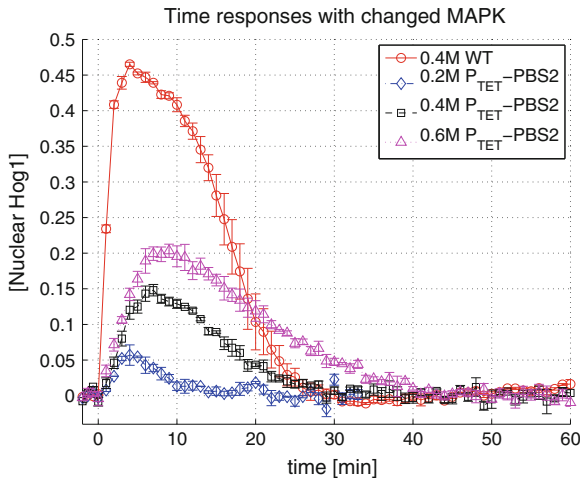
415 network topology. The authors represent in a subsystem all relevant reactions that  
 416 determine the activation of the MAPK signalling pathway and the nuclear import  
 417 of the activated protein Hog1 (MAPK signaling & nuclear import) and in another  
 418 subsystem all the Hog1-dependent mechanisms that promote the glycerol production  
 419 (Hog1-dependent mechanisms—such as the transcriptional activation of genes that  
 420 encode enzymes involved in the glycerol production and potential protein-protein  
 421 interaction initiated by Hog1 in the cytoplasm or nucleus that lead to glycerol accu-  
 422 mulation). In contrast in the model of Fig. 4.2, developed by [8], the HOG pathway  
 423 controller represents both the HOG signalling pathway, transcription/translation and  
 424 the synthesis of enzymes involved in glycerol production.

#### 4.5.1 Experimental Measurements for the Perfect Adaptation

426 In [26] the authors observed perfect adaptation of Hog1 nuclear enrichment in  
 427 response to step increases of the extracellular osmolyte concentration (see Fig. 4.11  
 428 where step inputs of NaCl with different amplitude are applied and Fig. S3 in [26]  
 429 where KCl and sorbitol are also used as osmolytes—in these and in the following  
 430 figures of this section, Hog1 nuclear enrichment is defined as the relative change from  
 431 the pre-shock level): this adaptation occurs with very low cell-to-cell variability and  
 432 is robust to the signalling fidelity of the MAPK cascade. In particular, for different  
 433 cells, the dynamics of Hog1 nuclear enrichment and cell volume are very similar  
 434 in response to a step osmotic stress, with trends that closely follow the population  
 435 average (see Fig. 2A, B in [26]). In fact, the cell-to-cell variability in unstressed cells  
 436 is comparable to the one in osmo-stressed cells as shown in Fig. S2 in [26], further  
 437 indicating that the intrinsic noise of signal propagation is low and suggesting that the  
 438 experimental setup itself may be the predominant source of noise in the experimental  
 439 data. Moreover, to demonstrate the robustness of this perfect adaptation, measure-  
 440 ments of the Hog1 response have been performed in cells with compromised MAPK  
 441 signalling, by controlling the expression of PBS2, which encodes the kinase of Hog1

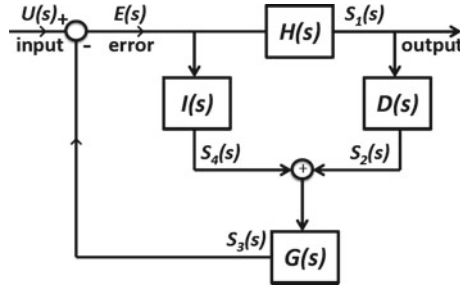


**Fig. 4.11** Time measurements of Hog1 nuclear enrichment and volume to hyperosmotic shocks with indicated concentrations of salt. Data taken from [26]



**Fig. 4.12** Time measurements of Hog1 nuclear enrichment after changing the signaling fidelity of the MAPK cascade by controlling the expression of PBS2. Data taken from [26]

442 (see Fig. 4.12). Also in this case Hog1 nuclear enrichment still perfectly adapts and  
 443 therefore we can say that the perfect adaptation is a robust property of the system  
 444 and not a consequence of *ad hoc* parameter tuning. From these results, together  
 445 with extensive theoretical analysis of adaptive systems in engineering, Muzzey et  
 446 al hypothesised that this system implements integral feedback control in order to  
 447 achieve robust perfect adaptation that does not require a precise tuning of system  
 448 parameters such as protein levels or rate constant (see [14, 34, 39]).



