Experiments in Fluids 23 (1997) 48–53 © Springer-Verlag 1997

A numerical study of the influence of the Basset force on the statistics of LDV velocity data sampled in a flow region with a large spatial velocity gradient

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Abstract The motion of small particles, such as those typically used as seeding particles for tracer particle flow velocity measurement techniques, is studied numerically for a flow region with a large spatial velocity gradient. The influence of the Basset history integral on the statistics of results of particle motion calculations which are based on multi-disperse particle size distributions is investigated. The biasing of the measured velocity data, with regard to the actual flow velocity, which results as a consequence of such particle size distributions is discussed. It is found that the net effect of the Basset integral on the calculations is indeed to reduce the maximum RMS deviation associated with the multi-disperse distribution and that the relative reduction increases with a decreasing particle density. The main result of this study is, thus, that it is desirable to use light tracer particles not only because they more readily adjust to a changed flow velocity but in particular also because they tend to contribute less to the overall RMS deviation of velocity data sampled in a region with a large spatial gradient of the flow velocity.

1 Introduction

The optimisation of non-intrusive flow velocity measurement techniques such as laser Doppler anemometry (LDA) or particle image velocimetry (PIV) has seen a rapid progress over the last two decades or so. Major improvements were in particular brought about by the advances in computer technology with increasingly powerful yet affordable computers becoming available and by the incorporation of fibre optic technology for LDA systems. The latter, for instance, led to the design of more versatile systems as a result of small transmitting and receiving optics becoming available which can be spatially separated from the laser. LDA has been firmly established as a standard tool in virtually all branches of experimental fluid dynamics for some years already and PIV will soon also be considered as such.

Whereas the technical aspects of both these measurement techniques have experienced major advances over the years one well known key problem associated with both of them on the other hand still remains. Both techniques are tracer techniques and, as such, rely on the velocity information obtained from small tracer particles contained in and transported with the flow that is investigated. Only if the tracer particles faithfully follow any changes of the flow velocity one can expect the measurements to yield velocity data which accurately represent the actual flow velocity. If the tracer particles are too large or if their density is too high then, as a result of their inertia, they may not respond to velocity changes sufficiently rapidly and may display a velocity lag instead. This, in turn, results in the collection of experimental velocity data which do not represent the actual flow velocity.

Because of its crucial importance to tracer type flow velocity measurement techniques the motion of tracer particles has, over the years, been the scope of many experimental and theoretical investigations. A relatively recent summary of some of the most relevant literature on this research can be found in Thomas (1991), Thomas (1992), Thomas and Bütefisch (1993) and Thomas et al. (1993) and it does not appear reasonable to repeat this summary here.

When experimenters work with techniques such as LDA the flow velocity at some point in a flow field is usually obtained by determining the mean value and the associated RMS deviation of a large number of individual single velocity measurements at that point. If the individual velocity data are collected from tracer particles with different sizes, at a point in the flow field with a constant flow velocity but which is located in a region with a sufficiently large spatial velocity gradient, then one necessarily measures velocity values which fluctuate around some mean value simply as a result of the different response behaviours of the particles to the changing flow velocity. These fluctuations do not indicate a turbulence of the flow; they cannot, however, be discerned from those fluctuations which do indeed result from a real turbulence of the flow. As a consequence of this the analysis of such experimental data becomes difficult and care has to be taken in order to avoid erroneous interpretations of the data.

The influence of the size distribution of tracer particles on velocity data collected in regions with a large spatial velocity gradient has been studied in detail, experimentally and numerically, in our previous publications Thomas (1991), Thomas and Bütefisch (1993) and Thomas et al. (1993). For the numerical simulations presented in these studies the contribution of the Basset history integral which appears in the equation describing the particle motion has been neglected. This integral term takes into account deviations of the flow pattern from the steady state and is generally interpreted as an additional flow resistance. The influence of the Basset history integral on the motion of single particles in such regions with

Received: *9 August 1996*/*Accepted*: *25 January 1997*

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large spatial velocity gradients has been studied numerically by Thomas (1992). It was shown there that it can have a considerable influence on the particle motion of light particles for instance in typical measurement applications across aerodynamic shocks in compressible flows. In our previous studies we did not, however, consider the overall effect of the Basset force on the mean values obtained from samples of tracer particles which contain particles with different sizes and which are subject to this type of flow.

