Experimental verification of Type-II-eigenmode destabilization in the boundary layer over a compliant rotating disk

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Destabilization of the Type-II eigenmode in boundary layers over compliant rotating disks was predicted theoretically by Cooper and Carpenter [J. Fluid Mech. **350**, 231 (1997)]. Their results showed that for relatively low levels of compliance the Type-II eigenmode was destabilized, to be stabilized and ultimately eliminated for higher levels of compliance. The goal of the present study was to obtain the first experimental verification of the prediction that the Type-II mode can be destabilized at low levels of compliance. To this end a new type of rotating-disk apparatus was designed and a new type of material was used to produce suitable compliant walls for the experiments. Background noise in the new facility is substantially reduced in comparison with that in facilities used in related previous studies. This enabled the detection of substantially cleaner hot-film signals. Although the mean base flow remained unchanged, noise characteristics have been improved and turbulence intensities are significantly reduced. The measurements reveal not only the comparatively strong signals from the Type-I (cross-flow vortices) instability mode but also clear evidence of the Type-II eigenmode. In agreement with the theory of Cooper and Carpenter the data analysis shows that relatively low levels of wall compliance destabilize the Type-II mode. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2202175]

I. INTRODUCTION

This article summarizes results of an experimental study investigating effects of wall compliance on the laminar-turbulent transition process of the boundary layer over a rotating disk. Cooper and Carpenter[1] previously studied these effects theoretically and some of their main results were confirmed experimentally by Colley *et al.*[2] However, the experimental apparatus of Colley *et al.* was inadequate for testing some other key predictions of Ref. 1. Here we describe new experiments that were obtained with a modified and substantially improved experimental facility. These experiments have now enabled us to test those predictions of Ref. 1 that had hitherto not been corroborated. In the remainder of this introduction we shall briefly review the background of our research together with the key concepts and terminologies required to follow the presentation and the discussion of the results.

For almost six decades rotating-disk flow has served as the foremost model problem for studying laminar-turbulent transition in fully three-dimensional (3D) boundary layers. From an applied point of view rotating-disk flow owes its importance to the fact that the transition of the rotating-disk boundary layer shares many similarities with the transition of other 3D boundary layers found in typical engineering applications. One example is, for instance, transition over highly swept wings. Comprehensive reviews of instability and transition in rotating-disk boundary-layer flow over rigid surfaces have been published by Reed and Saric[3] and, more recently, by Saric *et al.*[4]

The original motivation for investigating the effects of wall compliance on the transition of boundary layers dates back to suggestions[5] that dolphins achieve their high swimming speeds by controlling the boundary-layer flow over their compliant skin. The underlying idea being that the fluid flow over the dolphin's body is controlled by its interaction with the flexible, deformable dolphin skin in such a way that boundary-layer transition and turbulence is suppressed and, as a consequence, the skin-friction drag reduced. Although most of the quantitative estimates contained in Ref. 5 were shown to be inaccurate[6] it has, nevertheless, been established for many years[7–12] that wall compliance can postpone transition for the essentially two-dimensional (2D) boundary layer over a flat plate. Consequently compliant walls, or compliant coatings, are a viable method for drag reduction in such quasi-2D flow environments. However, most flows of engineering interest, such as the flow over swept wings, the flow past ships or, in fact, the flow over a dolphin's body are 3D flows. Consequently Cooper and Carpenter[1] extended previous theoretical work[10,12] from the 2D case to the rotating-disk flow as the classic 3D-flow paradigm. The state of the art for the use of compliant walls for flow control has recently been comprehensively summarized in Carpenter and Pedley,[13] which among other things contains a review of rotating flows over compliant walls.[14]

The 3D rotating-disk flow is not only of applied relevance. It is also very appealing to experimentalists and theoreticians in the context of more fundamental issues. For the experimentalist it represents a relatively easily accessible

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and low-noise environment for making detailed measurements—if compared with wind tunnels or water channels. Its attraction for the theoreticians results from the existence of a similarity solution that is an exact solution to the Navier-Stokes equation. The solution was first derived by Kármán[15] for the case where an infinite disk rotates under a fluid that is at rest far above the spinning disk. This flow configuration is now referred to as Kármán flow and it displays some differences in comparison with the base flow for the closely related cases of a stationary disk under a rotating fluid (Bödewadt flow), the case when disk and fluid co-rotate with approximately equal rotation rates (Ekman flow) and the case of the flow in the finite, fluid-filled gap between a rotating and a stationary disk or between two co-rotating disks (Batchelor flow). The differences between the radial and azimuthal flow-velocity profiles for these four flow configurations are discussed in detail in Owen and Rogers.[16] Despite existing differences of the base flow the transition process of the associated rotating-disk boundary layers share—in all four configurations—three different types of instability mode.

The most widely known mode is, probably, the Type-I cross-flow instability (or Class B in some older publications). This is an inviscid instability mode arising as a consequence of the existence of an inflection point (Rayleigh point-of-inflexion criterion[17]) in the velocity profile of the radial flow component. The Type-I mode leads to the well-known, characteristic spiral vortices easily observed in flow-visualization experiments within the laminar-turbulent transition region—see for instance photographs in Reed and Saric[3] or in Wimmer.[18] The Type-II (or Class A) mode is an viscous instability that arises in association with Coriolis forces. The discovery of the Type-I and Type-II modes dates back to Smith[19] and, in particular, to the seminal theory and experiments of Gregory *et al.*[20] and Faller.[21] The third instability mode (Type-III) was first identified by Mack.[22] Its significance, however, has only been identified relatively recently in the theoretical and experimental work of Lingwood[23–25] and she has shown that this mode can coalesce with Type-I to form an absolute instability.

The first experiments on the evolution of boundary-layer disturbances over a compliant rotating disk were carried out by Colley *et al.*[2] They showed that for flow over compliant disks the turbulence levels in the transitional and fully turbulent flow regimes are lower than the corresponding values for the rigid disk. In agreement with one of the key theoretical predictions of Cooper and Carpenter[1] the study of Colley *et al.*[2] revealed, in particular, that wall compliance has a stabilizing influence in the frequency range associated with the (inviscid) Type-I cross-flow instability. However, the overall influence of wall compliance was observed to be destabilizing, in that flow transition had in fact advanced to lower critical Reynolds numbers.

Colley *et al.*[2] argued that the reduced transitional Reynolds number for flow over compliant disks may have been due to the destabilizing effect of wall compliance for the range of lower wave numbers associated with the viscous Type-II instability mode. This destabilizing effect was another key theoretical prediction of Cooper and Carpenter.[1] The sampled hot-wire data of Colley *et al.*[2] were too noisy to identify the Type-II mode. Consequently, after the publication of Ref. 2 we developed modifications of the experimental facility that greatly reduced the environmental noise, thereby enabling us to investigate the effect of wall compliance on the Type-II eigenmode. Thus, the main purpose of the present article is to describe our new experimental results demonstrating that wall compliance can indeed destabilize the Type-II eigenmode, as predicted theoretically by Cooper and Carpenter.[1]

The original study of Colley *et al.*[2] followed fairly closely the methodology of Jarre *et al.*[26–28] developed to study corresponding flow phenomena over rigid disks. The design of their experimental facility was based on that of Jarre *et al.*[26–28] and the same data-analysis techniques were employed to determine growth rates and other characteristics of the inviscid Type-I disturbances developing in the boundary-layer flow. Although the type of apparatus and the techniques of Jarre *et al.*[26–28] proved to be adequate for resolving the effects of wall compliance on the Type-I mode we now know that they are not sufficient to investigate the corresponding effects on the Type-II mode.