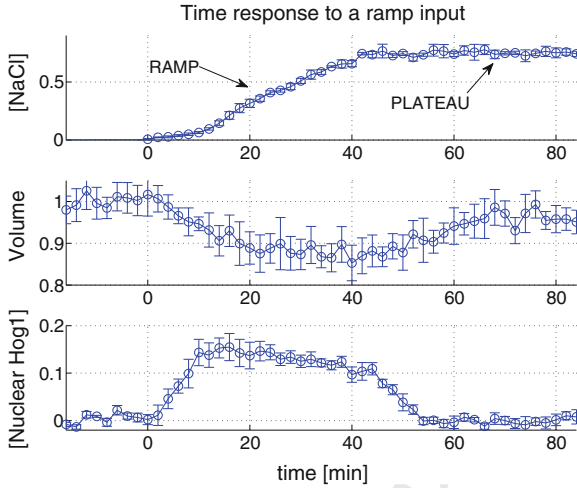
**Fig. 4.13** Block diagram of the osmoregulation system presented in [26].  $H$  represents all relevant reactions that link an osmotic disturbance at the membrane with Hog1 nuclear enrichment.  $D$  and  $I$  represent the Hog1-dependent and independent mechanisms for the glycerol accumulation, respectively.  $G$  represents the metabolic reactions involved in the glycerol synthesis

### 4.5.2 The Integral Feedback

The osmoregulation system is described by using the network diagram of Fig. 4.10, where the error indicates the deviation from the initial turgor pressure before applying the hyperosmotic stress. Figure 4.13 shows the corresponding block diagram of the osmosensing network of 4 subsystems denoted with  $G$ ,  $D$ ,  $H$  and  $I$ .  $H$  takes into account reactions that determine the activation of the MAPK signalling pathway and the nuclear import of the activated protein Hog1.  $D$  and  $I$  represent the Hog1-dependent and independent mechanisms that contribute to glycerol accumulation, respectively. Finally,  $G$  represents the metabolic reactions involved in the glycerol synthesis and any other reactions that promote glycerol accumulation. Approximating the network as being LTI, each subsystem can be described by a Laplace transform, or transfer function. In general a Laplace transform  $F(s)$  of a function  $f(t)$  is given by

$$F(s) = \int_0^{\infty} f(t) \exp^{-st} dt, \quad (4.19)$$

where  $s$  is a complex variable. The transfer function  $S(s)$  of a LTI system is defined as  $S(s) = Y(s)/U(s)$ , where  $U(s)$  and  $Y(s)$  are the Laplace transform of the system input,  $u(t)$ , and output  $y(t)$ , respectively (see [5] pp. 30–33). The Laplace transform has the useful property that many relationships and operations in the time domain that require calculus can instead be performed using linear algebra in the  $s$ -domain (the differential equations in the time domain can be transformed into algebraic equations in the  $s$ -domain using the Laplace transform—these are then much easier to solve). By applying the final-value theorem (see [32] p. 43), the steady-state input and output are related via  $y_{ss} = S(0)u_{ss}$ , so perfect adaptation of the system output ( $y_{ss} = 0$ )



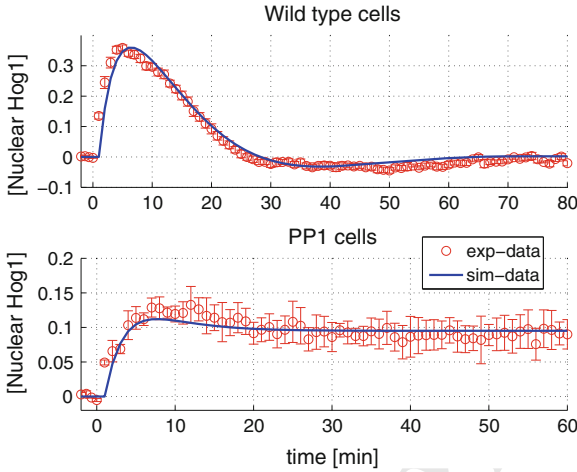
**Fig. 4.14** Measurements of Hog1 nuclear enrichment and volume in an experiment where the salt concentration ramps upward over time, reaching a plateau after nearly 45 min. Experimental data taken from [26]

472 for an LTI system is equivalent to  $S(0) = 0$ , since the input is constant and nonzero.  
 473 The input-error transfer function of the osmosensing system of Fig. 4.13 is given by

474 
$$S_{ue}(s) = \frac{E(s)}{U(s)} = \frac{1}{1 + G(s)(D(s)H(s) + I(s))}, \quad (4.20)$$

475 where  $G(s)$ ,  $D(s)$ ,  $H(s)$  and  $I(s)$  are the transfer functions of the four subsystems  
 476 in the network (see [5] pp. 42–46). We need that  $S_{ue}(0) = 0$  to achieve perfect  
 477 adaptation of the error to a step input. Therefore at least one of the four subsystems  
 478 implements an integrator (its transfer function is given by  $1/s$ —see [5] p. 31—  
 479 thereby allowing  $S_{ue}$  to be zero at  $s = 0$ ). In general, a system contains at least one  
 480 feedback loop with at least  $n + 1$  integrators connected in series in order to achieve  
 481 perfect adaptation to an input corresponding to the  $n$ -th integral of a step function,  
 482 where  $n$  is a positive integer. Perfect adaptation to a step input, where  $n = 0$ , requires  
 483 at least one integrator, perfect adaptation to a ramp input, where  $n = 1$  since the ramp  
 484 is the integral of a step, requires at least two integrators in series, and so on. In [26]  
 485 the authors showed that neither cell volume nor Hog1 perfectly adapt in response  
 486 to a ramp input, confirming that there is exactly one integrator in the osmosensing  
 487 network (see Fig. 4.14). Therefore, the perfect adaptation of the error to a step input  
 488 requires that only one of the four subsystems contains one integrator. Similarly, the  
 489 input-output transfer function is given by