Because of the undesired influences associated with the tracer particles' response behaviour to large velocity gradients experimenters generally wish to use mono-disperse samples of tracer particles in order to exclude a biasing of the data as a result of different particle sizes. Tracer particles whose size distribution is sufficiently narrow to be considered mono-disperse are generally available; one example are for instance latex spheres. Nevertheless, practice shows that these particles can often only be generated and then introduced in the flow at number rates insufficient for certain applications. Hence, in practice experimenters often have to revert to tracer particles with a size distribution which is far from mono-disperse such as for instance oil droplets or certain types of smoke generated by some sort of particle generator.

As the Basset force can have a considerable influence on the motion of single particles in region with a large spatial velocity gradient it is then of interest to determine its overall influence on the results sampled from multi-disperse tracer particle distributions. The purpose of the present short numerical parameter study is thus to investigate this influence for tracer particles which are distributed according to some known size distribution. For a typical practical situation which is encountered in similar form in many everyday measurement applications in compressible flows, i.e. for the flow across an aerodynamic shock, the mean velocity and the associated value of the RMS deviation that result from the different response behaviours of the particles contained in the size distribution are determined. The magnitude of the contribution due to the Basset force is determined and it is shown how this contribution varies with the size distribution and the particle density.

2
Theoretical description of the particle motion

The motion of a small (size of the order of μm), spherical, non-deformable particle can be described by the Basset–Boussinesq–Oseen (BBO) equation which is given in the notation of Soo (1967) by

$$\frac{4\pi}{3}r_p^3\rho_p\frac{d\mathbf{U}_p}{dt_p}=\frac{4\pi}{3}r_p^3\rho_p G(\mathbf{U}_F-\mathbf{U}_p)-\frac{4\pi}{3}r_p^3\frac{\partial P}{\partial \mathbf{r}}+\frac{1}{2}\frac{4\pi}{3}r_p^3\rho_F\frac{d}{dt_p}(\mathbf{U}_F-\mathbf{U}_p)+6r_p^2\sqrt{\pi\rho_F\mu_F}\int_{t_{p_0}}^{t_p}\frac{d/d\tau(\mathbf{U}_F-\mathbf{U}_p)}{\sqrt{t_p-\tau}}\,d\tau+\mathbf{F}_a \quad (1)$$

with

$$G=\frac{3}{8}C_D\frac{\rho_F}{\rho_p}\frac{1}{r_p}|\mathbf{U}_F-\mathbf{U}_p|, \quad [\mathrm{s}^{-1}] \quad (2)$$

In Eq. (1) and (2) $\mathbf{U}_p$ and $\mathbf{U}_F$ denote particle and fluid velocity respectively and ρ_p, ρ_F are the density of the particle material and the fluid. The radius of the particle is r_p, the viscosity of the fluid is denoted by μ_F and the drag coefficient of the particle is denoted by C_D.

The physical significance of the five terms on the right hand side of Eq. (1) is as follows: The first term represents the force acting on the particle due to a stationary viscous flow. (This term can be seen to be the well-known Stokes law for the viscous forces acting on a sphere if the drag coefficient is expressed as $C_D=24/Re$ and with the usual definition of the Reynolds number Re.) The second term is due to the pressure gradient in the surrounding fluid. The third term is referred to as added or apparent mass and represents the force required to accelerate the mass of the fluid surrounding the particle and moving with it; the increment for a sphere being one half the mass of the fluid displaced ($\frac{1}{2}m_F$). The fourth term is the Basset history integral which was already discussed in the introduction and whose contribution to the particle motion is the subject of this study. The last term on the right hand side of Eq. (1) denotes external forces such as gravity.