In this article we shall first describe a new experimental facility that was specifically designed to overcome the inadequacies of our previous apparatus. Second, we shall present experimental results, obtained with the new rig, clearly demonstrating that, in accordance with the theory of Cooper and Carpenter,[1] wall compliance can destabilize the Type-II eigenmode for moderately compliant walls.

Some of the main results discussed here were first presented in a paper and a poster presentation at the *4th EUROMECH Fluid Mechanics Conference*[29] but the present article also contains additional results of supporting nature that were obtained more recently.

II. EXPERIMENTAL SETUP AND TECHNIQUES

A. The rotating-disk facility

The original experimental facility used by Colley *et al.*[2] and the new facility developed for the present study are illustrated and compared with each other in Figs. 1(a) and 1(b). For additional technical details also refer to Colley *et al.*[2]

Briefly, in the original setup the disk rotated with a frequency f_D corresponding to a rotational velocity $\Omega = 2\pi f_D$ within an inner partial enclosure. This whole unit was contained in a larger tank filled with water as shown in Fig. 1(a). The enclosure consisted of a lid with an annular sidewall. The height of the lid above the surface was $h = 20$ mm.

The thickness δ of the boundary layer over a rotating disk is given by[16]

$$\delta = 5.5\left(\frac{\nu}{\Omega}\right)^{1/2}, \quad (1)$$

where ν represents the kinematic viscosity of the fluid medium. For the experiments summarized by Colley *et al.*[2] conditions were such that $h/\delta \approx 10$. This ratio was assumed to be sufficient to ensure that the flow above the rotating disk

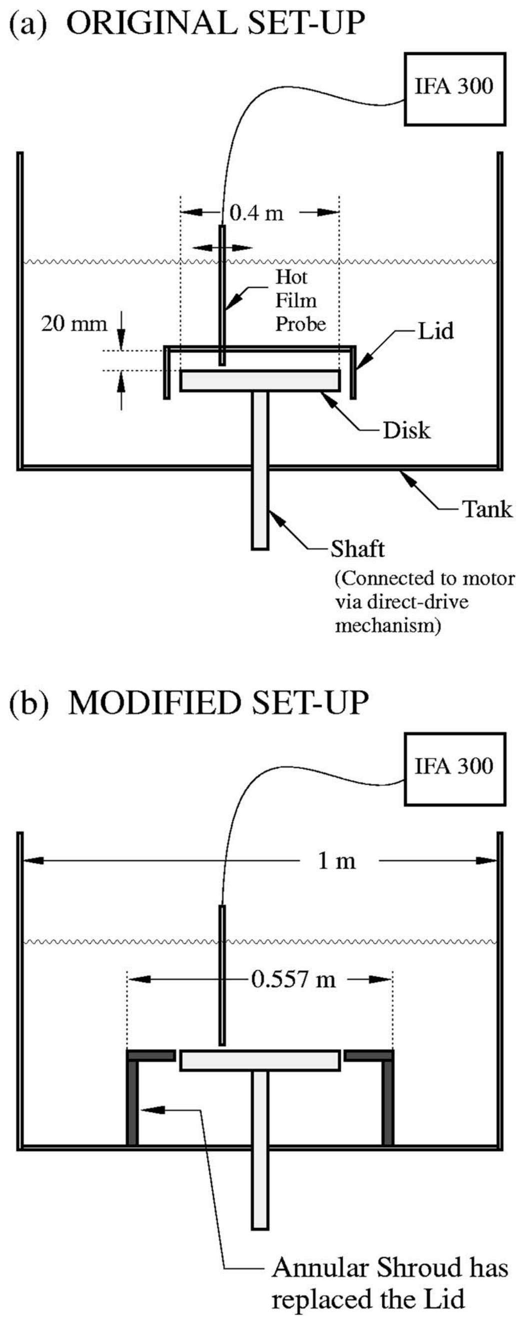

FIG. 1. (a) The original rotating-disk facility of Colley *et al.* (Ref. 2) and (b) the new facility of the present study (sketches not to scale).

resembles the Kármán type[15,16]—for which the fluid is at rest far above the disk.

Measurements were carried out with a calibrated TSI ISA 300 constant-temperature hot-film anemometry system. As discussed in Colley *et al.*[2] calibration of the system *in situ* is difficult; it required considerable technical effort to enable this to be done. We are not aware of any other existing rotating-disk boundary-layer velocity measurements with calibrated hot-film probes in water. The experimental configuration of Colley *et al.*[2] enabled them to successfully obtain their novel results. Nevertheless, at the time, they were unable to acquire any meaningful results without the lid being in place. Whenever the lid was removed the free surface of the water in the tank was highly disturbed making hot-film measurements impossible. However, the lid used in the original facility shown in Fig. 1(a) had several severe disadvantages. First, the detected hot-film signals were very noisy. Substantial noise was, no doubt, in part introduced as a result of turbulent flow leaving the disk and recirculating within the cavity between lid and disk. This source of noise had already been noted by Jarre *et al.*[26] whose facility also incorporated such a lid covering the disk. Second, the lid made it very inconvenient to use a hot-film probe because it restricts probe access to the measurement environment. The probe had to be recalibrated frequently but access was provided only through a radial slot in the lid. Further, the slot created additional disturbances affecting the flow in the cavity underneath it.

From our research carried out subsequent to the publication of Colley *et al.*[2] it emerged that the undesired strong disturbances to the free surface were mainly caused by the recirculation of the highly disturbed flow from *below* the rotating disk. The lid in the original setup of Fig. 1(a) was originally introduced to suppress influences on the boundary-layer flow arising from disturbances of the free surface. A superior and preferable experimental facility should dispense with the lid while simultaneously suppressing free-surface disturbances. Our new experimental facility has been designed accordingly.

The new configuration used here is shown in Fig. 1(b). The lid over the disk was removed and, instead, a stationary annular shroud surrounding the disk has been added as illustrated in Fig. 1(b). The diameter of the disk remains $d=0.4$ m as in the original facility. The diameter of the newly added annular shroud shown in Fig. 1(b) is $d_S=0.557$ m. The narrow radial gap between the rotating disk and the stationary shroud is approximately 1 mm wide. The experiments have shown that special care has to be taken to ensure that the shroud is aligned flush with the disk. If the shroud is not fitted correctly then the measurements can be affected.

Removing the lid eliminates the possibility of recirculating turbulent boundary-layer flow in the cavity between disk and lid. Introducing the shroud inhibits flow created in the region on the *underside* of the disk from disturbing the free surface and, thus, the flow above the test surface. Finally, it is significantly easier to make measurements in the new facility because access for the hot-film probe to the test surface is no longer restricted by the lid.

Note that a configuration similar to the shroud fitted to our facility has previously been used by Cham and Head.[30] Their rotating-disk experiments were, however, carried out in air whereas the present study investigates rotating-disk flow in water. Wilkinson and Malik,[31] who also studied rotating-disk flow in air, used a related technique whereby a flow-control panel with a circular hole was mounted at an elevation of 1.6 mm above the disk surface. The hole in the panel exposed the disk surface. The purpose of the panel was to duct the boundary layer leaving the disk away from the apparatus to avoid interactions with the incoming axial flow.