490 
$$S_{us_1}(s) = \frac{S_1(s)}{U(s)} = \frac{H(s)}{1 + G(s)(D(s)H(s) + I(s))}. \quad (4.21)$$



**Fig. 4.15** Time domain response of the system (4.26) to a step increase of 0.4M NaCl: comparison of the responses (sim.) predicted by the two models (wild type and PP1) vs the experimental (exp.) measurements taken from Fig. 5A in [26]

491 If  $H(s)$  were the only subsystem performing integration, then Hog1 would not perfectly  
 492 adapt ( $S_{us_1}(0) \neq 0$ ). Therefore one or more of the other subsystems must  
 493 contain an integrator to achieve perfect adaptation of Hog1, but the system only  
 494 contains one integrator. In [26] it is shown that the cells lose perfect adaptation with PP1,  
 495 a treatment to completely eliminate the Hog1 kinase activity, as the steady-state Hog1  
 496 accumulation ( $s_{1_{ss}}$  in Fig. 4.13) does not go back to the pre-stimulus level. Since the  
 497 presence of PP1 disconnects the  $D$  subsystem from Hog1, the input-output transfer  
 498 function of the system is modified as

$$499 \quad S_{us_1}(s) = \frac{H(s)}{1 + G(s)I(s)}. \quad (4.22)$$

500 In this case Hog1 does not perfectly adapt, then the product  $G(s)I(s)$  does not go to  
 501 infinity at  $s = 0$  ( $S_{us_1}(0) \neq 0$ ), which implies that either the  $G$  and  $I$  subsystems both  
 502 lack integrators, or one subsystem has an integrator but the other perfectly cancels  
 503 the integrator (it is a differentiator with a transfer function equal to  $s$ —see [5] p. 31).  
 504 If  $I$  contained the integrator, then the turgor pressure would perfectly adapt in the  
 505 presence of PP1, and Hog1 likely would as well, but both properties are not observed  
 506 in the data. If  $G$  were to act as an integrator, then cell volume and turgor pressure  
 507 would continue to perfectly adapt for a nonzero input to the  $G$  subsystem. But, in  
 508 the presence of PP1, the only input to subsystem  $G$  is the output from subsystem  $I$ ,  
 509 as subsystem  $D$  is disconnected. Thus, no volume recovery observed in PP1-treated  
 510 cells would only occur if the output of subsystem  $I$  prematurely goes to zero (i.e. if  
 511 it were a differentiator). As explained in [26], this observation would require that all  
 512 Hog1-independent mechanisms completely desensitize within approximately 20 min  
 513 (i.e. the time needed for Hog1 nuclear enrichment to reach steady state in PP1 cells—



see the lower plot of Fig. 4.15) despite persistence in their stimulus (i.e. the acute loss of turgor pressure). On the basis of this argument, it is extremely improbable that subsystem  $G$  acts an integrator. Therefore, the combination of all findings points to  $D$  as the subsystem with the only integrator in the feedback loop. Moreover, in PP1 cells, levels of total glycerol and extracellular glycerol are measured over time in the presence and absence of osmotic shock (see [26]): in the absence of salt shock, glycerol synthesis is increased as well as glycerol leakage; in the presence of osmotic shock, glycerol leakage is rapidly and transiently diminished, as in wild type cells, whereas the absence of Hog1 kinase activity prevents an increase in glycerol synthesis, unlike in wild type cells. These data suggest that Hog1 kinase activity plays a critical role in rapidly regulating glycerol synthesis but not its leakage as in [38].

Note from Fig. 4.12 (see also Fig. 3D in [26]) that the time-integral of the Hog1 scales linearly with the shock strength. If the system were composed only of reactions modelled with linear dynamics, then the result that  $D$  subsystem is an integrator would be trivial. However, this result is valid also when the other subsystems are nonlinear stable systems without integrators (see Fig. 4.12 where the fact that the peak Hog1 amplitude saturates as a function of salt is an evidence of nonlinear dynamics in the  $H$  subsystem). If it is assumed that the error perfectly adapts and the steady-state output of the  $I$  subsystem is zero when its steady-state input is zero, then 1) the net change induced by the system in the steady-state input of the  $G$  subsystem simply equals the time-integral of Hog1, 2) the net change in the output of  $G$  must equal the net change in the system input in order for the error to go to zero. If the  $G$  subsystem were perfectly linear, then its output would be directly proportional to its input at steady state and so the time-integral of Hog1 would be directly proportional to the magnitude of the osmotic stress (despite potential nonlinearities in the  $H$  and  $I$  subsystems). This relationship is almost exactly what Fig. 3D in [26] shows, except that the line relating the integral of Hog1 nuclear enrichment to the magnitude of the osmotic stresses does not cross the origin. This difference may be due to nonlinearities in the input-output steady-state function of subsystem  $G$  that become evident for osmotic stresses of small magnitude ( $<0.2$  M NaCl).