For the flow considered in this study, i.e. for the flow across an oblique shock, the pressure term disappears in the region downstream of the shock where the flow conditions are assumed constant. The present study is only concerned with the influence of the Basset integral on the particle motion calculations, thus the added mass term and influences of external forces will be neglected. It is noted that it is indeed justified to also neglect these terms under the flow conditions considered here if quantitative comparison between numerical simulation and experimental data were to be carried out; this has been discussed in detail in Thomas (1991).

The simplified equation for the particle motion across the shock is integrated numerically by means of the Bulirsch–Stoer method using routines (ODEINT, BSSTEP, RZEXTR, MMID) of Press et al. (1986). The integration by these routines incorporates an automatic, adaptive changing of the fundamental stepsize during the integration process in order to monitor numerical errors and to ensure accuracy. The overall required accuracy of the integration, set by the value of the parameter EPS which is passed to the integration routines from the main program, was for all runs of the simulation at least of the order 10^{-6}.

The calculations presented are based on the equation for the drag coefficient C_D by Henderson (1976). The results of particle motion calculations depend, of course, on the underlying C_D equation which is used for the simulations. A detailed general discussion of the magnitude of the influence of the C_D equation on such simulations was carried out by Walsh (1975) and Walsh (1976) for the motion of single particles; a brief discussion of the influence for the particular flow configuration considered here is included in Thomas (1991). However, the main purpose of the present study is to investigate the general qualitative influence of the Basset term on particle motion calculations and this should not depend on the underlying C_D model.

With regard to our earlier experimental and numerical investigations (Thomas 1991, Thomas 1992, Thomas and Bütefisch 1993, Thomas et al. 1993) the same typical measurement situation for the flow across on oblique aerodynamic shock is considered as in these publications. The flow geometry is indicated in Fig. 1 and the typical measurement situation is that the flow velocity has to be measured along the traverse $\overline{EF}$.

Because of the deflection of the flow across the shock a particle detected at any location along $\overline{EF}$ has originally crossed the shock at a position lying below the intersection of $\overline{EF}$ and the shock. A transformation of the particle motion into the measurement system of the LDA is obtained by simple geometric considerations (Thomas 1991); the results discussed are presented in the co-ordinate system x, y shown in Fig. 1. The flow velocity across the shock was modelled as a hyperbolic tangent profile (see Thomas, 1991).

The particle distribution on which the calculations are based is shown in Fig. 2. This distribution was originally obtained experimentally by a TSI particle sizer for a sample of olive oil particles which were used as seeding particles for some of the experiments summarised in our previous papers. For the present study the particle motion was calculated individually for $i = 33$ different size classes d_{p_i}. The results were then weighted and averaged in accordance with the contribution of each size class to the size distribution in order to obtain a mean