Corresponding to our experiments described in Colley *et al.*[2] the disturbances in all experiments summarized here were triggered by a small roughness element attached to the surface of the disk. As in Colley *et al.* the roughness element consisted of a small $(1\ \text{mm} \times 1\ \text{mm})$ piece of adhesive tape. The tape was approximately 0.1 mm thick and, thus, protruded into the boundary layer to a height of approximately 0.05δ. The dimensions of the roughness element were the same for the experiments with the rigid and the compliant disks such that the disturbances generated were also the same. The particular dimensions of the roughness element were chosen in order to produce disturbances that can be assumed to be of comparable magnitude to those generated in the study of Wilkinson and Malik.[31]

B. The compliant disks

The terminology "compliant," as used here, indicates that the surface of the disk is sufficiently flexible to deform in response to flow-generated pressure and shear stress when the disk rotates at a sufficiently high rate. The level of compliance is characterized, as usual, in terms of the modulus of elasticity, E. We have developed a precision apparatus for measuring this elastic modulus of our compliant coatings *in situ*, i.e., while the coatings are attached to the disk. The principle of operation of this apparatus is to press a small spherically shaped indenter (radius, $r_i = 5$ mm) onto the surface of the compliant material. The indentation depth, δ_i, is measured as a function of the restitutive force, N. The modulus of elasticity can then be determined according to classic Hertzian[32] theory from the expression $\delta_i = (9N^2/16E^2 r_i)^{1/3}$.

The compliant disk surfaces of Colley *et al.*[2] were manufactured from a single layer of room-temperature vulcanizing silicone rubber. For details relating to the manufacturing process refer to Ref. 2. The elastic modulus of their disk surfaces was in the range $290\ \text{kPa} \leq E \leq 380\ \text{kPa}$. For any given elastic modulus the effective level of compliance depends on the rotation rate—this is discussed in Refs. 1 and 2. The elastic modulus of the disks used by Colley *et al.*[2] corresponded to a comparatively low effective compliance at the rotational speed of $\Omega = 7.85\ \text{rad s}^{-1}$ at which their experiments were performed. According to the computations of Cooper and Carpenter[1] the response of these disks to the Type I cross-flow-vortex eigenmode should, in fact, have been almost indistinguishable from that of rigid walls.

In order to promote stronger effects of wall compliance on the boundary-layer flow above the disk we have used a new, much softer material in the present study. The compliant wall used here was manufactured from silicon gel (NuSil MED 6340, Polymer Systems Technology Ltd., High Wycombe, U.K.). The gel is supplied in two separate components. Varying the mixing ratio of these components enables compliant walls with different elastic moduli to be produced. The compliant disk manufactured for the present study had an elastic modulus close to $E = 80$ kPa; the mechanical properties were measured in the same way as previously done in Colley *et al.*[2] Note that the same material was used for the present study as for a recent study on the related problem of laminar-turbulent transition of torsional Couette flow.[33]

III. EXPERIMENTAL RESULTS

For the discussion of the experimental results the following conventional nomenclature is recalled. The Reynolds number RE associated with the rotating disk flow is defined as

$$\text{Re} = \frac{r}{\delta^*}, \tag{2}$$

where r is the radial distance from the center of the disk and the displacement thickness δ^* is given by

$$\delta^* = \sqrt{\frac{\nu}{\Omega}}. \tag{3}$$

With respect to Kármán's[15] similarity solution δ^* corresponds to that height over the disk where the azimuthal component of the flow velocity equals $v_\phi = 0.5\ \Omega r$.

The vertical height, z, over the disk is nondimensionalized as follows:

$$\zeta = \frac{z}{\delta^*}. \tag{4}$$

The radial component, v_r, and the azimuthal component, v_ϕ, of the flow velocity in the boundary layer are expressed in nondimensional form as

$$F(\zeta) = \frac{v_r}{\Omega r}; \quad G(\zeta) = \frac{v_\phi}{\Omega r}. \tag{5}$$

The theoretical values for $F(\zeta)$ and $G(\zeta)$ according to Kármán[15] can be found in tabulated form, for instance in Ref. 16.

Recall that the hot-film probe is stationary. The disk with its vortex-like disturbances (wavelength: λ)—which are stationary in the rotating frame of reference—rotates below the probe. Thus, a measured temporal frequency, f, is really a measure of an azimuthal disturbance wave number, $k = 2\pi/\lambda$, associated with the $n = kr$ wavelengths occupying the disk perimeter at radius r. In order to facilitate the comparison of our data with Cooper and Carpenter,[1] Colley *et al.*,[2] Malik *et al.*,[34] and with Lingwood[23–25] we define the azimuthal disturbance wave number, as introduced in Ref. 34, in terms of

$$\beta = k\delta^* = n/\text{Re}. \tag{6}$$

The new experiments were carried out under experimental conditions reproducing those of the study of Colley *et al.*[2] as closely as possible. As previously, the disk rotates at a frequency of $f_D = 1.25$ Hz corresponding to a rotational velocity $\Omega = 7.85\ \text{rad s}^{-1}$. Some results were also obtained for higher rotational speeds.

A. Boundary-layer velocity profiles for the base flow

In order to test whether the modifications made to the experimental facility have affected the mean, base flow of the rotating-disk boundary layer we measured the radial and azimuthal velocity components in the new facility and compared the results to those from the old apparatus.

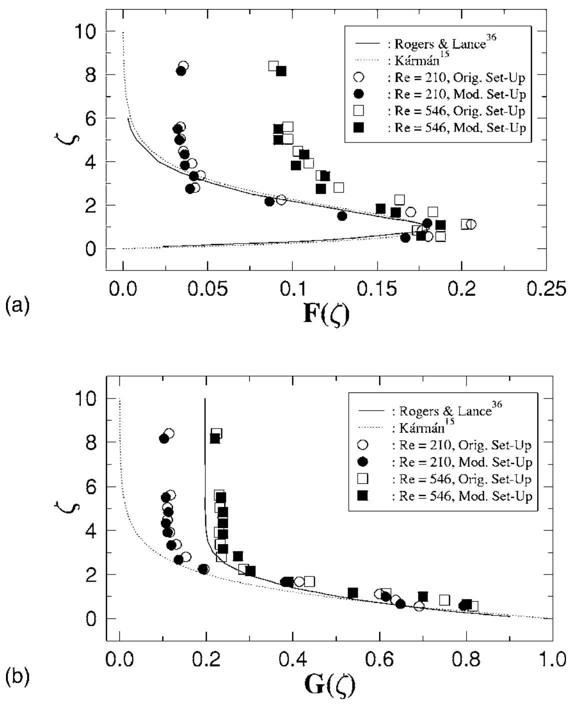

FIG. 2. Comparison between the measured and computed components of the mean flow velocity in the original and the new experimental facility. (a) Radial velocity component and (b) azimuthal velocity component.

Note there have been several previous experiments on disks spinning in water. Mean velocity profiles are, however, not reported in any of them apart from our own.[2] This is no doubt because of the difficulties of calibrating the hot-film probes. We went to considerable trouble and expense to do this—for technical details refer to Ref. 2.