Finally, in order to validate these results, a LTI system can be used to implement the concise model represented by the block diagram of Fig. 4.13. The subsystems of the osmosensing network can be represented as follows:  $H$  and  $G$  as first-order systems where the corresponding transfer functions  $H(s) = \frac{k_h}{s + \gamma_h}$ , with gain  $k_h$  and time constant  $\gamma_h^{-1}$ , and  $G(s) = \frac{1}{s + \gamma_g}$  with time constant  $\gamma_g^{-1}$ ,  $I$  as a scalar  $\alpha_i$  (i.e.  $I(s) = \alpha_i$ ) and  $D$  as an integrator with gain  $\alpha_d$  (i.e.  $D(s) = \frac{\alpha_d}{s}$ ). Therefore the Laplace transform of the output,  $S_1(s)$ , of the  $H$  subsystem is defined as:

$$S_1(s) = \frac{k_h}{s + \gamma_h} E(s) = \frac{k_h}{s + \gamma_h} (U(s) - S_3(s)), \quad (4.23)$$

where the Laplace function error  $E(s) = U(s) - S_3(s)$ , with  $U(s)$  and  $S_3(s)$  the Laplace functions of the input  $u(t)$  of the system and the output  $s_3(t)$  of subsystem

**Table 4.4** Best-fitting parameters for the system (4.26)

	$k_h$ [ $\text{min}^{-1}$ ]	$\gamma_h$ [ $\text{min}^{-1}$ ]	$\gamma_g$ [ $\text{min}^{-1}$ ]	$\alpha_d$ [ $\text{min}^{-1}$ ]	$\alpha_i$ [ $\text{min}^{-1}$ ]
Wild type	0.496	0.369	0.119	0.0106	0.0806
PP1	0.147	0.369	0.119	0	0.0806

555  $G$ , respectively. We can obtain the rate equation for the output  $s_1$  (corresponding to  
556 measured Hog1 nuclear enrichment) in the time domain applying the inverse Laplace  
557 transform of the following relation, by rewriting the Eq. (4.23):

$$558 \quad s S_1(s) = -\gamma_h S_1(s) - k_h S_3(s) + k_h U(s). \quad (4.24)$$

559 By applying the property that the derivative operator with respect to time correspond  
560 to a multiplication by  $s$  in the  $s$ -domain (see [5] p. 31), the inverse Laplace transform  
561 of (4.24) follows as:

$$562 \quad \dot{s}_1(t) = -\gamma_h s_1(t) - k_h s_3(t) + k_h u(t) \quad (4.25)$$

563 In the same way, we can obtain the rate equations for the outputs  $s_2$  and  $s_3$  of  
564 the corresponding subsystems  $D$  and  $G$ . Then the following system of differential  
565 equations is obtained:

$$566 \quad \begin{pmatrix} \dot{s}_1(t) \\ \dot{s}_2(t) \\ \dot{s}_3(t) \end{pmatrix} = \begin{pmatrix} -\gamma_h & 0 & -k_h \\ \alpha_d & 0 & 0 \\ 0 & 1 & -(\alpha_i + \gamma_g) \end{pmatrix} \begin{pmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \end{pmatrix} + \begin{pmatrix} k_h \\ 0 \\ \alpha_i \end{pmatrix} u(t). \quad (4.26)$$

568 Figure 4.15 shows the response of two strains (wild type and PP1 cells) to a  
569 step input of 0.4 M NaCl. Table 4.4 reports the best set of parameters that fit the  
570 experimental data. For the PP1 experiment we set  $\alpha_d = 0$  to break the connection  
571 between Hog1 and the  $D$  subsystem. The simulations show how the devised model  
572 is able to capture the dynamics of the system and produces an excellent match to the  
573 experimental data.

## 574 4.6 The Role of Ultrasensitivity

575 As shown above, systems and control theory provides a highly useful approach to  
576 abstract complex biological systems that seem to operate with similar goals as engi-  
577 neered control systems, and the osmoregulation system in yeast is a prime example  
578 of this. The models here presented, by combing proportional and integral feedback  
579 controllers capture the key dynamics of a homeostatic system like osmoregulation in  
580 yeast, but they do not shed light on how the evolution of such a biological control sys-  
581 tem can proceed to result in integral feedback control. In the following we explore  
582 the possible role of ultrasensitivity in osmoregulation. Indeed, it has been well-

documented that the upstream signalling pathways involved in this system implement high levels of ultrasensitivity, however, the role of such high gain in producing the observed perfect adaptation is not clear. Therefore, we extend the proportional controller model presented in [8] for this system with the implementation of ultrasensitivity, Montefusco [25].