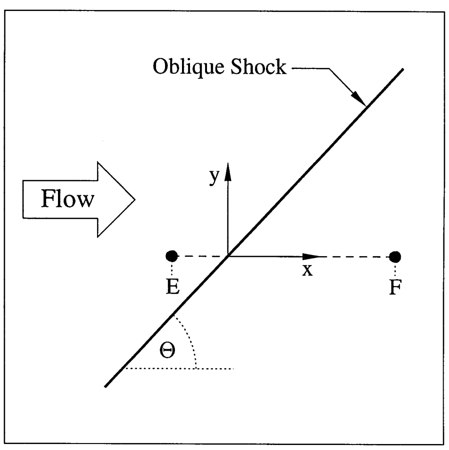

Fig. 1. Sketch of the flow geometry assumed for the numerical simulations

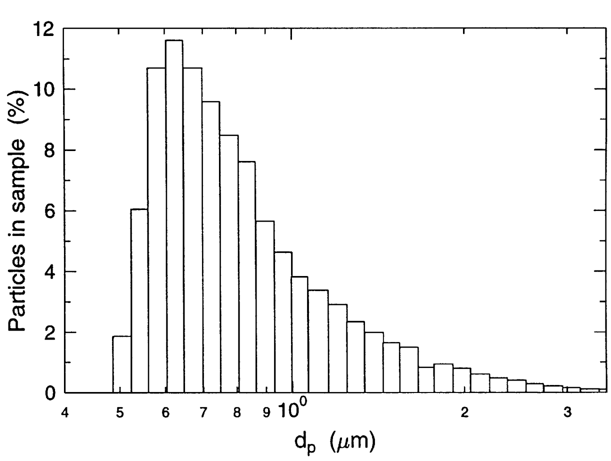

Fig. 2. The particle size distribution on which the numerical simulations are based

velocity curve and the associated curve for the RMS deviation for the velocity components in the main flow direction (x-axis) and perpendicular to it. In order to determine the influence of the Basset history integral on the results the calculations and the subsequent data averaging were carried out twice. The calculations were first based on an equation of motion for which the Basset integral was neglected and the whole process was then repeated but this time based on the equation of motion for which the Basset integral was retained.

3
Numerical results

The results presented below refer to the component of the flow velocity in the main flow direction. The RMS deviation is given in percent with respect to the constant free-stream velocity ahead of the shock. The results are based on the following flow conditions upstream of the shock: Mach number $M = 1.93$, total pressure $P_0 \approx 97416$ Pa, static pressure $P_1 \approx 13863$ Pa, total temperature $T_0 \approx 291.4$ K. The constant free-stream velocity upstream of the shock is $u_1 = 500.0$ m s^{-1}; it is assumed that all particles travel with the flow at this velocity ahead of the shock. The shock angle is $\Theta = 46^\circ$ and the shock thickness is about 7 μm in the direction perpendicular to the shock.

Figure 3a, and b show the calculated mean velocity and the associated RMS deviation as a function of the position downstream of the shock respectively. The results are based on the size distribution of Fig. 2. Figure 3a, and correspondingly Fig. 3b, shows three pairs of curves. Each pair corresponds to the results obtained for a different value of the particle density ρ_p. For each pair the solid curve represents the results obtained from the calculation for which the Basset integral was neglected. The corresponding curve for the calculation for which the Basset integral was retained is in each case the curve immediately below the particular solid line considered. It should be noted that unlike in our earlier papers the abscissa of the figures in this study is plotted on a logarithmic scale in order to be able to more clearly summarise the results for all three values of the particle density in one figure.

The curves for the mean velocity $\bar{u}$ displayed in Fig. 3a simply confirm qualitatively for the mean values the results which were already obtained for single particles in our earlier papers. These results were that, of course, the distance required by the particles to adjust to the reduced flow velocity downstream of the shock decreases with the particle density and further, and more importantly, that close to the shock the Basset force acts as an additional flow resistance while its overall effect is, indeed, to increase the distance required for an adjustment to the reduced flow velocity (compare Thomas 1992).

Figure 3b shows the associated curves for the RMS deviation. As has already been described and discussed in Thomas (1991) the maximum value of the RMS deviation for those simulations which are based on the equation of motion for which the Basset integral was neglected is essentially independent of the particle density. Additionally one obtains the following new results from Fig. 3b. One effect of the Basset force is to increase the RMS deviation in the region between the shock and some position just upstream (i.e. to the left) of the maxima of the curves. For locations