Figure 2(a) displays the comparison of the data for the radial flow velocity in the original and the new setup. The comparison for the azimuthal flow velocity is shown in Fig. 2(b). In each case corresponding data sets collected at two different radial locations characterized in terms of the Reynolds number are shown. Figures 2(a) and 2(b) reveal that the measured mean velocity profiles for both velocity components are identical within experimental error.

The solid lines superposed in Figs. 2(a) and 2(b) represent a Batchelor-type profile[2,16,35] computed by Rogers and Lance.[36] The data according to Rogers and Lance[36] were computed for $\omega/\Omega=0.2$, where Ω and ω are, respectively, the rotational velocity of the disk and the rotational velocity of the fluid far above the disk. The data were taken from Fig. 3.4(a) in Ref. 16. The dotted lines represent the similarity solution after Kármán[15] which—with regard to Batchelor flow—corresponds to the case when $\omega=0$. For the Kármán profiles we have used the numerical values tabulated by Owen and Rogers.[16] The comparison of experimental and theoretical data in Fig. 2(b) data clearly shows that there is some residual rotational motion in the flow outside the boundary layer in *both* experimental facilities. Reference to Fig. 2(a) further reveals that $F(\zeta)\neq 0$ when $\zeta\rightarrow\infty$ which is at odds with the theoretical predictions of Kármán[15] and Rogers and Lance.[36] We presently do not have a firm explanation for this nonvanishing radial velocity other than that it may reflect boundary effects arising from the finite flow domain.

Residual motion may not be too surprising for the flow in the original facility of Colley *et al.*[2] where the ratio of cavity height h and boundary-layer thickness δ was $h/\delta\approx 10$. This ratio might be considered insufficient to approximate flow in an infinite environment underneath a stationary fluid as is required for the Kármán-type solution. However, for the present experiments with the new setup the height between the rotating disk and the free surface was approximately $H=0.16$ m such that the corresponding ratio is $H/\delta\approx 80$. Consequently, our new results show that for rotating-disk flow in water, at comparatively high values of H/δ, the boundary-layer velocity profiles of the base flow do not resemble the Kármán-type solution when $\zeta\gtrsim 3.0$. Even for these high ratios H/δ the profiles bear closer resemblance to the Batchelor-type profiles associated with flow above a disk spinning inside a stationary enclosure. We suspect that the residual rotational motion is a feature of all rotating-disk experiments in water. It may be associated with the finite lateral dimensions of the water-filled chambers or drums containing the rotating disk.

It is possible to use the theoretical results of Lingwood[25] to make rough estimates of the effects of the residual rotational motion on the critical Reynolds number for Type I, Type II and absolute instability. To this end first consider our Fig. 2(b). Figure 2(b) shows that for the lower values of the Reynolds number (Re=210), before laminar-turbulent transition occurs, the ratio of the residual rotational flow speed outside the boundary layer to that of the disk is approximately $G(\zeta)=0.1$. Lingwood[25] defines a Rossby number, $\text{Ro}=(\Omega_F^*-\Omega_D^*)/\Omega^*$, where Ω_F^* and Ω_D^* are, respectively, the angular velocities of the fluid and the disk. The value Ω^* represents an appropriate system rotation rate that is given for the Kármán layer by $\Omega^*=\Omega_D^*$. Thus, Kármán flow, for which $\Omega_F^*=0$, is associated with a Rossby number of $\text{Ro}=(0-1)/1=-1$. For our experiments $G(\zeta)\equiv\Omega_F^*$ and, hence, $\text{Ro}=(0.1-1.0)/1.0=-0.9$. Lingwood[25] displays the neutral-stability curves for the Type-I mode in her Fig. 5(a) for values of Ro=−1 and Ro=−0.5. Additionally she lists the associated values for the critical Reynolds number at the nose of the marginal curves in her Table 2. Linear interpolation between the data for Ro=−1, Re=290.1 and Ro=−0.5, Re=160.9 in Table 2 and/or Fig. 5(a) of Lingwood[25] gives a drop in critical Re for Type I from 290.1 to approximately 264 and for Type II [identified as the second local maximum of the neutral curve in Fig. 5(a) of Lingwood[25]] from approximately 460 to approximately 420. A somewhat more accurate estimate for the absolute instability of Lingwood[25] can be obtained by interpolating between the given data for Ro=−1.0, Re=507.3 and Ro=−0.8, Re=434.8 in Table 3 of Lingwood.[25] This reveals that for our experiments, with Ro=−0.9, the critical Reynolds number drops from 507.3 to approximately 471.1.

Finally, Figs. 2(a) and 2(b) reveal that there is a slight undulatory variation of the radial velocity $F(\zeta)$ and the azimuthal velocity $G(\zeta)$ at $\zeta\approx 3$ and lower values of Re. This variation is also indicative of residual rotational motion. It is consistent with the theory for Kármán flows with finite azi-

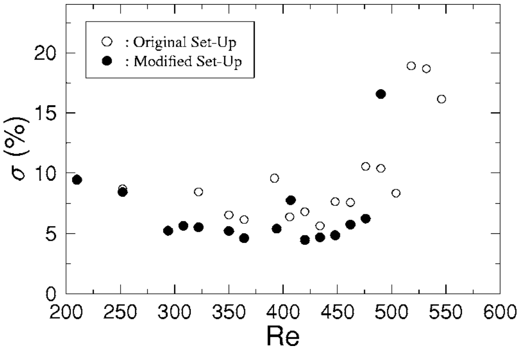

FIG. 3. Turbulence intensity σ at height $\zeta=1.29$ above the disk as a function of the radial position, r, which is expressed in terms of the Reynolds number, Re. Comparison of turbulence intensities for a rigid disk in the original and the new experimental facility.

muthal velocity as $\zeta \to \infty$ (see Ref. 25). Such flows are characterized by the type of undulatory variation measured in our experiments.

B. Turbulence intensity

The root-mean-square turbulence level σ is quantified as a percentage of the mean flow velocity measured at any particular location. Figure 3 compares the turbulence intensities in the original facility with those of the new facility. In Fig. 3 the radial position is expressed in terms of the Reynolds number Re defined in Eq. (2). All data points were collected in a height $\zeta=1.29$ above the disk surface.

Figure 3 shows that the data for the turbulence intensities in the new facility are consistently below the corresponding values of the original apparatus. The values for the new facility are typically 2–5 percentage points lower than those from the original setup. Typical errors for the turbulence intensity, σ, in the original setup were found to be approximately 0.02σ at $\mathrm{Re}\approx 250$ and they increase, approximately linearly, to just under 0.08σ at $\mathrm{Re}\approx 500$. For the modified setup the corresponding errors were approximately 0.02σ at $\mathrm{Re}\approx 250$ with a roughly linear increase to just under 0.05σ at $\mathrm{Re}\approx 500$. This means that typical error bars for the σ values, at lower Reynolds numbers, are smaller than the actual diameter of the circular markers identifying the data points. In the worst case—i.e., for high Reynolds numbers in the original facility—the overall extent of the error bar is no larger than approximately twice the marker diameter. Thus, the reductions in the turbulence intensities between the original and the modified facility in Fig. 3 lie substantially outside the error bounds and they are, thus, significant.