#### 4.6.1 Ultrasensitive Model and Parameters

The mathematical model used for our analysis is the same as that presented in Sect. 4.3, apart from allowing the Hog controller to be non-linear (see Fig. 4.2). Indeed, in this case, the output of the HOG branch, which corresponds to the HOG pathway dependent glycerol production, is expressed as

$$u_{HOG}(t) = \begin{cases} k_{HOG} \cdot f(e), & e(t) > 0 \\ 0, & \text{otherwise,} \end{cases} \quad (4.27)$$

where the control function is given by

$$f(e) = \frac{e(t)^n}{\beta e(t)^n + K^n}, \quad (4.28)$$

with  $\beta = 1$  and  $K$  and  $n$  being the nonlinear Hill function variables. We have thus modified the control law for the HOG pathway, compared to the model in [8], to allow for a non-linear controller response. This is inspired by the fact that MAPK systems, of which the HOG pathway is an example, often show Hill type responses, Huang and Ferrell [13]. The performance of the nonlinear controller is contrasted with the proportional controller given in [8], where  $\beta = 0$  and  $K = n = 1$ . Our model contains 16 parameters as reported in Table 4.5. However, four of these are dependent parameters which do not need to be constrained. The other parameters are estimated by simulating the model with different osmotic shocks and minimising the error, defined by Eq. (4.5), and time adaptation corresponding to the time required by the cell to approximately return to its volume before the stress (see the definition in the next subsection). For the optimization, we use a hybrid Genetic Algorithm (GA) (see [18]), that combines the most well-known type of evolutionary algorithm with local gradient-based algorithms (see [7, 10]). We use the function *ga* from the MATLAB Global Optimization Toolbox, MATLAB [23], and *fmincon* from the MATLAB Optimization Toolbox, MATLAB [22], as the local algorithm. By the optimisation procedure some parameters do not significantly change their values, therefore, they are fixed equal to the values estimated in [8], except for  $V^{P_i=0}$ , which is set to 0.8, the value of the volume at zero  $P_i$  according to a recent study presented in [31].

The cost function used for the parameter estimation is given by

**Table 4.5** Model parameters: all volumes are scaled such that the initial volume of the cell is 1

Parameters		Bounds
$k_{p1}$	Water perm. coeff.	[0.0052 160] Osm <sup>-1</sup>
$k_{p2}$	Fps1 control const.	[0 10]
$T_d$	Time delay	[5 30] min
$k_{HOG}$	HOG control const.	[0 2] Osm <sup>-1</sup>
$K$	Hill const.	[0 0.01 2]
$n$	Hill exponent	[0 4]
Fixed parameters		Value
$Gly(0)$	Initial $Gly$	$2 \times 10^{-4}$
$P_i(0)$	Initial $P_i$	0.636 Osm
$P_e(0)$	Initial $P_e$	0.24 Osm
$V_b$	Non osmotic volume	0.368
$V^{P_i=0}$	$V$ when $P_i = 0$	0.8
$V_e$	External volume	$4.79 \times 10^3$
Dependent parameters		Value
$V(0)$	Initial $V$ - relative volume	1
$Gly_e(0)$	Initial $Gly_e$	$\frac{V_e Gly(0)}{V(0) - V_b}$
$P_i(0)$	Initial $P_i$	$P_i(0) - P_e(0)$
$s$	No. of osmolytes other than $Gly$	$P_i(0)(V(0) - V_b)$ $- Gly(0)$

Both  $Gly$  and  $Gly_e$  represent number of molecules (mol scaled by  $V(0)$ )

$$\min_x J, \quad (4.29)$$

where

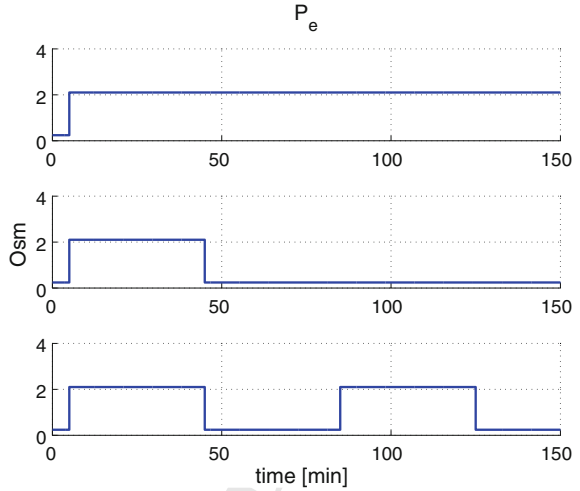
$$J = J_p + J_v + J_t \quad (4.30)$$

is a sum of three scalar functions:  $J_p$  is the turgor pressure error,  $J_v$  is the difference between the desired and the effective volume and  $J_t$  is the response time of the system after the perturbation.

## 4.6.2 Results: Ultrasensitive Versus Proportional Controller

In our adaptation of the model developed by [8], we particularly consider the observed ultrasensitivity in the HOG branch of the system. This branch was originally modeled as a proportional control in [8], which we have replaced here by a Hill-type function to model ultrasensitivity (see Eqs. (4.27) and (4.28)). We then compare the performance of this new model against the original model. In particular, we evaluate the two different controllers—proportional (Pr) and ultrasensitive (Us)—by simulating their dynamics with different stress inputs (see Fig. 4.16) and optimizing their

**Fig. 4.16** Different osmotic stresses. *Upper plot* a constant step of 1M NaCl at  $t = 5$  min corresponding to an increase of  $P_e$  equal to 1.96 Osm. *Middle plot* single pulse signal at  $t=5$  min with duration of 40 min of 1M NaCl. *Lower plot* double pulse signal at  $t_1 = 5$  and  $t_2 = 85$  min, both with duration of 40 min and amplitude of 1M NaCl