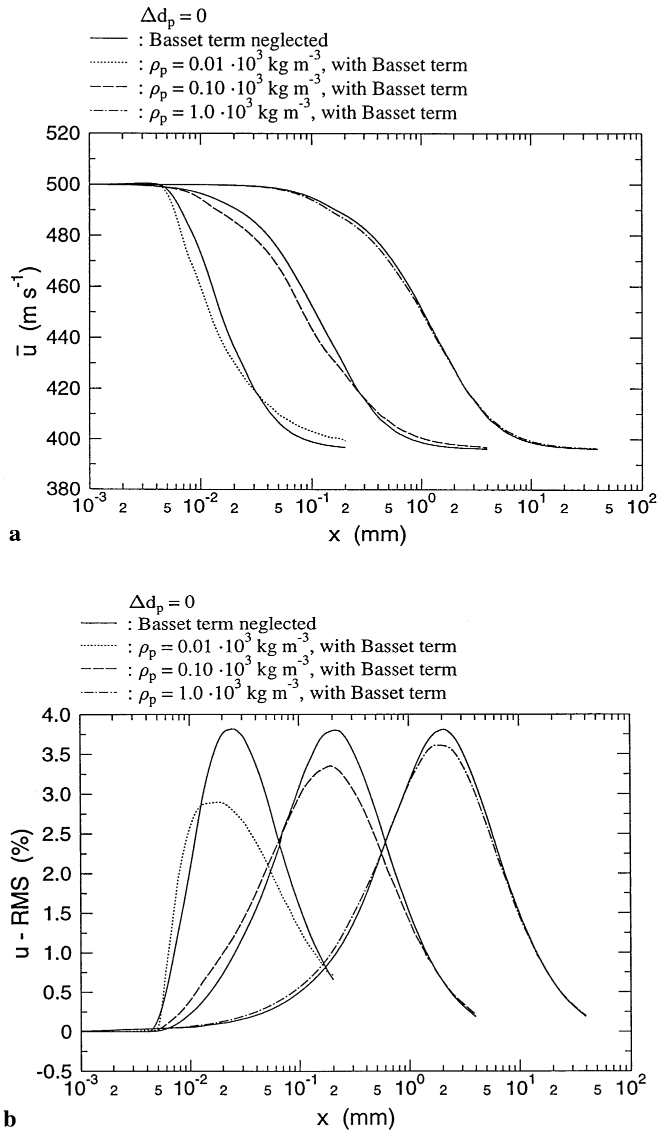

Fig. 3. **a** mean velocity $\bar{u}$ and **b** associated RMS deviation as function of the downstream distance x from the shock for a numerical simulation based on the particle size distribution of Fig. 2 and for three different values of the particle density ρ_p

downstream of this position the effect of the Basset force is, on the other hand, to decrease the RMS deviation. Additionally, the inclusion of the Basset term also shifts the position of the maxima slightly upstream with regard to their positions if the Basset force is neglected. The main new result one obtains from Fig. 3b is, however, that the inclusion of the Basset term reduces the RMS deviation at the position of the maxima and that the magnitude of this reduction increases with decreasing particle density. For a particle density of $\rho_p = 1.0 \times 10^3$ kg m^{-3} one finds, for instance, a RMS deviation of approximately 3.8% at the position of the maxima if the Basset term is neglected and a value of approximately 3.6% for the corresponding calculation for which the Basset term was retained. This absolute decrease of 0.2% corresponds to a relative decrease of approximately 5%. Correspondingly one finds for a particle density of $\rho_p = 0.01 \times 10^3$ kg m^{-3} a RMS deviation of approximately 3.8% if the Basset term is neglected and a value of approximately 2.9% for the calculation for which Basset term was retained; the absolute decrease of 0.9% corresponds in this case to a relative decrease of approximately 23%.

In order to determine how the Basset term affects the results if the calculations are based on a different but similar size distribution with larger particles the calculations were repeated after increasing the particle diameter of each of the i size classes d_{p_i}, by $\Delta d_p = 1$ μm to a new value of $d_{p_i} + \Delta d_p$. The results obtained with this modified size distribution are shown in Fig. 4a, b. The results are qualitatively identical to those of Fig. 3a, b. Nevertheless, the overall value of the RMS deviation is lower for the results displayed in Fig. 4b which are based on the larger particles – this was already observed and discussed in Thomas (1991), Thomas and Bütefisch (1993) and Thomas et al. (1993).