In both facilities the turbulence intensities, σ, are observed to increase in a qualitatively similar way once the hot-film probe enters the transition zone at approximately $\mathrm{Re}>425$. However, it should be noted that these turbulence-intensity measurements were made some time after the original measurements[29] displayed in Figs. 4 and 5. Therefore we cannot guarantee that the adjustments made to the apparatus, in particular the exact location of the shroud, were precisely identical for the two sets of experiments. Accordingly, the levels of turbulence intensity may have been slightly different in the two cases.

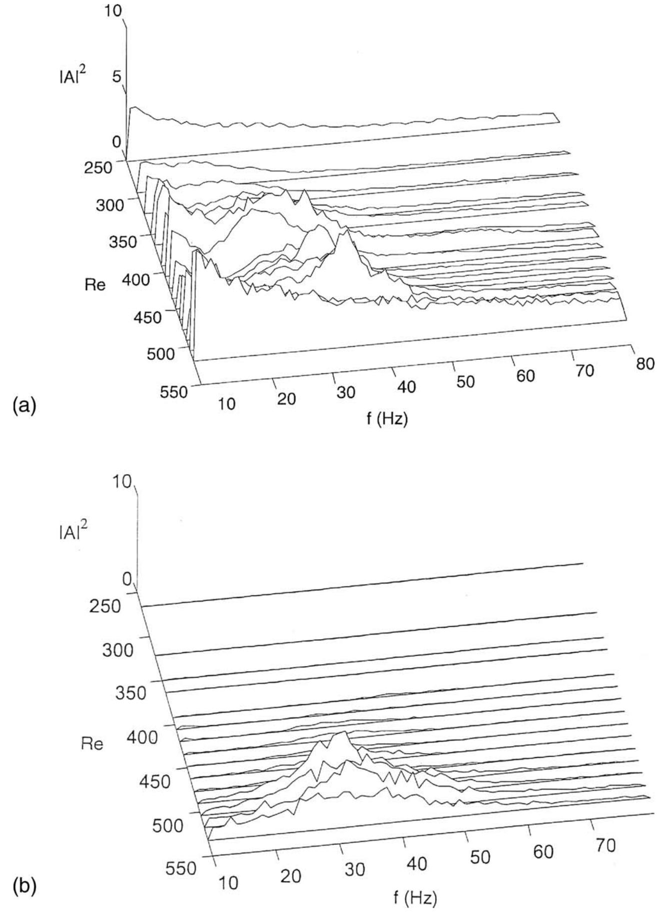

FIG. 4. Three-dimensional view of the evolution of the Fourier energy spectra with the radial position, r, expressed in terms of the Reynolds number, Re. The data were collected in height $\zeta=1.29$ above the disk. Comparison for flow over a rigid disk rotating at $f_D=1.25$ rev s^{-1} in (a) the original facility and (b) the new facility.

In summary, the reduction of the turbulence levels expressed by Fig. 3, together with the data for the mean flow velocity discussed in Sec. III A confirm that the modifications made to our rotating-disk facility have substantially improved our experimental setup without measurably affecting the base flow.

C. Fourier spectra over rigid disk

Following the same methods as Colley *et al.*[2] successive Fourier spectra for a range of radial positions were determined for the flow over the rigid disk in the new facility. The measurements were taken at a disk speed of $\Omega = 7.85$ rad s^{-1} and at height $\zeta=1.29$ above the disk.

For each radial position 60 separate time series of length 1.0 s were recorded with a resolution of 1024 data points per series. The typical spin-up time scale required for the water in the tank to adopt quasistationary conditions is no longer than a few minutes. The time delay between the disk spin-up and the measurement of the first time series was always sufficiently large to ensure that the data were not biased by transient spin-up effects. For each individual time series the Fourier spectrum was calculated. The resulting 60 spectra were ensemble averaged to yield one average spectrum for

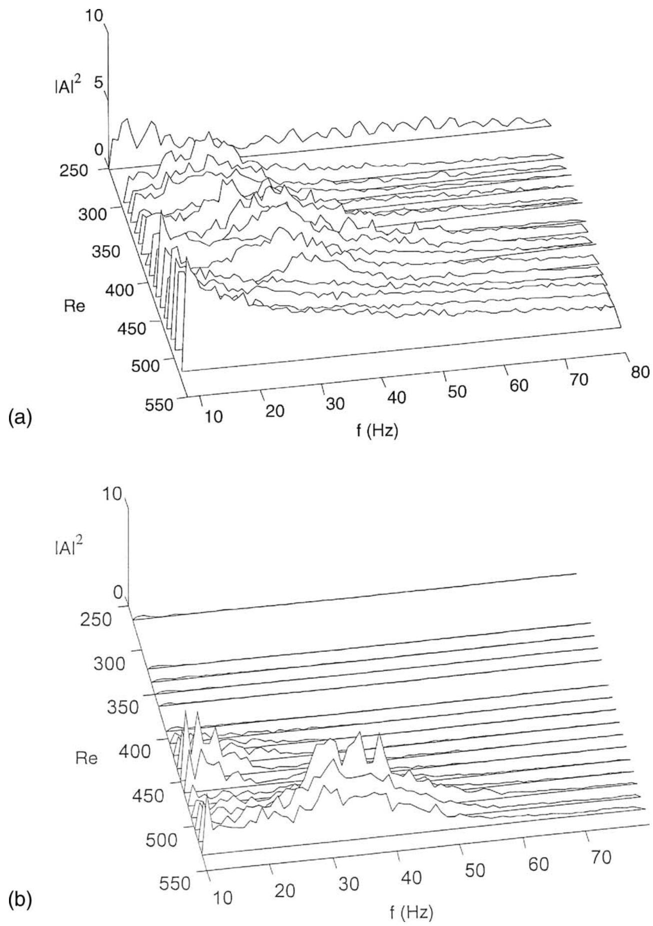

FIG. 5. Three-dimensional view of the evolution of the Fourier energy spectra with the radial position, r, expressed in terms of the Reynolds number, Re. The data were collected at height $\zeta=1.29$ above the disk. Comparison for flow over a compliant disk rotating at $f_D=1.25$ rev s^{-1} in (a) the original facility and (b) the new facility.

the particular measuring location. The ensemble-averaged spectra are summarized and compared to the corresponding spectra measured with the original facility in Figs. 4(a) and 4(b). The radial position is again expressed in terms of the Reynolds number Re.

The new rigid-disk data from the new facility [Fig. 4(b)] reveal the existence of a distinct, single peak developing for Reynolds numbers above approximately Re=450 and being centered around disturbance frequencies of $30 \leq f \leq 40$ Hz. Note that the final spectrum in Fig. 4(b) for a Reynolds number of Re≈533 is broadband which is indicative of fully turbulent flow.

As previously discussed in Colley *et al.*[2] the single peak in Fig. 4(b) represents experimental evidence of the inviscid Type-I cross-flow-vortex instability. As the frequency peak in Fig. 4(b) is centered around frequencies $30 \leq f \leq 40$ Hz, and because the disk is rotating with a frequency of $f_D = 1.25$ Hz, the data suggest that the number of spiral vortices, $n=f/f_D$, in the boundary layer is approximately in the range $24 \leq f/f_D \leq 32$. Thus, for Re=450 the azimuthal wave numbers, $\beta=n/\mathrm{Re}$, of the disturbances are in the range $0.053 \leq \beta \leq 0.071$. This is in excellent agreement with the computational and experimental results of Malik *et al.*[34] and Wilkinson and Malik.[31] The computations summarized in Ref. 34 showed that β is almost constant with a value of $\beta \simeq 0.0698$.