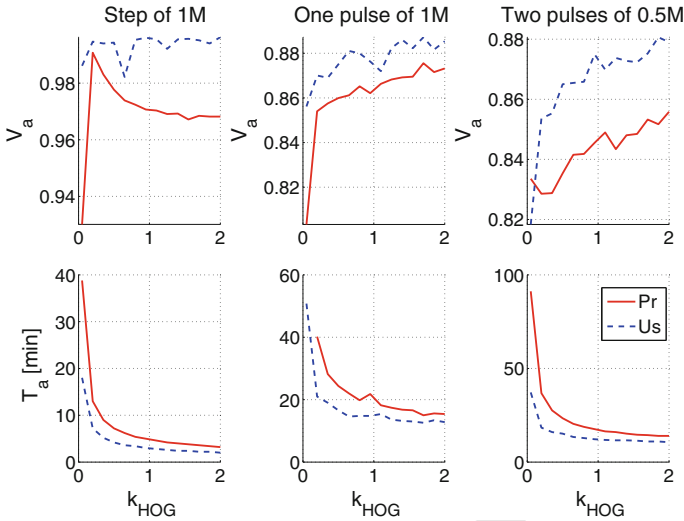
631 parameters for optimum response (i.e. minimal deviation of cell volume and turgor  
 632 pressure in presence of an osmo-schock, see Sect. 4.6.1 for details). We repeat this  
 633 procedure for different levels of overall sensitivity (i.e. gain) of the HOG branch and  
 634 different types of osmo-shock sequences and evaluate the tests by using two differ-  
 635 ent performance indices: adaptation precision and adaptation time. The adaptation  
 636 precision is defined as

$$637 \quad X_a = \prod_i X_{s,i}, \quad (4.31)$$

638 where  $X_{s,i}$  is the steady state value of the variable  $X$  (volume  $V$  or turgor pressure  
 639  $P_t$ ) after the  $i$ -th perturbation. Since the initial volume is set to unity, this measure  
 640 gives 1 for perfect adaptation. Deviations from 1 indicate inability of the system to  
 641 perfectly adapt volume to pre-perturbation levels. The time adaptation,  $T_a$ , defined as

$$642 \quad T_a = \sum_i t_{a,i}, \quad (4.32)$$

643 where  $t_{a,i}$  is the time required by the system to reach 85% of the volume  $V$  after the  
 644  $i$ -th osmotic stress. Figure 4.17 shows the results of the two controllers by applying  
 645 three different osmotic stresses: constant step, single pulse and double pulse. For  
 646 all different inputs the ultrasensitive controller achieves better and faster adaptation  
 647 irrespective of the level of overall gain. The better performance is particularly signif-  
 648 icant when overall gain is limited to lower values, where the ultrasensitive controller  
 649 achieves almost 2-fold faster responses. Indeed, using a Hill function within the  
 650 HOG branch allows us to effectively achieve a steeper response from this branch  
 651 compared to a linear function for any given error (see Eqs. (4.27) and (4.28)). Thus,  
 652 the controller acts faster and more strongly, allowing quicker and fuller recovery of

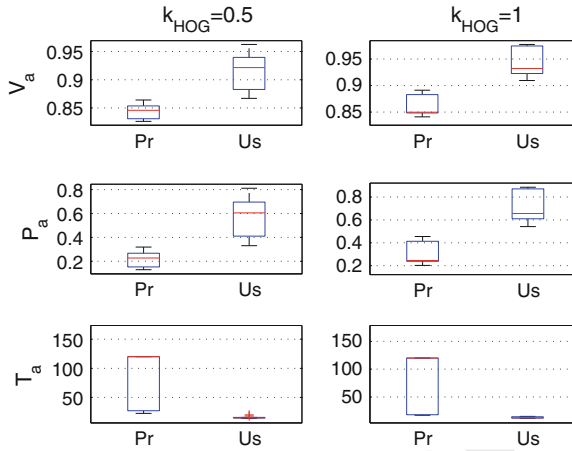


**Fig. 4.17** Performance comparison between the proportional (Pr) and ultrasensitive (Us) controllers by applying different shocks: first column, a step of 1M of NaCl; second column, 1 pulse of 1M; third column, 2 pulses of 0.5M.  $V_a$  close to 1 indicates the capability of the system to adapt.  $T_a$  indicates the time adaptation

**Table 4.6** Optimized parameters for a given  $k_{HOG}$  with a double pulse signal of 1M of NaCl

$k_{HOG}$	Optimized parameters—Pr/Us						Us	
	$k_{p1}$		$T_d$		$k_{p2}$		$\bar{K}$	$n$
	Pr	Us	Pr	Us	Pr	Us		
0.65	93	155	5	5	0.43	0.96	0.17	3.53
1.1	159	124	5	5	0.69	1.17	0.23	3.78
1.55	0.36	134	5	5	0.9	1.61	0.23	3.8
2	155	159	5	5	1	1.54	0.25	3

