Near wall Turbulence of fully developed Turbulent Channel flow

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Recap

- Turbulence statistics in fully developed Channel are only function of y
- Mean stream wise pressure gradient is uniform across the flow

Basic Relationships

- $$\tau(y) = \tau_w(1 - \frac{y}{\delta})$$
- $$\tau_t = \mu\frac{du}{dy} - \rho < u^{'} v^{'} >$$
- $$u_\tau = \sqrt{\frac{\tau_w}{\rho}}$$
- $$y^+ = \frac{u_\tau y}{\nu}$$
- $$\frac{d\tau_t}{dy} = \frac{dp_w}{dx}$$

Mean velocity profiles

Fully Developed Channel flow is fully described by ρ, ν, μ and pressure gradient.

$$u_\tau = (-\frac{\delta}{\rho}\frac{dp_w}{dx})^{1/2}$$

$$\frac{d\tau}{dy} = -\frac{\tau_w}{\delta}$$

$$\tau_w = -\delta\frac{dp_w}{dx}$$

$$< U >= u_\tau F_o(\frac{y}{\delta}, Re_\tau)$$

$$\frac{d < U >}{dy} = \frac{u_\tau}{y}\Phi(\frac{y}{\delta}, y^+)$$

Law of Wall

Only Valid in Viscous Sub layer $y^{+} < 5$

$$\frac{d < U >}{dy} = \frac{u_{\tau}}{y}\Phi_{I}(y^{+}) \qquad for \quad \frac{y}{\delta} << 1$$

$$u^{+} = \frac{U}{u_{\tau}}$$

$$\frac{u^{+}}{dy^{+}} = \frac{1}{y^{+}}\Phi(y^{+})$$

By Integration

$$u^{+} = f_{w}(y^{+})$$

Where

$$f_{w}(y^{+}) = \int \frac{1}{y^{'}}\Phi(y^{'})d(y^{'})$$

Law of Wall

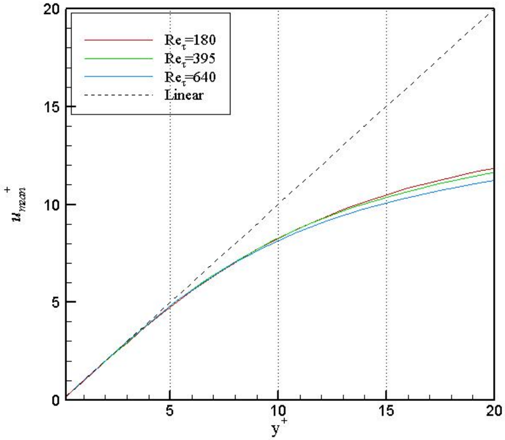


Figure: Wall law for Different Reynolds Number

Log Law

Only Valid in range $y^+ > 5, y/\delta < 0.3$

$$\Phi_l(y^+) = \frac{1}{\kappa}$$

$$\frac{du^+}{dy^+} = \frac{1}{\kappa y^+}$$

$$u^+ = \frac{1}{\kappa} lny^+ + B$$

Where

$$\kappa = 0.41, \qquad B = 5.2$$

Log Law

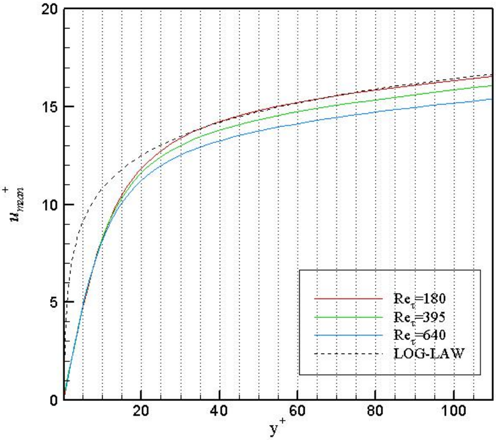


Figure: Log law for Different Reynolds Number

Log Law

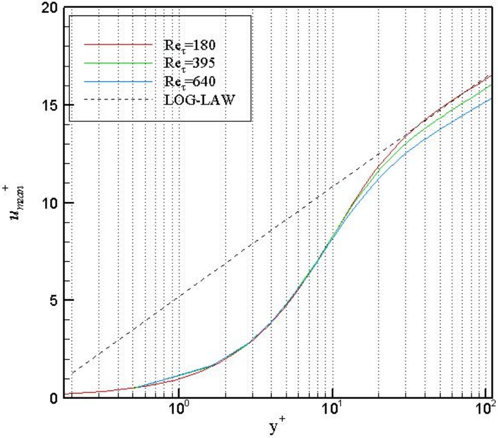


Figure: Log law for Different Reynolds Number

Wall regions and layer and their properties

Region	Location	Defining property
Inner layer	$y/\delta < 0.1$	$\langle U \rangle$ determined by u_τ and y^+, independent of U_0 and δ
Viscous wall region	$y^+ < 50$	The viscous contribution to the shear stress is significant
Viscous sublayer	$y^+ < 5$	The Reynolds shear stress is negligible compared with the viscous stress
Outer layer	$y^+ > 50$	Direct effects of viscosity on $\langle U \rangle$ are negligible
Overlap region	$y^+ > 50,\ y/\delta < 0.1$	Region of overlap between inner and outer layers (at large Reynolds numbers)
Log-law region	$y^+ > 30,\ y/\delta < 0.3$	The log-law holds
Buffer layer	$5 < y^+ < 30$	The region between the viscous sublayer and the log-law region