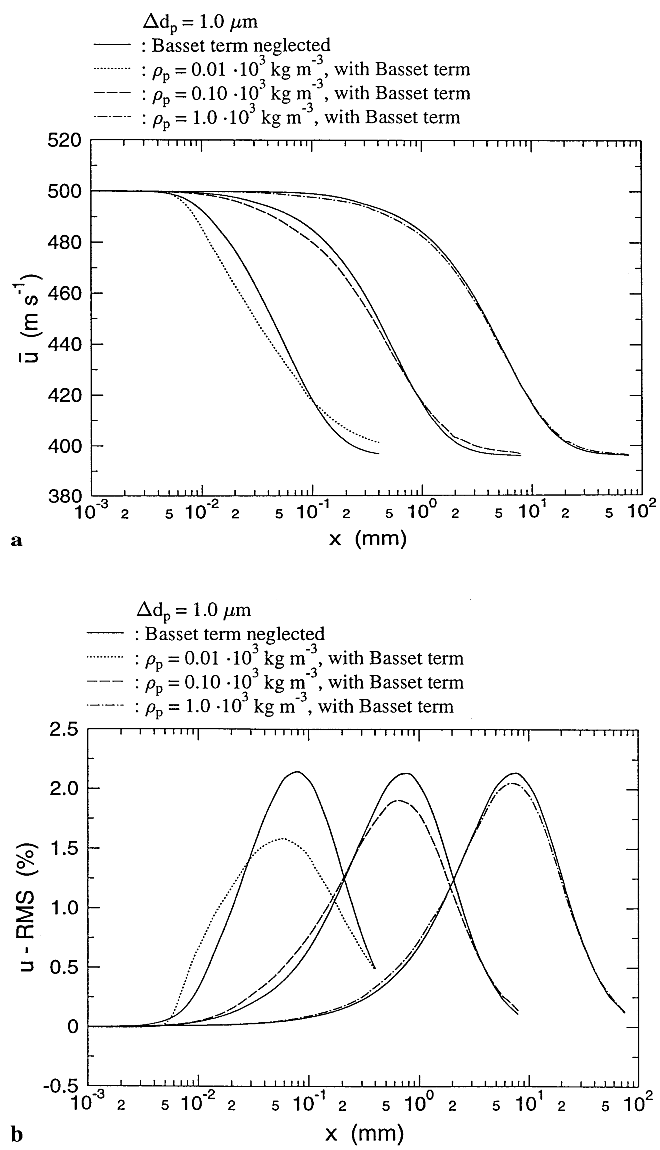

Fig. 4. **a** mean velocity $\bar{u}$ and **b** associated RMS deviation as function of the downstream distance x from the shock for a numerical simulation based on the particle size distribution of Fig. 2 which is shifted by Δd_p towards larger particle sizes and for three different values of the particle density ρ_p

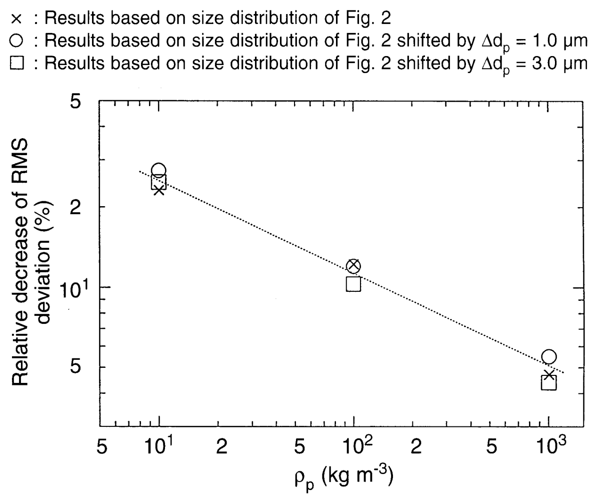

Fig. 5. Relative decrease of the RMS deviation at the position of the maxima of the RMS curves if the Basset term is retained in the numerical simulations as a function of particle density ρ_p