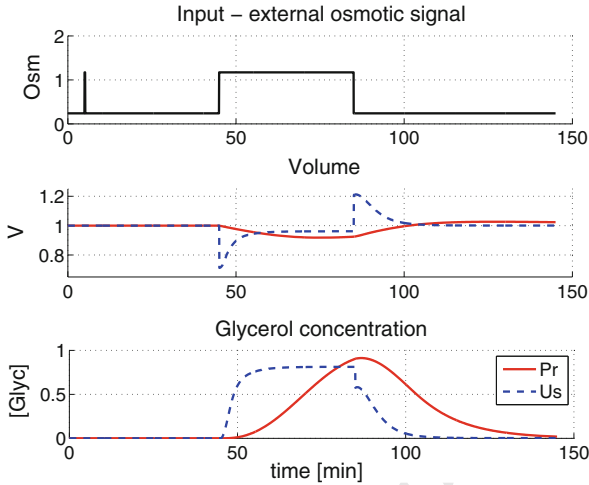
653 the system. This insight is in line with the optimized parameters for both controllers  
 654 as reported in Table 4.6: in most cases, the optimal parameters for the ultrasensitive  
 655 controller result in a very steep Hill function that produces maximal outputs for even  
 656 small error values. Of the other free parameters of the model, we note that certain  
 657 parameters are optimized differently for the two controllers. For example, the per-  
 658 meability coefficient  $k_{p1}$ , which controls water flow in the model (see Eq. (4.2)) is  
 659 usually optimized to higher values in the ultrasensitive controller compared to the  
 660 proportional controller. This parameter affects the sensitivity of the system, as faster  
 661 water movement can allow both a high volume reduction for a given osmo-shock and  
 662 also fast recovery. Given its fast dynamics, the ultrasensitive controller can “afford”  
 663 this parameter to become higher compared to the proportional controller.



**Fig. 4.18** Sensitivity analysis using the box-and-whisker representation (*median–middle line*, the 25th and 75th percentile—*lower and upper lines* of the box, and the outliers (crosses)) by fixing  $k_{HOG}$  and Hill function parameters ( $K = 0.05$ ,  $n = 3.5$ ) and applying one pulse signal of 1M of NaCl. The first column of each boxplot reports the results for the proportional (Pr) controller, the second column the results for the ultrasensitive (Us) controller. The system robustly adapts if  $V_a$  and  $P_a$  are close to 1.  $T_a$  indicates the time adaptation. Similar results are obtained with different Hill function parameters ( $K = 0.2$ ,  $n = 2$ )—see Fig. 7 in [25]

664 Such differences between the optimal model parameters for the two controllers  
 665 suggest that implementation of ultrasensitivity might allow more freedom in the other  
 666 parameters of the model or allow them to be in a more favorable regime. To test the  
 667 former possibility, we perform a simple sensitivity analysis for the two controllers.  
 668 Given a certain gain, and Hill function parameters, we evaluate the adaptation preci-  
 669 sion and time of the two controllers for a set of 100 randomly generated parameters.  
 670 Figure 4.18 shows that the ultrasensitive controller achieves much more robust adap-  
 671 tation performance than the proportional controller according to these two criteria.

672 As discussed above, the performance increase of the ultrasensitive controller over  
 673 the proportional one stems from its high sensitivity to the error due to the Hill function.  
 674 The incorporation of the Hill function, however, can also allow development of  
 675 thresholds in the system. In particular, the ultrasensitive controller can be tuned as a  
 676 filter allowing responses only to signals of certain magnitude or duration. To test this  
 677 hypothesis, we devise an alternative cost function for the optimization procedure and  
 678 optimize the system towards functioning as a filter. The new cost function is given by  
 679  $J_n = J - J_{glyc}$ , where  $J$  is defined by the Eq. (4.30) and  $J_{glyc}$  represents the glycerol  
 680 production upon the signal of limited duration. Figure 4.19 shows the performance for  
 681 a signal with a first short and then long duration pulse. The ultrasensitive controller  
 682 ignores the first pulse and responds to the second by tuning the Hill parameters,  
 683 whereas the proportional controller model is not able to response to the second  
 684 signal (the permeability coefficient  $k_{p1}$ , that affects the sensitivity of the system, is  
 685 equal to the lower bound).



**Fig. 4.19** Response to a signal with a short and long pulse duration, assuming  $k_{HOG} = 0.25$ . *Upper plot* external osmotic signal. *Second plot* the volume response for the proportional (Pr) controller and the ultrasensitive (Us) controller model. *Lower plot* the glycerol concentration for both the models