A Type-I peak corresponding to that in Fig. 4(b) can be identified in Fig. 4(a) for the rigid-disk data collected in the original facility. However, the data from the original facility display a substantial amount of noise surrounding the Type-I peak. Hence, the comparison of Fig. 4(a) to Fig. 4(b) clearly demonstrates that one obtains substantially improved Fourier spectra from measurements in the new facility.

In summary, the modifications made to our facility have essentially eliminated any discernible trace of disturbance to the free surface. The facility appears to be much superior to the original setup as it provides a substantially cleaner measurement environment.

D. Effect of compliance on the Type-II instability mode

The compliant disk surfaces of Colley *et al.*[2] had a low effective compliance. Nevertheless the detailed data analysis in Ref. 2, following the procedures of Jarre *et al.*,[27,28] did reveal that even such low levels of compliance had a stabilizing effect on the Type-I cross-flow-vortex eigenmode. As discussed in Sec. I the overall effect of compliance was, however, found to advance transition. Thus, it was speculated that earlier transition may have been caused by the Type-II mode which, according to the theory of Cooper and Carpenter,[1] can be destabilized substantially by relatively low levels of wall compliance. However, the ensemble-averaged Fourier spectra that were collected for the relatively stiff compliant surfaces in the original, comparatively noisy facility were not adequate for revealing evidence of the existence of Type-II-eigenmode destabilization. We have now carried out new experiments with a softer disk (E=80 kPa) which is expected to promote the destabilization of the Type-II eigenmode.

Corresponding to Sec. III C, Figs. 5(a) and 5(b) display summaries of ensemble-averaged Fourier spectra for the original and the new facility, this time for flow over the compliant disk ($\Omega=7.85$ rad s^{-1}, $\zeta=1.29$). As with Fig. 4 in Sec. III C one can see that the spectra for the new facility shown in Fig. 5(b) contain substantially less background noise than the spectra for the original facility in Fig. 5(a).

Most importantly, however, the summary of Fourier spectra for the compliant-disk data from the new facility in Fig. 5(b) reveal the existence of *two* distinct, well-defined peaks. The first being the Type-I peak as discussed earlier in Sec. III B. This peak was, of course, also observed over the rigid disk. The second peak develops for Reynolds numbers above approximately $420 \leq \mathrm{Re} \leq 480$ and it is centered around disturbance frequencies of $14 \leq f \leq 20$ Hz. This second peak was not observable in the compliant-disk spectra from the original facility displayed in Fig. 5(a), nor is it present over the rigid disk. If the frequency components associated with the second peak were present in the flow in the original facility then they were masked completely by background noise as is suggested by comparing Fig. 5(a) to Fig. 5(b).

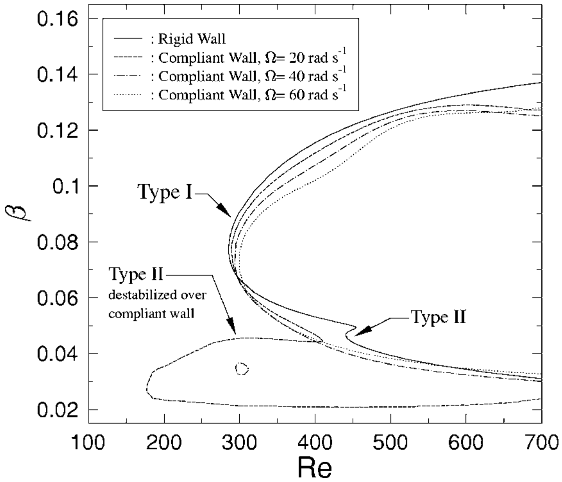

FIG. 6. Neutral curves for stationary disturbances over rigid and compliant rotating disks—based on Fig. 2(a) of Cooper and Carpenter (Ref. 1). The variation of $\beta=n/\mathrm{Re}$ with Reynolds number, Re, according to the theory of Cooper and Carpenter (Ref. 1) is shown. Note that the number of azimuthal disturbances, $n=\beta\mathrm{Re}$, should strictly be an integer.

A comparison of Fig. 5(b) to Fig. 4(b) from Sec. III C reveals that our new experimental results collected with the new facility are sufficient to qualitatively corroborate the destabilization of the Type-II eigenmode predicted by Cooper and Carpenter.[1] To this end consider the theoretical results displayed in Fig. 6.

Figure 6 demonstrates the influence of a successively increasing rotational velocity Ω on the theoretically predicted neutral-stability curve for the Type-I and the Type-II instability modes of Cooper and Carpenter.[1] An increasing rotational disk speed Ω can be interpreted as an increased effective wall compliance.[1,2] Hence, Fig. 6 shows that small changes of wall-compliance can result in a modest stabilization of the Type-I mode in the wave-number range $0.05 \leq \beta \leq 0.128$. This is the wave-number range measured and discussed in Sec. III B and the existence of the Type-I stabilization was already verified experimentally in Colley *et al.*[2] Figure 6 further reveals that corresponding small levels of the effective wall-compliance can result in a substantial destabilization of the Type-II mode in the wave-number interval $0.02 \leq \beta \leq 0.048$. Figure 6 shows, for instance, that the critical Reynolds number for the Type-II mode is approximately Re=450 for the rigid disk whereas it is reduced to approximately Re=180 for the compliant disk with the lowest rotational velocity of $\Omega=20$ rad s^{-1} corresponding to the lowest effective level of compliance. A rise in rotational speed, i.e., a rise in effective compliance, results in a shrinking of the unstable region. A relatively low level of wall compliance is destabilizing, but higher levels of compliance are stabilizing.

The disk in the present experiments rotates at $f_D = 1.25$ Hz. The frequencies of the second peak in Fig. 5(b) are in the range $14 \leq f \leq 20$ Hz. Hence, one finds that disturbance numbers $n=f/f_D$ lie in the interval $11.2 \leq n \leq 16$ Hz. Consequently, for Reynolds numbers of $420 \leq \mathrm{Re} \leq 480$ the wave number, $\beta=n/\mathrm{Re}$, is within the range $0.023 \leq \beta \leq 0.038$. A comparison with the computational results of Fig. 6 shows that this is precisely the region of values of β where destabilization of the Type-II mode is predicted by the computations of Cooper and Carpenter.[1]

Note, however, that one cannot expect to find an agreement between the Reynolds numbers associated with the experimentally observed values of $0.023 \leq \beta \leq 0.038$ for the Type-II peak in Fig. 5(b) and the corresponding Reynolds numbers shown for values of β in this range by the computational curves in Fig. 6. In Fig. 5(b) the Type-II peak is centered approximately at $420 \leq \mathrm{Re} \leq 480$ whereas in Fig. 6 the computed curve for $\Omega=20$ rad s^{-1} shows that waves with $\beta \approx 0.03$ become unstable above approximately Re=180. There are two reasons for this apparent discrepancy.