In order to summarise the influence of the Basset term on the calculations the relative decrease of the RMS deviation at the positions of the maxima, as obtained from the data of Fig. 3b and Fig. 4b, is plotted in Fig. 5 as a function of the particle density ρ_p. Figure 5 additionally includes three data points which were obtained for a simulation for a size distribution which was modified by shifting the original distribution of Fig. 2 by $\Delta d_p = 3$ μm. The maximum value of the RMS deviation for the case when the Basset term was neglected is approximately 1.13% for this simulation and, as previously, independent of the particle density. The curves for the mean velocity $\bar{u}$ and the associated RMS deviation are qualitatively identical to those shown in Fig. 3 and 4 and it did not appear necessary to include them here. Figure 5 shows that the relative decrease of the RMS deviation, for a fixed value of the particle density, is approximately the same for the three different particle size distributions considered. This indicates that the decrease does not depend strongly on the exact statistical properties of the distribution for the flow conditions assumed for this study. The data of Fig. 5 are displayed in a double logarithmic representation. In this plot all data points can be summarised approximately as a straight line which indicates a dependence of the relative decrease of the RMS deviation on the particle density in form of a power law as ρ_p^m. From a least squares fit to the data points of Fig. 5 one finds a value of $m \approx -0.35$ for the exponent. This result can, of course, not be considered to be more than a rule of thumb as the exponent might take on different values if the simulations are based on other assumptions for the flow conditions etc. Nevertheless, it is reasonable to expect to find a similar dependence as long as these conditions are not too different from those assumed here.

4
Discussion and conclusion

The results of a numerical parameter study involved with the motion of small particles through a region in a moving fluid with a large spatial velocity gradient have been described. The relative contribution of the Basset history history integral to the mean velocity and the associated RMS deviation obtained from particle motion calculations based on multi-disperse particle size distributions has been determined. The magnitude of the relative contribution of the Basset term has been studied in dependence of the particle density and the particle size distribution. The main result of the study is that the Basset term can have a considerable influence on the RMS deviation for particles with a low density and for the flow conditions considered. It was found that the Basset term decreases the maximum RMS deviation. The relative decrease was found to be of the order of up to 25% for light particles with $\rho_p = 0.01 \times 10^3$ kg m^{-3} and it was found to scale with the particle density approximately as $\rho_p^{-0.35}$ under the assumptions made for the simulations.

It is unfortunately not possible to draw many general conclusions from a numerical parameter study such as the present one. As any experimenter will be well aware of there are numerous other contributing factors which influence the actual data collected in measurement applications and this consequently significantly complicates the problem. For instance, not all the tracer particles present in the flow field might be detected. Be it because of the scattering properties of the particles, or as a result of a selective sensitivity of the measurement device which might favour the larger particles, or be it because of certain particles not moving into some areas of the flow field. The results of the numerical simulations also depend on the flow conditions and to a certain degree on assumptions such as those made for the equation used for the drag coefficient. Because of this it does not appear reasonable to extend the present investigation and to include any further parameter variations as the study would necessarily always remain incomplete.

However, there is one important general conclusion that can be drawn from the results presented and discussed here. This study shows that while the Basset force becomes increasingly more important for small light particles its net effect on multi-disperse tracer particle distributions is indeed to reduce the maximum RMS deviation resulting from the different response behaviour of particles of different size to a gradient of the flow velocity. It is thus important to realise that it is not only desirable to use small and light tracer particles because they more readily adjust to a changing flow velocity but in particular also because particles with a low density effectively tend to reduce the biasing of the experimental data caused by a multi-disperse particle size distribution.

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