686 In conclusion, we show, using a previously developed proportional control model  
 687 of the osmoeregulation incorporating two main and distinct branches (HOG and chan-  
 688 nel branches), that ultrasensitivity in the HOG branch allows better overall perfor-  
 689 mance. We find that the primary effect of ultrasensitivity in the HOG branch is  
 690 an increase in the response speed of the system and consequently in its adapta-  
 691 tion precision. In addition to this, however, we find that ultrasensitivity provides  
 692 also a non-trivial flexibility to the system parameters. By increasing the speed of  
 693 overall system responses, ultrasensitivity in the HOG branch allows sensitivity to  
 694 be increased in the other branch of the system. In the absence of ultrasensitivity,  
 695 fast (i.e. highly sensitive) regulation of the glycerol exchange branch limits the cell's  
 696 adaptability through the HOG branch (i.e. glycerol production). With ultrasensitivity  
 697 in glycerol production, the other system parameters can be increased or varied more  
 698 freely, without compromising performance. Moreover, by increasing the gain of the  
 699 HOG branch, the system with a proportional HOG controller is able to improve the  
 700 performance in terms of adaptation, but there is a presence of overshoot in the system  
 701 response, whereas ultrasensitivity in the HOG branch allows to avoid this phenomena  
 702 (we do not consider the overshoot to compute the performance). Note that for large  
 703 values of the error ( $e > 1$ ), a proportional branch may have a higher gain than an  
 704 ultrasensitive one and, if  $K > 1$ , the gain of the proportional controller will always  
 705 be higher, but this is not the case here because the error never goes above 1, given the  
 706 system parameters (the absolute maximum value of the error is  $P_f(0)$ ). The ultrasen-  
 707 sitive response in the HOG branch also allows tuning of the overall system response  
 708 towards certain signal regimes. In other words, the control system can be tuned to  
 709 filter out signals below a threshold and respond only when volume decreases cross



710 this threshold. Considering that glycerol production is potentially highly costly for  
711 the cell, this ability of the system could give an evolutionary advantage by allowing  
712 cells to ignore short lived or low doses of osmo-shock.

## 713 4.7 Conclusions

714 The results illustrated in this chapter demonstrate the power of applying engineer-  
715 ing principles to the analysis of the osmoregulation system in yeast. Gennemark  
716 et al. [8] proposed a simple model that describes the essential physics and biology of  
717 osmoregulation. This model has been abstracted from another more detailed model,  
718 developed by [15], by focusing on fewer components which allow the reproduction  
719 of the main dynamics of the system: the cell controls the biophysical system (in  
720 particular in terms of volume and turgor pressure) by using two proportional con-  
721 trollers, which act in parallel and regulate the glycerol production and the glycerol  
722 outflow (see Fig. 4.2). This simple model captures the main dynamical features of  
723 the osmoadaptive response by predicting the behaviour of different strains (wild type  
724 and modified) with different inputs and confirming the existence of two mechanisms  
725 of control (see Sect. 4.3). Note, however, that in general the volume adapts while  
726 the turgor pressure does not, because the model parameters are estimated using only  
727 glycerol concentration measurements which are not sufficient for complete recov-  
728 ery of both volume and turgor pressure. Therefore the model does not show robust  
729 adaptation, since the adaptation requires a careful tuning of the system parameters.

730 The group of van Oudenaarden, using frequency domain analysis, identified a  
731 minimal model represented by a LTI system with only two dynamics variables (see  
732 Sect. 4.4). Then, they estimated the biological quantities corresponding to the two  
733 relevant variables of the LTI model and, using these results, deduced the network  
734 diagram of Fig. 4.10. Using biological measurements and engineering principles,  
735 they showed that the robust perfect adaptation of Hog1 nuclear enrichment and cell  
736 volume (as turgor pressure) results from one integrating mechanism that requires  
737 Hog1 kinase activity and regulates the glycerol synthesis (see Sect. 4.5).

738 The models of Figs. 4.2 and 4.10 seem similar at a “formal” level but they are  
739 quite different from the system theoretical point of view. The model of the group  
740 of Van Oudenaarden is inferred by employing the Hog measurements (the output  
741 of the model) and contains one branch of control modelled with exactly one in-  
742 tegrator. Instead, in Gennemark’s model, the Hog protein cannot be observed (it is  
743 not a variable of the model and the Hog controller does not have a direct biological  
744 correspondence) and the two branches of control are modelled using simple propor-  
745 tional controllers. The model could be modified by adding measurable variables, for  
746 example Hog1, but this would obviously increase the complexity of the model.

747 Interesting additional results were recently presented in [20], where the authors  
748 investigated which network topologies in a generic signalling network are capable  
749 of robust adaptation. In particular, they used a network of three nodes as a minimal  
750 framework, where there is a first node that receives the input, a second that transmits

751 the output and a third that can play diverse regulatory roles. They found that all  
 752 the networks containing one of the following two motifs achieve adaptation: neg-  
 753 ative feedback loop with a buffering node and incoherent feedforward loop with a  
 754 “proportioner” node.

755 Despite the many striking insights that have been produced into the yeast osmoreg-  
 756 ulation system by the above analyses, it is still not clear how the evolution of biologi-  
 757 cal control systems of this type can result in integral feedback, and in our recent work  
 758 we investigated a heretofore largely unexplored alternative control system which also  
 759 appears to be able to achieve perfect adaptation. In particular, we extended the pro-  
 760 portional control model developed by [8] with the implementation of ultrasensitivity  
 761 and found that a proportional controller implementing ultrasensitivity allows more  
 762 precise and faster adaptation of cell volume following an osmo-shock. Further, the  
 763 ultrasensitive controller can be tuned as a filter, where the proportional controller  
 764 could not, and thereby allows responses to signals only above a certain threshold  
 765 (see Sect. 4.6). These results provide new insights on the potential role of gain in  
 766 biological systems and should be of interest to synthetic biologists attempting to  
 767 design robust biomolecular control systems.

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