First, the theoretical results of Cooper and Carpenter[1] (see Fig. 6) show that the value of the critical Reynolds number for the Type II instability is very sensitive to the precise level of compliance and that the manner in which it changes with the compliance level is very subtle. In comparison with the rigid disk, the critical Reynolds number for Type II initially falls when the disk's level of compliance is low to moderate. However, when the disk becomes more compliant then the critical Reynolds number rises again. Further complexity arises from the fact that the effective level of compliance varies with the value of the linear speed of the fluid above the compliant material and, therefore, with the radial location on the rotating disk.

Thus, it is actually the *lowest* level of compliance, corresponding to $\Omega=20$ rad s^{-1} in Fig. 6, that has by far the greatest destabilizing effect. For the disk with $\Omega = 40$ rad s^{-1}, i.e., for a disk that has a higher level of compliance, the Type II eigenmode has almost disappeared—except for a small, isolated region of instability centered around, approximately, Re=305 and $\beta=0.034$. Finally, at the highest level of compliance, corresponding to $\Omega=60$ rad s^{-1} in Fig. 6, all signs of the Type II eigenmode have entirely disappeared. Owing to the complexity of the problem it is, in summary, not possible to quantitatively relate the theoretical results shown in Fig. 6 to the experiment in any simple way. Consequently it is not possible to make quantitative comparisons between theory and experiment, except for the critical wave number which does not appear to vary greatly with the level of compliance. All that we can conclude from the theory, as regards the critical Reynolds number for the Type II instability, is that it will be markedly lower in our experiments with the compliant disk then for the rigid disk. It is not yet possible to make quantitative comparisons between the expected and the measured decrease of critical Reynolds numbers for any particular value of the elastic modulus. Note that these issues were also addressed in Colley *et al.*[2]

Second, the Reynolds-number range, around which the Type-II peak in Fig. 6 is centered, has little to do with the measured *onset* of wave amplification. For instance, a critical value of Re=180—as for the computational curve for $\Omega = 20$ rad s^{-1} at $\beta=0.03$ in Fig. 6—identifies a certain radial distance away from the center of the spinning disk. This Reynolds-number value, or equivalently this radial position, only represents that particular location from which flow conditions are such that the small disturbance with $\beta=0.03$ can begin to grow. The growing disturbance is then convected radially outwards where it can only be detected by measurement equipment once its amplitude has grown to sufficiently large values. A peak will appear in the spectrum for a Reynolds number where the amplitude of the wave reaches its maximum value. This Reynolds number is necessarily higher than the critical value one would determine from a curve in Fig. 6. After reaching a maximum value the amplitude of that particular mode can decrease again even though the flow becomes fully turbulent.

The theoretical results of Cooper and Carpenter[1] show that for a fixed elastic modulus destabilization of the Type-II eigenmode is only observed for relatively low rotational speeds. There is evidently an optimum rotational speed for destabilization, although its precise value was not determined in Ref. 1. As rotational speeds rise above this optimum value the region of instability shrinks and ultimately the Type-II is completely eliminated. When the elastic modulus is fixed, the rotational speed can be considered a measure of wall compliance, the higher the rotational speed, the higher the degree of compliance. Strictly, though, it is the square of the linear speed $\Omega^2 r^2$ that is proportional to wall compliance. Thus for a fixed rotational speed and elastic modulus, the effective wall compliance varies as r^2. We can thereby see that the theory of Ref. 1 implies that there will only be a finite range of r (or, equivalently, Re) for which wall compliance will be destabilizing for the Type-II eigenmode. This is exactly what one would expect for a viscous instability mode. It is also precisely what we observe in the experiments, as illustrated in Fig. 5(b). The final, and most important result of our study can be inferred as follows. The Type-I peak is present for both the rigid disk as well as the compliant-disk data from the new facility in Figs. 4(b) and 5(b), respectively. The second peak, however, is only present for the compliant-disk data in Fig. 5(b). As all experimental parameters are the same for both summaries of spectra the appearance of the second peak in Fig. 5(b) must be attributed to wall compliance. As discussed previously the wave numbers associated with the frequencies of this second peak are precisely those where destabilization of the Type-II eigenmode is expected on the basis of the theory of Cooper and Carpenter.[1] Finally the Type-II eigenmode is only seen for a finite range of Re, as would be expected for a viscous instability and as found by the theory of Ref. 1. Thus, it can be concluded that the presence of the second peak in Fig. 5(b) represents an experimental verification of Type-II-eigenmode destabilization by relatively low levels of wall compliance. This is the main result of the present article.

IV. SUMMARY AND CONCLUSION

The goal of this study was to obtain the first experimental verification of the existence of Type-II-eigenmode destabilization by wall compliance in boundary-layer flow over a rotating disk that was predicted theoretically by Cooper and Carpenter.[1] To this end we have designed a new type of experimental rotating-disk facility which is far superior to the configuration used by Colley *et al.*[2] or Jarre *et al.*[26–28] in the past. Our measurements have shown that the mean, base flow is—within measurement accuracy—the same in both types of facilities. However, the noise characteristics are greatly improved in the new facility and turbulence intensities are substantially reduced. The new facility greatly improves the accessibility of the measurement environment by hot-film probes and this greatly simplifies experimental work. The experimental results obtained with the new facility clearly show a destabilization of the Type II eigenmode for a range of relatively low Reynolds numbers. This is qualitatively consistent with the theoretical predictions of Cooper and Carpenter.[1] A more detailed quantitative comparison with their theory is not possible, except for the value of the critical wave number which, according to the theory, does not vary greatly with the level of compliance. In this case there is reasonably good agreement between theory and experiment.

We suggest that future experimental facilities intended for the study of Kármán-type flow over a disk rotating underneath a liquid should employ the type of facility introduced by us here—rather than the type of facility used by Colley *et al.*[2] and Jarre *et al.*[26–28] However, it would be advisable to increase the size of the water tank (see Fig. 1) which houses the rotating disk. It appears that the tank of our facility may not be big enough to ensure that a pure Kármán-type flow is, in fact, established above the rotating disk. Our measurements of the mean, base flow summarized in Sec. III A have shown that there is some residual rotational motion above the disk. The discussion has shown that the velocity measurements yield results which bear a closer resemblance to Batchelor-type profiles[2,16,35,36] rather than to the expected Kármán-type profiles.[2,15,16] We believe that this result may emerge as a consequence of the finite size of the water tank in our facility. Large-scale flow recirculation driven by the rotating disk may affect the flow above the measurement environment over the disk. However, the fact that the velocity profiles measured on the new and the old setup are very similar seems to suggest that the generation of Batchelor-type flow is not strongly dependent on the size of the water tank.

In our current experimental study the transitional Reynolds number appears to be slightly higher for the compliant disk then for the rigid one. This can be seen by comparing the previous Reynolds numbers which the spectral peaks for the Type-I mode in Figs. 4(b) and 5(b) become broadband. In Fig. 4(b), for the rigid disk, this transition occurs for Reynolds numbers exceeding approximately 505 whereas the corresponding value for the compliant disk, as estimated from Fig. 5(b), is approximately 520. However, the differences probably do not exceed the experimental error in locating transition and, hence, we do not claim a definite transition delay. What can be safely claimed, however, is that in our present study wall compliance does not advance transition to lower Reynolds numbers, which was the case in the experiments described by Colley *et al.*[2] There transition over the rigid disk occurred in the Reynolds-number range between 504 and 518 whereas for the compliant disk it occurred in the range between 448 and 462.