Reynolds Stresses

For fixed x,z,t and small y values

$$u = a_1 + b_1 y + c_1 y^2 + ...,$$
$$v = a_2 + b_2 y + c_2 y^2 + ...,$$
$$w = a_3 + b_3 y + c_3 y^2 + ...,$$

All coefficients are zero-mean random variables

At Lower Wall,

$$u = 0, \quad \Rightarrow \quad a_1 = 0$$
$$v = 0, \quad \Rightarrow \quad a_2 = 0$$
$$w = 0, \quad \Rightarrow \quad a_3 = 0$$

Reynolds Stresses

$$(\frac{\partial u}{\partial x})_{y=0} = 0$$

$$(\frac{\partial w}{\partial x})_{y=0} = 0$$

$$(\frac{\partial v}{\partial x})_{y=0} = b2 = 0$$

Reynolds stresses

$$< u^2 > = < b_1^2 > y^2 + ...,$$

$$< v^2 > = < c_c^2 > y^4 + ...,$$

$$< w^2 > = < b_3^2 > y^2 + ...,$$

$$< uv > = < b_1 c_2 > y^3 + ...,$$

Reynolds Stresses

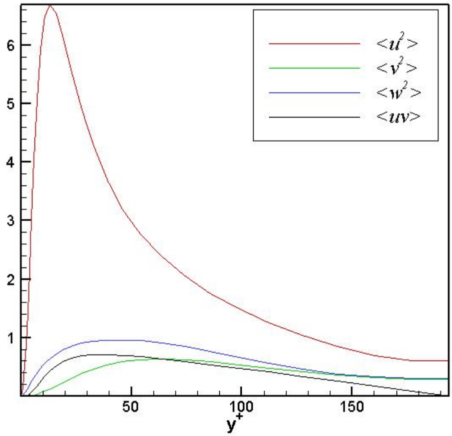
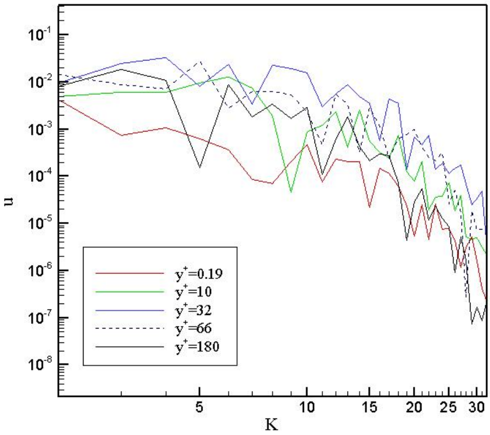
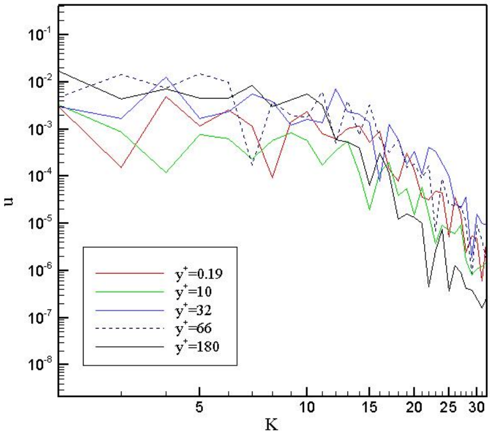


Figure: Reynolds stresses behavior near wall

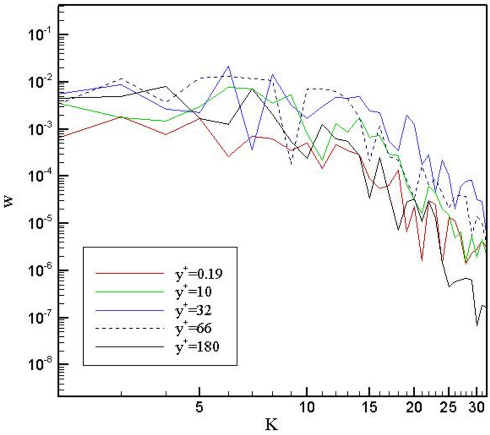
Instantaneous U



Instantaneous V



Instantaneous W



Instantaneous Pressure