In order to develop compliant coatings as a technology for drag reduction it is required that a coating delays transition. Although the data of our present experiments are not sufficiently supportive to make a definitive claim that a transition delay did exist we would, however, like to conclude by briefly drawing attention to the related experimental study of Cros *et al.*[33] They used the same compliant material as in the present study to investigate the influence of wall compliance on the flow in a very narrow gap (gap width: 1.5–3 mm) between a compliant rotating disk and a stationary wall (torsional Couette flow). In this narrow-gap flow configuration various other types of instabilities can be observed experimentally.[33] Cros *et al.*[33] showed that the compliant material used here and in their experiments does have a stabilizing effect on some of the instabilities associated with torsional Couette flow.

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[1] A. J. Cooper and P. W. Carpenter, "The stability of rotating-disc boundary-layer flow over a compliant wall. Part 1. Type I and Type II instabilities," J. Fluid Mech. **350**, 231 (1997).
[2] A. J. Colley, P. J. Thomas, P. W. Carpenter, and A. J. Cooper, "An experimental study of boundary-layer transition over a rotating, compliant disk," Phys. Fluids **11**, 3340 (1999).
[3] H. L. Reed and W. S. Saric, "Stability of three-dimensional boundary layers," Annu. Rev. Fluid Mech. **21**, 235 (1989).
[4] W. S. Saric, H. L. Reed, and E. B. White, "Stability and transition of three-dimensional boundary layers," Annu. Rev. Fluid Mech. **35**, 413 (2003).
[5] J. Gray, "Studies in animal locomotion, VI. The propulsive powers of the dolphin," J. Exp. Biol. **13**, 192 (1936).
[6] P. W. Carpenter, C. Davies, and A. D. Lucey, "Hydrodynamics and compliant walls: Does the dolphin have a secret?" Curr. Sci. **79**, 758 (2000).
[7] M. O. Kramer, "Boundary layer stabilization by distributed damping," J. Am. Soc. Nav. Eng. **72**, 25 (1960).
[8] T. B. Benjamin, "Effects of a flexible boundary on hydrodynamic stability," J. Fluid Mech. **9**, 513 (1960).
[9] M. T. Landahl, "On the stability of a laminar incompressible boundary layer over a flexible surface," J. Fluid Mech. **13**, 609 (1962).
[10] P. W. Carpenter and A. D. Garrad, "The hydrodynamic stability of flow over Kramer-type compliant surfaces. Part 1. Tollmien-Schlichting instabilities," J. Fluid Mech. **155**, 465 (1985).
[11] M. Gaster, "Is the dolphin a red herring?" in *Proceedings of the IUTAM Symposium on Turbulence Management and Relaminarization*, Bangalore, India, edited by H. W. Liepmann and R. Narasimha (Springer, New York, 1987), pp. 285–304.
[12] A. D. Lucey and P. W. Carpenter, "Boundary-layer instability over compliant walls: Comparison between theory and experiment," Phys. Fluids **7**, 2355 (1995).
[13] *Flow Past Highly Compliant Boundaries and in Collapsible Tubes*, edited by P. W. Carpenter and T. J. Pedley (Kluwer, Dordrecht, The Netherlands, 2003).
[14] P. W. Carpenter, P. J. Thomas, and M. Nagata, "Rotating flows over compliant walls," in *Flow Past Highly Compliant Boundaries and in Collapsible Tubes*, edited by P. W. Carpenter and T. J. Pedley (Kluwer, Dordrecht, The Netherlands, 2003), p. 167.
[15] Th. von Kármán, "Über laminare und turbulente Reibung," Z. Angew. Math. Mech. **1**, 233 (1921).
[16] J. M. Owen and R. H. Rogers, *Flow and Heat Transfer in Rotating-Disc Systems, Volume 1—Rotor-Stator Systems* (Research Studies, Taunton, Somerset, U.K., 1989).
[17] H. Schlichting and K. Gersten, *Boundary-Layer Theory*, 8th ed. (Springer, New York, 2000).
[18] M. Wimmer, "Viscous flows and instabilities near rotating bodies," Prog. Aerosp. Sci. **25**, 43 (1988).
[19] N. H. Smith, "Exploratory investigation of laminar-boundary-layer oscillations on a rotating disk," NACA Technical Note No. 1227 (1947).
[20] N. Gregory, J. T. Stuart, and W. S. Walker, "On the stability of three-dimensional boundary layers with application to the flow due to a rotating disk," Philos. Trans. R. Soc. London, Ser. A **248**, 155 (1955).
[21] A. J. Faller, "An experimental study of the instability of the laminar Ekman boundary layer," J. Fluid Mech. **15**, 560 (1963).
[22] L. M. Mack, "The wave pattern produced by a point source on a rotating disk," AIAA Paper 85-0490 (1985).
[23] R. J. Lingwood, "Absolute instability of the boundary layer on a rotating disk," J. Fluid Mech. **299**, 17 (1995).
[24] R. J. Lingwood, "An experimental study of absolute instability of the rotating-disk boundary-layer flow," J. Fluid Mech. **314**, 373 (1996).
[25] R. J. Lingwood, "An absolute instability of the Ekman layer and related rotating flows," J. Fluid Mech. **331**, 405 (1997).
[26] S. Jarre, P. Le Gal, and M. P. Chauve, "Experimental analysis of the instability of the boundary layer over a rotating disk," Europhys. Lett. **14**, 649 (1991).
[27] S. Jarre, P. Le Gal, and M. P. Chauve, "Experimental study of rotating disk instability. I. Natural flow," Phys. Fluids **8**, 496 (1996).
[28] S. Jarre, P. Le Gal, and M. P. Chauve, "Experimental study of rotating disk instability. II. Forced flow," Phys. Fluids **8**, 2985 (1996).
[29] A. J. Colley and P. W. Carpenter, "Experiments on boundary-layer transition over a rotating compliant disc," in *Book of Abstracts, 4th EUROMECH Fluid Mechanics Conference*, 19–23 November (Eindhoven University of Technology, Eindhoven, The Netherlands, 2000), p. 158.
[30] T-S. Cham and M. R. Head, "Turbulent boundary-layer flow on a rotating disk," J. Fluid Mech. **37**, 129 (1969).
[31] S. P. Wilkinson and M. R. Malik, "Stability experiments in the flow over a rotating disk," AIAA J. **23**, 588 (1985).
[32] *Principles of Tribology*, edited by J. Halling (MacMillan, London, 1975).
[33] A. Cros, R. Ali, P. Le Gal, P. J. Thomas, L. Schouveiler, P. W. Carpenter, and M. P. Chauve, "Effects of wall-compliance on the laminar-turbulent transition of torsional Couette flow," J. Fluid Mech. **481**, 177 (2003).
[34] M. R. Malik, S. P. Wilkinson, and S. A. Orszag, "Instability and transition in rotating disk flow," AIAA J. **19**, 1131 (1981).
[35] G. K. Batchelor, "Note on a class of solutions of the Navier-Stokes equations representing steady rotationally-symmetric flow," Q. J. Mech. Appl. Math. **4**, 29 (1951).
[36] M. H. Rogers and G. N. Lance, "The rotationally symmetric flow of a viscous fluid in the presence of an infinite rotating disk," J. Fluid Mech. **7**, 617 (1960).